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INTUITIONISTIC FUZZY $\widehat{\beta}$-GENERALIZED CLOSED SETS AND ITS APPLICATIONS

M. Umadevi[1*], K. Arun Prakash[2] and S. Vengataasalam[3]

[1]Department of Mathematics, Cheran College of Engineering, Karur – 639111, Tamil Nadu – India.

[2]Department of Mathematics, Kongu Engineering College, Perundurai - 638 052, Tamil Nadu – India.

[3]Department of Mathematics, Kongu Engineering College, Perundurai - 638 052, Tamil Nadu – India.

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ABSTRACT

The purpose of this paper is to introduce a new class of closed sets namely intuitionistic fuzzy $\widehat{\beta}$-generalized closed sets in intuitionistic fuzzy topological space. We have studied the relations between the new class of set with the other existing intuitionistic fuzzy closed sets and some of its properties were discussed. Also some applications of intuitionistic fuzzy $\widehat{\beta}$-generalized closed sets were defined and some characterizations were discussed.

Keywords and Phrases: *Intuitionistic fuzzy set, Intuitionistic fuzzy topological space, intuitionistic fuzzy $\widehat{\beta}$-generalized closed set, intuitionistic fuzzy $\widehat{\beta}$-generalized open set and intuitionistic fuzzy $\widehat{\beta}$-generalized $T_{1/2}$ space.*

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1. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh in his classical paper [12] in 1965. Using the concept of fuzzy sets, Chang [2] introduced the concept of fuzzy topological space. In [1], Atanassov introduced the notion of intuitionistic fuzzy sets in 1986. Using the notion of intuitionistic fuzzy sets, Coker [3] defined the notion of intuitionistic fuzzy topological spaces in 1997. This approach provided a wide field for investigation in the area of intuitionistic fuzzy topology and its applications. One of the directions is related to the properties of intuitionistic fuzzy sets introduced by Gurcay [4] in 1997. After this a large amount of research work had been carried out in intuitionistic fuzzy topological spaces. Many closed sets that exist in general topology have been extended to intuitionistic fuzzy topological space.

In this paper section 3 define the notion of intuitionistic fuzzy $\widehat{\beta}$-generalized closed sets in intuitionistic fuzzy topological space. We discuss the relationship between intuitionistic fuzzy $\widehat{\beta}$-generalized closed sets with the other existing intuitionistic fuzzy closed sets. Section 4 defines intuitionistic fuzzy $\widehat{\beta}$-generalized open set and some of its properties are studied. The last section deals with some applications of intuitionistic fuzzy $\widehat{\beta}$-generalized closed set and characterization theorem were studied.

2. PRELIMINARIES

Definition: 2.1 [1] An intuitionistic fuzzy set (IFS, for short) A in X is an object having the form

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x \in X\}$$

Corresponding author: M. Umadevi[1]*
[1]Department of Mathematics, Cheran College of Engineering, Karur – 639111, Tamil Nadu – India.

where the functions $\mu_A: X \to [0,1]$ and $\gamma_A: X \to [0,1]$ denote the degree of the membership (namely $\mu_A(x)$) and the degree of non- membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A respectively, $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for each $x \in X$.

Definition: 2.2 [1] Let A and B be IFS's of the forms
$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle / x \in X\}$

Then,
(a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$ for all $x \in X$,
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(c) $\overline{A} = \{\langle x, \gamma_A(x), \mu_A(x)\rangle / x \in X\}$,
(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x),\ \gamma_A(x) \vee \gamma_B(x)\rangle / x \in X\}$
(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x),\ \gamma_A(x) \wedge \gamma_B(x)\rangle / x \in X\}$
(f) $0_\sim = \{\langle x, 0, 1\rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle / x \in X\}$
(g) $\overline{\overline{A}} = A$, $\overline{1_\sim} = 0_\sim$, $\overline{0_\sim} = 1_\sim$.

Definition: 2.3 [3] An intuitionistic fuzzy topology (IFT for short) on X is a family τ of IFS's in X satisfying the following axioms:
(i) $0_\sim, 1_\sim \in \tau$,
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(iii) $\cup G_i \in \tau$ for any arbitrary family $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS for short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X. The complement $\bar{A}$ of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X.

Definition: 2.4 [3] Let (X, τ) be an IFTS and $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x \in X\}$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of A are defined by

$$int(A) = \cup \{G \mid G \text{ is an IFOS in X and } G \subseteq A\},$$
$$cl(A) = \cap \{K \mid K \text{ is an IFCS in X and } A \subseteq K\}.$$

Note that, for any IFS A in (X, τ), we have

$$cl(\bar{A}) = \overline{int(A)} \quad \text{and} \quad int(\bar{A}) = \overline{cl(A)}$$

Proposition: 2.5 [3] Let (X, τ) be an IFTS and A, B be IFSs in X. Then the following properties hold:
(a) $int(A) \subseteq A$
(b) $A \subseteq cl(A)$
(c) $A \subseteq B \Rightarrow int(A) \subseteq int(B)$
(d) $A \subseteq B \Rightarrow cl(A) \subseteq cl(B)$
(e) $int(int(A)) = int(A)$
(f) $cl(cl(A)) = cl(A)$
(g) $cl(A \cup B) = cl(A) \cup cl(B)$
(h) $int(A \cap B) = int(A) \cap int(B)$

Definition: 2.6 [1] Let $\alpha, \beta \in [0,1]$ with $\alpha+\beta \le 1$. An *intuitionistic fuzzy point* (IFP), written as $p_{(\alpha,\beta)}$, is defined to be an IFS of X given by

$$p_{(\alpha,\beta)} = \begin{cases} (\alpha,\beta), & if\ x = p \\ (0,1), & otherwise \end{cases}$$

Definition: 2.7 An IFS $A = \{< x, \mu_A(x), \gamma_A(x) > / x \in X\}$ in an IFTS (X, τ) is called an
(i) *intuitionistic fuzzy semiclosed set* (IFSCS) if $int(cl(A)) \subseteq A$ [4],
(ii) *intuitionistic fuzzy α-closed set* (IFαCS) if $cl(int(cl(A))) \subseteq A$ [4],
(iii) *intuitionistic fuzzy preclosed set* (IFPCS) if $cl(int(A)) \subseteq A$ [4],
(iv) *intuitionistic fuzzy regular closed set* (IFRCS) if $A = cl(int(A))$ [4],

(v) *intuitionistic fuzzy γ-closed set* (IFγCS in short) if cl(int(A)) ∩ int(cl(A)) ⊆A [5].
(vi) *intuitionistic fuzzy weakly generalized closed set* (IFWGCS) if cl(int(A)) ⊆ U whenever A ⊆U, U is IFOS in X [6],
(vii) *intuitionistic fuzzy w-closed set* (IFWCS) if cl(A) ⊆ O whenever A ⊆ O and O is IFSOS in X [10],
(viii) *intuitionistic fuzzy regular generalized closed set* (IFRGCS) if cl(A) ⊆ O whenever A ⊆ O and O is an IFROS in X [8],
(ix) *intuitionistic fuzzy semi-preclosed* if there exists an intuitionistic fuzzy preclosed set B such that int(B) ⊆ A⊆ B [11],
(x) *intuitionistic fuzzy g-closed*, if cl(A) ⊆ O whenever A ⊆ O and O is intuitionistic fuzzy open in X [9],
(xi) intuitionistic fuzzy λ-closed set (IF λCS) if A ⊇ cl(U) whenever A ⊇U and U is intuitionistic fuzzy open set in X [7].

The complements of above intuitionistic fuzzy closed sets are respectively their intuitionistic fuzzy open sets.

3. INTUITIONISTIC FUZZY $\widehat{\beta}$-GENERALIZED CLOSED SETS

In this section we introduce intuitionistic fuzzy $\widehat{\beta}$ - generalized closed sets and investigate some of its properties.

Definition: 3.1 An IFS A of an IFTS (X,τ) is said to be an intuitionistic fuzzy $\widehat{\beta}$-generalized closed set (IF$\widehat{\beta}$GCS) if $cl\left(int\left(cl(A)\right)\right) \subseteq U$, whenever $A \subseteq U$ and U is an IFOS in X.

Example: 3.2 Let X = {a, b} and let $A = \langle x, \left(\frac{a}{0.6}, \frac{b}{0.5}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}\right)\rangle$. Then τ = {0~, 1~, A} is an IFT on X. Let $B = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.3}\right), \left(\frac{a}{0.7}, \frac{b}{0.6}\right)\rangle$ be any IFS in X.

Now $cl\left(int\left(cl(B)\right)\right) = 0_{\sim}$ Since $cl\left(int\left(cl(B)\right)\right) \subseteq U$, whenever $B \subseteq U$, where U is an IFOS. Hence B is an IF$\widehat{\beta}$GCS in X.

Example: 3.3 Let X = {a, b} and let $A = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.5}\right)\rangle$. Then τ = {0~, 1~, A} is an IFT on X. Let $B = \langle x, \left(\frac{a}{0.1}, \frac{b}{0.4}\right), \left(\frac{a}{0.6}, \frac{b}{0.5}\right)\rangle$ be any IFS in X. Now $cl\left(int\left(cl(B)\right)\right) = \bar{A}$.

Since $B \subseteq A$ and $cl\left(int\left(cl(B)\right)\right) \nsubseteq A$, B is not an IF $\widehat{\beta}$ GCS in X.

Theorem: 3.4 Every IFCS in an IFTS (X,τ) is an IF $\widehat{\beta}$ GCS, but not conversely.

Proof: Let A be an IFCS in X such that $A \subseteq U$, where U is an IFOS in X. Since A is an IFCS $cl(A) = A,\ cl(A) \subseteq U$.

But $cl\left(int(A)\right) \subseteq cl(A), cl\left(int\left(cl(A)\right)\right) = cl\left(int(A)\right) \subseteq cl(A) \subseteq U$.

Therefore $cl\left(int\left(cl(A)\right)\right) \subseteq U$. Hence A is an IF $\widehat{\beta}$ GCS in X.

Example: 3.5 Let X= {a, b} and let $A = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}\right), \left(\frac{a}{0.6}, \frac{b}{0.5}\right)\rangle$. Then τ = {0~, 1~, A} is an IFT on X. Let $B = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.4}\right), \left(\frac{a}{0.1}, \frac{b}{0.6}\right)\rangle$ be any IFS in X. Now $cl\left(int\left(cl(B)\right)\right) = 1_{\sim}$

Since $cl\left(int\left(cl(B)\right)\right) \subseteq U$, whenever $B \subseteq U$, where U is an IFOS, B is an IF $\widehat{\beta}$ GCS in X. But B is not an IFCS in X.

Theorem: 3.6 Every IFRCS in an IFTS (X,τ) is an IF $\widehat{\beta}$GCS, but not conversely.

Proof: Let A be an IFRCS in X such that $A \subseteq U$, where U is an IFOS in X. By hypothesis $cl(int(A)) = A$, since A is an IFCS, $cl\left(int(cl(A))\right) = cl(int(A)) = A \subseteq U$. Therefore $cl\left(int(cl(A))\right) \subseteq U$. Hence A is an IF$\widehat{\beta}$GCS in X.

Example: 3.7 Let X= {a, b} and let $A = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}\right), \left(\frac{a}{0.6}, \frac{b}{0.5}\right)\rangle$.Then τ = {$0_\sim$, $1_\sim$, A} is an IFT on X. Let $B = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.4}\right), \left(\frac{a}{0.1}, \frac{b}{0.6}\right)\rangle$ be any IFS in X. Now $cl\left(int(cl(B))\right) = 1_\sim$ Since $cl\left(int(cl(B))\right) \subseteq U$, whenever $B \subseteq U$, where U is an IFOS, B is an IF $\widehat{\beta}$GCS in X. But B is not an IFRCS in X, since $cl(int(B)) = 0_\sim \neq B$.

Theorem: 3.8 Every IFαCS in an IFTS (X, τ) is an IF $\widehat{\beta}$GCS, but not conversely.

Proof: Let A be an IFαCS in X such that A⊆U, where U is an IFOS in X. By hypothesis $cl\left(int(cl(A))\right) \subseteq A$. Therefore $cl\left(int(cl(A))\right) \subseteq U$. Hence A is an IF $\widehat{\beta}$GCS in X.

Example: 3.9 Let X = {a, b} and let $A = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}\right), \left(\frac{a}{0.7}, \frac{b}{0.6}\right)\rangle$. Then τ = {$0_\sim$, $1_\sim$, A} is an IFT on X. Let $B = \langle x, \left(\frac{a}{0.4}, \frac{b}{0.3}\right), \left(\frac{a}{0.5}, \frac{b}{0.6}\right)\rangle$ be any IFS in X.Now $cl(int(cl(B))) = \bar{A} \subseteq 1_\sim$ and $B \subseteq 1_\sim$ only, hence B is an IF $\widehat{\beta}$GCS in X. Also $cl(int(cl(B))) = \bar{A} \nsubseteq B$ implies B is not an IFαCS in X.

Remark: 3.10 The concepts of IFPCS and IF $\widehat{\beta}$GCS are independent to each other as seen from the following examples.

Example: 3.11 Let X = {a, b} and let $A = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.5}\right)\rangle$.Then τ = {$0_\sim$, $1_\sim$, A} is an IFT on X. Let $B = \langle x, \left(\frac{a}{0.1}, \frac{b}{0.4}\right), \left(\frac{a}{0.6}, \frac{b}{0.5}\right)\rangle$ be any IFS in X. Now cl(int(B)) = $0_\sim$⊆B, therefore B is an IFPCS in X. But cl(int(cl(B)))=$\bar{A} \nsubseteq A$, when $B \subseteq A$. Hence B is not an IF $\widehat{\beta}$GCS in X.

Example: 3.12 Let X = {a, b} and let $A = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.3}\right), \left(\frac{a}{0.4}, \frac{b}{0.6}\right)\rangle$. Then τ = {$0_\sim$, $1_\sim$, A} is an IFT on X. Let $B = \langle x, \left(\frac{a}{0.6}, \frac{b}{0.5}\right), \left(\frac{a}{0.4}, \frac{b}{0.5}\right)\rangle$ be any IFS in X.

Now $cl\left(int(cl(B))\right) = 1\sim$. Since $B \subseteq 1_\sim$ only, B is an IF $\widehat{\beta}$GCS in X. But cl(int(B)) = $\bar{A} \nsubseteq B$. Hence B is not an IFPCS in X.

Remark: 3.13 The concepts of IFγCS and IF $\widehat{\beta}$GCS are independent to each other as seen from the following examples.

Example: 3.14 Let X = {a, b} and let $A = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.3}\right), \left(\frac{a}{0.8}, \frac{b}{0.7}\right)\rangle$.Then τ = {$0_\sim$, $1_\sim$, A} is an IFT on X. Let $B = \langle x, \left(\frac{a}{0.1}, \frac{b}{0.2}\right), \left(\frac{a}{0.9}, \frac{b}{0.8}\right)\rangle$ be any IFS in X. Now cl(int(B))=$0_\sim$ and int(cl(B))=A. Therefore cl(int(B))∩int(cl(B)) = $0_\sim$∩A=$0_\sim$⊆B. Therefore cl(int(B))∩int(cl(B))⊆B. Hence B is IF γCS in X. But cl(int(cl(B)))=$\bar{A} \nsubseteq A$, when $B \subseteq A$. Hence B is not an IF $\widehat{\beta}$GCS in X.

Example: 3.15 Let X = {a, b} and $A = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.3}\right), \left(\frac{a}{0.4}, \frac{b}{0.6}\right)\rangle$. Then τ = {0~, 1~, A} is an IFT on X. Let $B = \langle x, \left(\frac{a}{0.6}, \frac{b}{0.5}\right), \left(\frac{a}{0.4}, \frac{b}{0.5}\right)\rangle$ be any IFS in X. Now $cl\left(int\left(cl(B)\right)\right) = 1_{\sim}$.Hence B is an IF$\widehat{\beta}$GCS in X. But cl(int(B)) = $\bar{A}$and int(cl(B))=1~.
Hence cl(int(B))∩ int(cl(B))=$\bar{A}$∩1~=$\bar{A}$ $\not\subseteq$ B. Hence B is not an IFγCS in X.

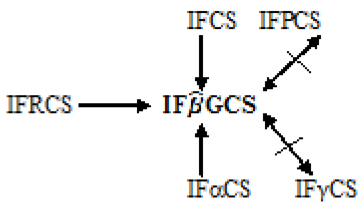
Theorem: 3.16 Union of two IF $\widehat{\beta}$ GCS is an IF $\widehat{\beta}$ GCS.

Proof: Let A and B be any two IF $\widehat{\beta}$ GCSs. Let U be an IFOS in X, such that A ∪ B ⊆ U.
Then A ⊆ U and B ⊆ U. Then by definition of IF $\widehat{\beta}$ GCS, we have $cl\left(int\left(cl(A)\right)\right) \subseteq U$ and $cl\left(int\left(cl(B)\right)\right) \subseteq U$,which implies$cl\left(int\left(cl(A \cup B)\right)\right) \subseteq U$. Hence $A \cup B$ is an IF$\widehat{\beta}$GCS in X.

Remark: 3.17 Intersection of two IF $\widehat{\beta}$ GCS in X is not an IF $\widehat{\beta}$ GCS in X in general as seen from the following example.

Example: 3.18 Let X = {a, b} and let $A = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.4}\right), \left(\frac{a}{0.5}, \frac{b}{0.6}\right)\rangle$. Then τ = {0~, 1~, A} is an IFT on X. Let $B = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.3}\right), \left(\frac{a}{0.6}, \frac{b}{0.6}\right)\rangle$ and $C = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.7}\right), \left(\frac{a}{0.7}, \frac{b}{0.3}\right)\rangle$ be any two IFSs in X. Then $B \cap C = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.3}\right), \left(\frac{a}{0.7}, \frac{b}{0.6}\right)\rangle$
and $cl(B \cap C) = \bar{A}, int\left(cl(B \cap C)\right) = A, cl\left(int\left(cl(B \cap C)\right)\right) = cl(A) = \bar{A} \not\subseteq A$, since $B \cap C \subseteq A$.
Hence $B \cap C$ is not an IF$\widehat{\beta}$GCS in X.

The following diagram shows the relationships between intuitionistic fuzzy $\widehat{\beta}$ - generalized closed set with other existing intuitionistic fuzzy closed sets.



The reverse implications in the diagram are not true in general as seen from the above illustrated examples.

Theorem: 3.19 If A is an IF $\widehat{\beta}$ GCS in an IFTS (X, τ) and B be an IFS such that A ⊆ B ⊆ cl(A), then B is also an IF$\widehat{\beta}$GCS in an IFTS (X, τ).

Proof: Let A be an IF $\widehat{\beta}$ GCS and U be an IFOS in X such that B ⊆ U. Since A ⊆ B, A ⊆ U. Then by definition of IF$\widehat{\beta}$GCS $cl\left(int\left(cl(A)\right)\right) \subseteq U$. By hypothesis
$cl(B) \subseteq cl(A)$ implies $int\left(cl(B)\right) \subseteq int\left(cl(A)\right)$, $cl\left(int\left(cl(B)\right)\right) \subseteq cl\left(int\left(cl(A)\right)\right) \subseteq U$.

Thus we have $cl\left(int\left(cl(B)\right)\right) \subseteq U$, where U is an IFOS in X. Hence B is an IF $\widehat{\beta}$ GCS in X.

Theorem: 3.20 If A is both an IFROS and an IFRCS, then A is an IF $\widehat{\beta}$ GCS in X.

Proof: Let A be an IFROS and IFRCS in X. Let A ⊆ U, where U is an IFOS in X. Since A is an IFRCS, cl(int(A)) ⊆ U.
Since A is an IFROS, cl(int(int(cl(A)))) ⊆ U which implies cl(int(cl(A))) ⊆ U. Hence A is an IF $\widehat{\beta}$ GCS in X.

Theorem: 3.21 If A is an IFROS and an IF $\hat{\beta}$ GCS, then A is an IFWCS in X.

Proof: Let A be an IFROS and an IF $\hat{\beta}$ GCS and U be an IFOS in X. By hypothesis cl(int(cl(A))) $\subseteq$ U. Since A is an IFROS in X, int(cl(A)) = A. Therefore cl(A) $\subseteq$ U. Since every IFOS is an IFSOS, U is an IFSOS and cl(A) $\subseteq$ U. Hence A is an IFWCS.

Theorem: 3.22 If A is both an IFWGCS and an IFCS, then A is an IF $\hat{\beta}$ GCS in X.

Proof: Let A be an IFWGCS in X. Then cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is an IFOS in X. Let A be an IFCS in X. Therefore cl(A) = A. Hence cl(int(cl(A))) $\subseteq$ U whenever A $\subseteq$ U and U is an IFOS in X. Hence A is an IF $\hat{\beta}$ GCS in X.

Theorem: 3.23 If A is both an IFCS and an IFSCS in an IFTS (X,τ), then A is an IFGCS in X.

Proof: Assume that A is an IFCS and an IFSCS in X. Then by definition int(cl(A)) $\subseteq$ A and since A is an IFCS, cl(A) = A. Then cl(int(cl(A))) $\subseteq$ cl(A) = A $\subseteq$ U. Thus cl(int(cl(A))) $\subseteq$ U. Hence A is an IF $\hat{\beta}$ GCS in X.

Theorem: 3.24 If A is both an IFCS and an IFGCS in an IFTS (X,τ), then A is an IF $\hat{\beta}$ GCS in X.

Proof: Let A be an IFCS and an IFGCS in X and A $\subseteq$ U, where U is an IFOS in X. By hypothesis cl(A) $\subseteq$ U. Then cl(int(A)) $\subseteq$ cl(A) $\subseteq$ U. Since A is an IFCS, cl(A) = A which implies cl(int(cl(A)) = cl(int(A)) $\subseteq$ cl(A) $\subseteq$ U. Hence A is an IF$\hat{\beta}$GCS in X.

Theorem: 3.25 If A is both an IF γ CS and an IFROS, then A is an $IF\hat{\beta}GCS$ in (X, τ).

Proof: Let A $\subseteq$ U, where U is an IFOS. Since A is an IF γ CS, cl(int(A)) $\cap$ int(cl(A)) $\subseteq$ A.

Since A is an IFROS, int(cl(A)) = A. Therefore cl(int(A)) $\subseteq$ A. Then cl(int(int(cl(A)))) $\subseteq$ A, cl(int(cl(A)))) $\subseteq$ A. Since A $\subseteq$ U and cl(int(cl(A)))) $\subseteq$ U, where U is IFOS. Hence A is $IF\hat{\beta}GCS$ in (X, τ) .

Theorem: 3.26 If A is both an IFWGCS and an IFROS then A is an $IF\hat{\beta}GCS$ in (X, τ).

Proof: Let A $\subseteq$ U where U is IFOS. Since A is an IFWGCS, cl(int(A)) $\subseteq$ U. Since A is IFROS int(cl(A)) = A, cl(int(int(cl(A))) $\subseteq$ U .Therefore cl(int(cl(A))) $\subseteq$ U, whenever A $\subseteq$ U and U is IFOS in (X, τ) .Hence A is $IF\hat{\beta}GCS$ in (X, τ) .

Theorem: 3.27 If A is both an IF λ CS and an IFGCS, then A is an $IF\hat{\beta}GCS$ in (X, τ).

Proof: Let A $\subseteq$ U, where U is an IFOS. Since A is an IFGCS, cl(A) $\subseteq$ U, int(cl(A)) $\subseteq$ cl(A) $\subseteq$ U. Since A is IF λ CS, cl(int(cl(A)) $\subseteq$ cl(U) $\subseteq$ A. Since A $\subseteq$ U, cl(int(cl(A))) $\subseteq$ A $\subseteq$ U. Hence cl(int(cl(A))) $\subseteq$ U, where U is IFOS in X. Hence A is $IF\hat{\beta}GCS$ in (X, τ).

Theorem: 3.28 If A is both an IFROS and an IFRGCS, then A is an $IF\hat{\beta}GCS$ in (X, τ).

Proof: Let A $\subseteq$ U, where U is an IFOS. Since A is an IFROS, int(cl(A)) = A. By hypothesis cl(int(cl(A))) = cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is IFROS in X. Since every IFROS is an IFOS, cl(int(cl(A))) $\subseteq$ U, where U is IFOS in X. Hence A is $IF\hat{\beta}GCS$ in (X, τ).

Theorem: 3.29 If A is both an IFSCS and an IFGCS then A is an $IF\hat{\beta}GCS$ in (X, τ).

Proof: Let A $\subseteq$ U where U is IFOS. Since A is IFSCS, int(cl(A)) $\subseteq$ A. Since A is an IFGCS, cl(A) $\subseteq$ U which in turn implies cl(int(cl(A))) $\subseteq$ cl(A) $\subseteq$ U. Hence A is $IF\hat{\beta}GCS$ in (X, τ).

Theorem: 3.30 If A is both IF γ CS and IFGCS then A is $IF\hat{\beta}GCS$ in (X, τ).

Proof: Let A $\subseteq$ U where U is IFOS.Since A is an IF γ CS, cl(int(A)) $\cap$ int(cl(A)) $\subseteq$ A, int(cl(A)) $\subseteq$ A, cl(int(cl(A))) $\subseteq$ cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFOS in X, as A is IFGCS. Hence A is $IF\hat{\beta}GCS$ in (X, τ).

Theorem: 3.31 If A is both an IFPOS and an $IF\hat{\beta}GCS$ then A is an IFGCS in (X, τ).

Proof: Since A is an IFPOS, A $\subseteq$ int(cl(A)), cl(A) $\subseteq$ cl(int(cl(A))). Since A is an $IF\hat{\beta}GCS$, cl(A) $\subseteq$ cl(int(cl(A))) $\subseteq$ U. Thus cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is an IFOS. Hence A is IFGCS in X.

Theorem: 3.32 If A is both an IF λ CS and an $IF\hat{\beta}GCS$ then A is an IF α CS in (X, τ).

Proof: Let U be an IFOS in X. Since A is an $IF\hat{\beta}GCS$, cl(int(cl(A))) $\subseteq$ U, cl(cl(int(cl(A)))) $\subseteq$ cl(U), cl(int(cl(A)))) $\subseteq$ cl(U). Since A is an IF λ CS cl(int(cl(A)))) $\subseteq$ cl(U) $\subseteq$ A. Thus cl(Int(cl(A))) $\subseteq$ A. Hence A is IF α CS in X.

Theorem: 3.33 If A is both an IFSPOS and an $IF\hat{\beta}GCS$ then A is an IFGCS in (X, τ).

Proof: Let U be an IFOS in X. Since A is an IFSPOS in X, A $\subseteq$ cl(int(cl(A))), cl(A) $\subseteq$ cl(cl(int(cl(A))). Since A is an $IF\hat{\beta}GCS$ in X, cl(A) $\subseteq$ cl(int(cl(A))) $\subseteq$ U. Thus cl(A) $\subseteq$ U, whenever U is an IFOS in X. Hence A is an IFGCS in X.

Theorem: 3.34 If A is both an IFROS and an $IF\hat{\beta}GCS$ then A is an IFGCS in (X, τ).

Proof: Let A be an IFROS and an $IF\hat{\beta}GCS$ in X and U be an IFROS in X. Since A is an $IF\hat{\beta}GCS$ in X, cl(int(cl(A))) $\subseteq$ U. Since A is an IFROS, cl(A) $\subseteq$ U where U is IFROS in X. Hence A is an IFRGCS in X.

4. INTUITIONISTIC FUZZY $\hat{\beta}$ GENERALIZED OPEN SETS

In this section, we introduce intuitionistic fuzzy $\hat{\beta}$ - generalized open sets and studied some of its properties.

Definition: 4.1 An IFS A is said to be an intuitionistic fuzzy $\hat{\beta}$-generalized open set (IF $\hat{\beta}$ GOS) in (X, τ) if the complement A^c is an IF $\hat{\beta}$ GCS in X.

The family of all IF $\hat{\beta}$ GOSs of an IFTS (X, τ) is denoted by IF $\hat{\beta}$ GO(X).

Example: 4.2 Let X = {a, b} and $A = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.3}\right), \left(\frac{a}{0.4}, \frac{b}{0.6}\right)\rangle$. Then τ = {0~, 1~, A} is an IFT on X. Let $B = \langle x, \left(\frac{a}{0.4}, \frac{b}{0.5}\right), \left(\frac{a}{0.3}, \frac{b}{0.2}\right)\rangle$ be any IFS in X. Now $cl\left(int\left(cl(B^c)\right)\right) = A^c \subseteq U$. Hence B^c is an IF $\hat{\beta}$ GCS in X. Clearly B is an IF $\hat{\beta}$ GOS in X.

Theorem: 4.3 An IFS A of an IFTS (X,τ) is an IF $\hat{\beta}$ GOS if and only if U $\subseteq$ int(cl(int(A))), whenever U $\subseteq$ A and U is an IFCS in X.

Proof: Necessity: Suppose that A is an IF $\hat{\beta}$ GOS in X. Let U be an IFCS in X such that U $\subseteq$ A. Then U^c is an IFOS in X such that $A^c \subseteq U^c$. Since A^c is an IF $\hat{\beta}$ GCS, cl(int(cl(A^c))) $\subseteq$ U^c, $(int(cl(int(A)))^c \subseteq U^c$. This implies U $\subseteq$ int(cl(int(A))).

Sufficiency: Let A be an IFS of X and let U ⊆ int(cl(int(A))), whenever U ⊆ A, U is an IFCS in X. Then $A^c \subseteq U^c$ and by hypothesis, $(int(cl(int(A)))^c \subseteq U^c$. Hence $cl(int(cl(A^c))) \subseteq U^c$. Thus A^c is an IF $\hat{\beta}$ GCS in X which implies A is an IF $\hat{\beta}$ GOS in X.

Theorem: 4.4 Let (X,τ) be an IFTS. If A is an IF $\hat{\beta}$ GOS and B be an IFS in X and int(A) ⊆ B ⊆ A, then B is also an IF $\hat{\beta}$ GOS in X.

Proof: Let A be an IF $\hat{\beta}$ GOS in X and B be an IFS in X such that int(A) ⊆ B ⊆ A. Then A^c is an IF $\hat{\beta}$ GCS and $A^c \subseteq B^c \subseteq (int(A))^c = cl(A^c)$. By theorem 3.19, B^c is an IF $\hat{\beta}$ GCS in X and hence B is an IF $\hat{\beta}$ GOS in X.

Theorem: 4.5 An IFS A of an IFTS (X,τ) is an IF $\hat{\beta}$ GOS if and only if F ⊆ int(A), whenever F is an IFCS and F ⊆ A.

Proof: Necessity: Assume that A is an IF $\hat{\beta}$ GOS in X. Let F be an IFCS such that F ⊆ A. Then F^c is an IFOS and $A^c \subseteq F^c$. By hypothesis A^c is an IF $\hat{\beta}$ GCS, $cl(int(cl(A^c))) \subseteq F^c$, $cl(A^c) \subseteq int(cl(A^c)) \subseteq cl(int(cl(A^c))) \subseteq F^c$. Therefore $cl(A^c) \subseteq F^c$, $(int(A))^c \subseteq F^c$, F ⊆ int(A).

Sufficiency: Assume that F is an IFCS and F ⊆ A. Then by hypothesis F ⊆ int(A). Now F ⊆ int(A) ⊆ int(cl(A)), F ⊆ int(A) ⊆ int(cl(A)) ⊆ int(cl(int(A))). Thus F ⊆ int(cl(int(A))). Hence A is an IF $\hat{\beta}$ GOS in X.

5. APPLICATIONS OF INTUITIONISTIC FUZZY $\hat{\beta}$ - GENERALIZED CLOSED SET

In this section, we introduce $IF_{\hat{\beta}g}T_{1/2}$ space which utilize IF $\hat{\beta}$ GCS and its characterizations are proved.

Definition: 5.1 An IFTS (X,τ) is called an intuitionistic fuzzy $\hat{\beta}$- generalized T $_{1/2}$ ($IF_{\hat{\beta}g}T_{1/2}$) space if every IF $\hat{\beta}$ GCS is an IFCS in X.

Theorem: 5.2 An IFTS (X,τ) is an $IF_{\hat{\beta}g}T_{1/2}$ space if and only if IF $\hat{\beta}$ G O(X) = IFO(X).

Proof: Necessity: Let A be an IF $\hat{\beta}$ GOS in X, then A^c is an IF $\hat{\beta}$ GCS in X. By hypothesis A^c is an IFCS in X. Hence A is an IFOS in X. Thus IF $\hat{\beta}$ GO(X) = IFO(X).

Sufficiency: Let A be an IF $\hat{\beta}$ GCS in X. Then A^c is an IF $\hat{\beta}$ GOS in X. By hypothesis A^c is an IFOS in X. Therefore A is an IFCS in X. Hence (X, τ) is an $IF_{\hat{\beta}g}T_{1/2}$ space.

Definition: 5.3 An IFTS (X,τ) is called an intuitionistic fuzzy pre - $\hat{\beta}$ generalized $T_{1/2}$ ($IF_{p\hat{\beta}g}T_{1/2}$) space if every IF $\hat{\beta}$ GCS is an IFPCS in X.

Definition: 5.4 An IFTS (X,τ) is called an intuitionistic fuzzy $\alpha\hat{\beta}$ generalized $T_{1/2}$ ($IF_{\alpha\hat{\beta}g}T_{1/2}$) space if every IF $\hat{\beta}$ GCS is an IFαCS in X.

Theorem: 5.5 Every $IF_{\hat{\beta}g}T_{1/2}$ space is an $IF_{\alpha\hat{\beta}g}T_{1/2}$ space but not conversely.

Proof: Let (X,τ) be an $IF_{\alpha\hat{\beta}g}T_{1/2}$ space and let A be an IF $\hat{\beta}$ GCS in X. By hypothesis A is an IFCS in X. Since every IFCS is an IFαCS, A is an IFαCS in X. Hence (X, τ) is an $IF_{\alpha\hat{\beta}g}T_{1/2}$ space.

Theorem: 5.6 Let (X,τ) be an IFTS and X is an $IF_{\hat{\beta}g}T_{1/2}$ space. Then A is an IFSOS in X

Proof: Let A be an IF $\hat{\beta}$ GOS in X. Since X is an $IF_{\hat{\beta}g}T_{1/2}$ space, A is an IFOS in X. Since every IFOS is an IFSOS, A is an IFSOS in X.

Theorem: 5.7 Let (X,τ) be an IFTS and X is an $IF_{\hat{\beta}g}T_{1/2}$ space .Then

(i) Any union of IF $\hat{\beta}$ GCS is an IF $\hat{\beta}$ GCS

(ii) Any intersection of IF $\hat{\beta}$ GOS is an IF $\hat{\beta}$ GOS.

Proof: (i) Let$\{A_i\}_{i\in J}$ be a collection of IF $\hat{\beta}$ GCS .Since(X, τ) is on $IF_{\hat{\beta}g}T_{1/2}$ space, every IF $\hat{\beta}$ GCS is an IFCS. But union of IFCS is on IFCS. Hence the union of IF $\hat{\beta}$ GCS is an IF $\hat{\beta}$ GCS in X.

(ii) It can be proved by taking complement in (i).

Theorem: 5.8 Let (X,τ) be an $IF_{\hat{\beta}g}T_{1/2}$ space. Then A is an $IF\hat{\beta}GOS$ in X if and only if for every IFP $p_{(\alpha,\beta)} \in A,$ there exists an $IF\hat{\beta}GOS$ in X such that $p_{(\alpha,\beta)} \in B \subseteq A.$

Proof: Necessity: If A is an $IF\hat{\beta}GOS$ in X, then we can take B = A, so that $p_{(\alpha,\beta)} \in B \subseteq A$ for every IFP $p_{(\alpha,\beta)} \in A.$

Sufficiency: Let A be an IFS in (X,τ) and assume that there exists an $IF\hat{\beta}GOS$ B in X such that $p_{(\alpha,\beta)} \in B \subseteq A.$ Since X is an $IF_{\hat{\beta}g}T_{1/2}$ space, B is an IFOS in X. Then $A = \cup_{p(\alpha,\beta)\in A}\ p_{(\alpha,\beta)} \subseteq \cup_{p(\alpha,\beta)\in A}\ B \subseteq A.$ Therefore $A = \cup_{p(\alpha,\beta)\in A}\ B$ which is an IFOS. Thus A is an $IF\hat{\beta}GOS$ in X.

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