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AN IDEAL BASED ZERO DIVISOR GRAPH OF GAMMA NEAR-RINGS

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ABSTRACT

In this paper, we study the notion of ideal based zero divisor graph structure of Gamma Near- ring M with respect to reflexive ideal I of M denoted by $\Gamma_I(M)$ whose vertices are the set $\{x \in M - I /$there exists $y \in M$-I such that $x\,\Gamma\, y \subseteq I\}$ with distinct vertices x and y are adjacent if and only if $x\,\Gamma\, y \subseteq I$.

Keywords: *ideal, graph, zero-divisor, diameter, cycle, Girth, clique.*

INTRODUCTION

The concept of a Gamma near –rings [9] was introduced by Satyanarayana and the ideal theory in Gamma near- rings was studied by Bh. Satyanarayana and G.L.Booth.

Let (M, +) be a group (not necessarily abelian) and Γ be a nonempty set. Then M is said to be a Γ- near ring if there exists a mapping $M \times \Gamma \times M \rightarrow M$ (denote the image of (m_1, α_1, m_2) by $m_1\alpha_1 m_2$ for $m_1, m_2 \in M$ and $\alpha_1 \in \Gamma$) satisfying the following conditions.

1. $(m_1 + m_2)\alpha_1 m_3 = m_1\alpha_1 m_3 + m_2\alpha_1 m_3$ and
2. $(m_1\alpha_1 m_2)\,\alpha_2 m_3 = m_1\alpha_1(m_2\alpha_2 m_3)$ for all $m_1, m_2, m_3 \in M$ and $\alpha_1, \alpha_2 \in \Gamma$.

Furthermore, M is said to be a zero symmetric Γ- near ring if mα0 = 0 for all $m \in M$ and $\alpha \in \Gamma$(where 0 is an additive identity in M.)

A normal subgroup L of M is called a left (resp right) ideal of M if u α(x + v) – u α v∈ L (resp x α u $\in L$) for all x $\in L, \alpha \in \Gamma$ and $u, v \in M$. A normal subgroup I of M is called an ideal if I is a both left and right ideal of M. An ideal I of M is said to be reflexive if $a\gamma b \in I \Rightarrow b\gamma a \in I$ for $a, b \in M$ and $\gamma \in \Gamma$. A proper ideal P of M is said to be prime if for any ideals A, B of M such that $A\,\Gamma\, B \subseteq P$,we have $A \subseteq P$ or $B \subseteq P$. An ideal P is called completely prime if $a\,\Gamma\, b \subseteq P$ implies $a \in P$ or $b \in P$. It is clear that if I is a reflexive ideal of M then I is prime iff I is completely prime. For any two nonempty subsets A, B of M, we write the set (A: B) = $\{m \in M / m\,\Gamma\, B \subseteq A\}$. We denote by I(a) the ideal of M generated by a. In [3], Beck introduced the concept of a zero divisor graph of a commutative ring with identity, but this work was mostly concerned with coloring of rings. In[2], Anderson and Livingston associate to a commutative ring with identity a (simple) graph Γ(R), whose vertex set is Z(R)* = Z(R) – {0}, the set of non zero divisor of R, in which two distinct x, y ∈ Z(R)* are joined by an edge if and only if xy = 0. They investigated the interplay between the ring theoretic properties of R and the graph theoretic properties of $\Gamma(R)$. Let I be a completely reflexive ideal ((ie.,) ab ∈ I implies ba ∈ I for a, b ∈R) then the ideal based zero divisor graph, denoted by $\Gamma_I(R)$, is the graph whose vertices are the set $\{x \in R - I / x\,\Gamma\, y \in I$ for some $y \in R -$ I} with distinct vertices x and y are adjacent if and only if $x\,\gamma\, y \in I,\ \gamma \in \Gamma$.

In this paper, we study the undirected graph $\Gamma_I(M)$ of Gamma near rings for any completely reflexive ideal I of M. Throughout this paper M stands for a non zero Gamma near -ring with zero element and I is a completely reflexive ideal of M. For distinct vertices x and y of a Graph G, let d(x, y) be the length of the shortest path from x to y. The diameter of a connected graph is the supremum of the distances between vertices. For any graph G, the girth of G is the length of a shortest cycle in G and is denoted by gr(G). If G has no cycle, we define the girth of G to be infinite. A clique of a graph is a maximal complete subgraph and the number of graph vertices in the largest clique or graph G, denoted by ω(G) is called the clique number of G. A graph G is bipartite with vertex classes V_1, V_2if the set of all vertices of G is $V_1 \cup V_2$, $V_1 \cap V_2 = \emptyset$, and edge of G joins a vertex from V_1 to a vertex of V_2.

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A complete bipartite graph is a bipartite graph containing all edges joining the vertices of V_1 and V_2. A complete bipartite graph on vertex sets of size m an n is denoted by $K^{m,n}$ for any positive integer, $K^{1,n}$ is called a star graph.

Theorem: 1
a) If I = (0) then $\Gamma_I(M)$ = Γ(M)
b) Let I be a nonzero completely reflexive ideal of M. Then $\Gamma_I(M)$ = Φ if and only if I is a completely prime ideal of M.

Proof:
a) Proof is Obivious.
b) Suppose that I is a completely prime ideal of M. For$\alpha \in \Gamma$, Then $x\,\alpha\, y \in I \Rightarrow x \in I$ or $y \in I$. Hence the vertex set $\Gamma_I(M)$ is empty.

Conversely suppose that $\Gamma_I(M)$ = Φ. Therefore if $x \in M - I$and $x\,\alpha\, y \in I,\ \alpha\ \in \Gamma$ $for\ some$ $y \in M$. We must have $y \in I$. (otherwise x is a vertex of $\Gamma_I(M)$). Hence I is a completely prime ideal of M.

Theorem: 2 Let I be a completely reflexive ideal of a gamma near-ring M. Then $\Gamma_I(M)$ is connected with diam($\Gamma_I(M)$) ≤ 3. Furthermore if $\Gamma_I(M)$ contains a cycle, then gr($\Gamma_I(M)$) ≤ 7.

Proof: Let x and y be distinct vertices of $\Gamma_I(M)$. Then there exists $z \in M - I$ and $w \in M - I$ with $x\,\Gamma\, z \subseteq I$ and $w\,\Gamma\, y \subseteq I$. If $x\,\Gamma\, y \subseteq I$ then x – y is a path of length 1. If $x\,\Gamma\, y \nsubseteq I$and $z\,\Gamma\, w \subseteq I$, then x-z-w-y is a path of length 3. If $x\,\Gamma\, y \nsubseteq I$ and $z\,\Gamma\, w \nsubseteq I$then there exists $\gamma \in \Gamma$ such that x-z γw-y is a path of length 2. Thus $\Gamma_I(M)$ is connected and diam($\Gamma_I(M)$) ≤ 3.

For any undirected graph G, gr(G) ≤ 2diam(G) + 1, if G contains a cycle. Thus gr(G) ≤ 2(3) + 1= 7.

Therefore gr($\Gamma_I(M)$) ≤ 7.

Theorem: 3 Let I be a completely reflexive ideal of M. For any x, y ∈ $\Gamma_I(M)$, if x-y is an edge in $\Gamma_I(M)$, then for each m ∈ M – I, either m-y or x-y' is an edge in $\Gamma_I(M)$ for some y′∈<y> - I

Proof: Let $x,\ y \in$ M – I, with x-y be an edge in $\Gamma_I(M)$ and suppose that m-y is not an edge in $\Gamma_I(M)$ for some m ∈ M – I.Then $x_1\,\Gamma\, y_1 \in I$ for some $x_1\ \in <x> - I$, $y_1 \in <y> - I$ and $m\,\Gamma\, y_1 \notin I$. But $m\,\Gamma\, y_1 \Gamma\, x_1 \in I$. So x-y ' is an edge in $\Gamma_I(M)$for some y'∈ <y> - I

Theorem: 4 Let I be a completely reflexive ideal of M and if a-x-b is a path in $\Gamma_I(M)$, then either I∪ $\{x_1\}$ is an ideal of M for some $x_1\ \in <x> - I$ or a-x-b is contained in a cycle of length ≤ 4.

Proof: Let a-x-b be a path in $\Gamma_I(M)$. Then there exists $x_1,\ x_2\ \in <x> - I$,$a_1 \in <a> - I$ and $b_1 \in <b> - I$ such that $a_1\Gamma x_1 \in I$ and $b_1\Gamma x_2 \in I$. If $a'\Gamma b' \in I$ for some $a'\ \in <a> - I$ and $b'\ \in <b> - I$. Then a-x-b-a is contained in a cycle of length ≤ 4. So let us assume that $a_1\Gamma b_1 \notin I$ for all $a_1 \in <a> - I$ and $b_1 \in <b> - I$

Case: (i) Let $x_1 = x_2$ then either $I_{a_1} \cap I_{b_1} = I \cup \{x_1\}$ or there exists c ∈ $I_{a_1} \cap I_{b_1}$ such that $c \notin I \cup \{x_1\}$. Then c$\Gamma a_1, c\Gamma b_1 \in I$. In the first case, $I \cup \{x_1\}$ is an ideal.In the second case a-x-b-c-a is contained in a cycle of length ≤ 4.

Case: (ii) Let $x_1 \neq x_2$, then clearly $<a_1> \cap <b_1> \nsubseteq$ I. Then for each z ∈ $<a_1> \cap <b_1> -$ I. We have, zΓ $x_1 \in <a_1><x_1> \subseteq I$ and zΓ $x_2 \in I$. Clearly either $x_1 \neq x$ or $x_2 \neq$ x. Say $x_1 \neq x$. Then we have a path a- x_1 -b and hence a-x-b- x_1 -a is contained in a cycle of length ≤ 4

Theorem: 5 Let I be a completely reflexive ideal of M. Then $\Gamma_I(M)$ can be neither a pentagon nor a hexagon

Proof: Suppose that $\Gamma_I(M)$ is a-b-c-d-e-a a pentagon. Then by theorem:4, For one of the vertices say (b_1), $I \cup \{b_1\}$ is an ideal of M for some $b_1 \in <b> - I$. Then in the pentagon,there exists $d_1 \in <d> - I$ and $e_1 \in <e> - I$ such that $d_1\Gamma\, e_1 \subseteq I$.Since $I \cup \{b_1\}$ is an ideal,$b_1\gamma d_1 = b_1 = b_1\gamma_1 e_1$ for some γ, γ_1 ∈ Γ. But $b_1\gamma(d_1\gamma_1 e_1) \in I$,$\gamma_1$ ∈ Γ. Then $b_1 = b_1\gamma_1 e_1$= $(b_1\gamma d_1)\gamma_1 e_1$= $b_1\gamma(d_1\gamma_1 e_1) \in I$. (ie).,$b_1 \in I$ which is a contradiction. The proof for the hexagon is the same

Theorem: 6 Let I be an reflexive ideal of a Gamma near ring M and let x, y $\in M - I$ Then
1. If x+I is adjacent to y+I in Γ $\left(\frac{M}{I}\right)$then x is adjacent to y in $\Gamma_I(M)$
2. If x is adjacent to y in $\Gamma_I(M)$ and x+I ≠ y+I then x+I is adjacent to y+I in Γ $\left(\frac{M}{I}\right)$
3. If x is adjacent to y in $\Gamma_I(M)$and x+I = y+I then x^2, $y^2 \in I$

Clearly there is a strong relationship between $\Gamma_I(M)$ and $\Gamma\left(\frac{M}{I}\right)$

Let I be an ideal of a gamma near- ring M. One can verify that the following method can be used to construct a graph $\Gamma_I(M)$. Let $\{a_\lambda\}_{\lambda\in\wedge} \subseteq$ R be a set of coset representatives of the vertices of $\Gamma\left(\frac{M}{I}\right)$. For each i $\in I$, define a graph G_i with vertices $\{a_\lambda + i/\lambda \in\wedge\}$where edges are defined by the relationship $a_\lambda + i$ is adjacent to $a_\beta + i\ in\ G_i iff a_\lambda + I$ is adjacent to $a_\beta + I$ in $\Gamma\left(\frac{M}{I}\right)$((ie.,)$a_\lambda\Gamma a_\beta \in I$)

Theorem: 7 Let I be a completely reflexive ideal of M. Then the following are hold

i. If M has identity, then $\Gamma_I(M)$has no cut vertices.

ii. If M has no identity and if I is a nonzero completely reflexive ideal of M then $\Gamma_I(M)$ has no cut vertices.

Proof: Suppose that the vertex x of $\Gamma_I(M)$ is cut vertex. Let u-x-w be a path in $\Gamma_I(M)$. Since x is a cut vertex, x lies in every path from u to w.

i) Assume that M is Gamma near ring with identity. For any u, v $\in \Gamma_I(M)$, there exists a path u-1-w which shows x$(\neq 1)$ in $\Gamma_I(M)$ is not a cut vertex. Suppose x = 1. Then there exists $u_1 \in< u > - I, w_1 \in< w > - I, \gamma \in \Gamma$ and $t_{1\,,}\ t_2 \in M - I$ such that $u_1\gamma t_1, w_1\gamma t_2 \in I$ which implies $u_1, w_1 \in \Gamma_I(M)$. Since $\Gamma_I(M)$ is connected, there exists m, $m_1 \in M - I \cup \{x\}$ such that $u_1 - m - w_1$(or)$u_1 - m - m_1 - w_1$ is a path in$\Gamma_I(M)$ which implies $u - m - w - 1 - u\ (or) u - m - m_1 - w - 1 - u$ is a cycle in $\Gamma_I(M)$ contradicting x=1 is a cut vertex

ii) Let M be a Γ-near-ring without identity and I be a non zero completely reflexive ideal of M. Since $u - x - w$ is a path from u $-w$, then there exists $u_1 \in< u > -I$, $w_1 \in< w > -I$ and $x_1, x_2 \in< x > -I$, $\gamma \in \Gamma$ such that $u_1\gamma x_1 \in I\ and\ w_1\gamma x_2 \in I$.

Case: (i) $x_1 = x_2$

If $u_1 + \mathrm{I} = x_1 + \mathrm{I}$ then $u_1\gamma w_1 \in \mathrm{I} \Longrightarrow$ u is adjacent to w. Similarly, If $x_2 + I = w_1 + \mathrm{I}$, u is adjacent to w. So assume that $u_1 + I \neq x_1 + \mathrm{I}$ and $x_2 + \mathrm{I} \neq w_1 + \mathrm{I}$. Let $0 \neq i \in I$. Then $u_1\gamma w_1 \in \mathrm{I}$ and $w_1\gamma x_2 \in I$ which implies $u_1\gamma(x_1 + i,\ w1\gamma x1+i\in I$. If x = $x1+i$ then x $\neq x1 \Rightarrow$ u- $x1$-w is path in $\Gamma I/M$. otherwise, u-$x1+i$-w is a path in $\Gamma I/M$. Thus there exists a path from u to w not passing through x which is a contradiction.

Case: (ii) Either x_1 or x_2 equal to x.

Without loss of generality, let us assume that $x_1 = \mathrm{x}$ and $x_2 \neq x$. Then $u_1\gamma x \in I$ and $x_2\gamma w_1 \in I$ =>$u_1\gamma x_1 \in I$ and $x_2\gamma w_1 \in I$. Also we have a path u- x_2-w which is a contradiction

Case: (iii) Neither x_1 nor x_2 equal to x.

If $x_1\gamma x_2 \in I$ then we have a path u- $x_1 - x_2$-wwhich is a contradiction. So assume that $x_1\gamma x_2 \neq x$, then we have a path u-$x_1\gamma x_2$–w which is a contradiction.

Thus x cannot be a cut vertex.

Definition: 8 Using the notation as in the above construction, we call the subset $a_\lambda + I$ a column of $\Gamma_I(M)$. If ${a_\lambda}^2 \in I$ then we call $a_\lambda + I$a connected column of $\Gamma_I(M)$.

Lemma: 9 Let I be an reflexive ideal of a Gamma near- ring M. Then gr($\Gamma_I(M)$) ≤ gr($\Gamma(\frac{M}{I})$). Inparticular if $\Gamma(\frac{M}{I})$ contains a cycle then so does $\Gamma_I(M)$and therefore gr($\Gamma_I(M)$) ≤ gr($\Gamma(\frac{M}{I})$) ≤ 4.

Proof: If gr($\Gamma(\frac{M}{I})$) = ∞ we are done. So suppose gr($\Gamma(\frac{M}{I})$) = n $<$ ∞.

Let $x_1 + \mathrm{I} - x_2 + \mathrm{I} - \cdots - x_\mathrm{n} + \mathrm{I} - x_1 + \mathrm{I}$ be a cycle in $\Gamma(\frac{M}{I})$ through n distinct vertices

Then $x_1 - x_2 - \cdots - x_\mathrm{n} - x_1$is a cycle in $\Gamma_I(M)$ of length n. Hence gr($\Gamma_I(M)$) ≤ n.

Lemma: 10 Let I be an reflexive ideal of a gamma near ring M. If $|I| \geq 3$ and $\Gamma_I(M)$ contains a connected column, then gr($\Gamma_I(M)$) = 3

Proof: Let x +I be a connected column of $\Gamma_I(M)$. Then $x^2 \in I$. Let i, j $\in I-\{0\}$. Then x-(x+i)-(x+j)-x is a cycle of length 3 in $\Gamma_I(M)$.

Lemma: 11 Let I be a reflexive ideal of a gamma near ring M. If I ≠ 0 and $\Gamma(\frac{M}{I})$ has only one vertex, then

$$\mathrm{gr}\Gamma_I(M) = \begin{cases} 3 \, if |I| \geq 3 \\ \infty \, if |I| = 2 \end{cases}$$

Proof: If $\Gamma(\frac{M}{I})$ has only one vertex then $\Gamma_I(M)$ consist of a single connected column. Thus $\Gamma_I(M)$ is a complete graph, and therefore has a cycle of length 3 unless $\Gamma_I(M)$ has only two vertices.

Lemma: 12 Let I be a reflexive ideal of a gamma near ring M. If I has two elements, $\Gamma(\frac{M}{I})$ has at least two vertices and $\Gamma_I(M)$ has at least two vertices, and $\Gamma_I(M)$ has at least one connected column, then $\mathrm{gr}(\Gamma_I(M)) = 3$

Proof: Let x+I be a connected column of $\Gamma_I(M)$.Then $x^2 \in I$.Let y+I be a vertex adjacent to x+I in $\Gamma(\frac{M}{I})$. Write I = {0,i}. Then y-x-x+i-y is a cycle of length 3 in $\Gamma_I(M)$

Theorem: 13 Let I be a nonzero reflexive ideal of a gamma near ring M that is not a completely prime ideal. Then
$\mathrm{Gr}(\Gamma_I(M) = \infty$ *if* $\Gamma\left(\frac{M}{I}\right)$ has only one cut vertex &|I| = 2

$$\begin{cases} 4 \text{ if } \mathrm{gr}(\Gamma(\frac{M}{I}) > 3 \text{ and } \Gamma_I(M) \text{ has no connected columns} \\ 3 \text{ otherwise} \end{cases}$$

Proof: The only remaining case is I ≠ (0), $\Gamma_I(M)$ has no connected columns, and $\mathrm{gr}((\Gamma(\frac{M}{I}))>3$.

Since $\Gamma_I(M)$has no connected columns, $\Gamma\left(\frac{M}{I}\right)$must have at least two vertices. By lemma 9, $\mathrm{Gr}(\Gamma_I(M)) \leq 4$. Assume x-y-z-x is a cycle in $\Gamma_I(M)$ of length 3 and we provide a contradiction. Since $\mathrm{gr}\left(\Gamma\left(\frac{M}{I}\right)\right) > 3$,x+I-y+I-z+I-x+I cannot be a cycle in $\Gamma\left(\frac{M}{I}\right)$.Therefore we have either x+I=y+I, y+I =z+I (or) z+I=x+I. If x+I =y+I, then$(x+I)^2 = (x+1)(y+$ *I=0+I* and so x+I is a connected column of Γ*I/M.* But this is a contradiction. We get a similar contradiction if y+I=z+I (or) z+I=x+I. Hence $\mathrm{gr}(\Gamma_I(M))$ = 4

Theorem: 14 Let I be a nonzero reflexive ideal of a gamma near ring M. Then $\Gamma_I(M)$ is bipartite if and only if either
a) $\mathrm{gr}(\Gamma_I(M)) = \infty$ (*or*)
b) $\mathrm{gr}(\Gamma_I(M)) = 4$ *and* $\Gamma\left(\frac{M}{I}\right)$is bipartite.

Proof: Suppose that $\Gamma_I(M)$ is bipartite.Since $\Gamma\left(\frac{M}{I}\right)$ is isomorphic to a subgraph (or) $\Gamma_I(M)$, $\Gamma\left(\frac{M}{I}\right)$ is bipartite (or a single vertex). By theorem 13, $\mathrm{gr}(\Gamma_I(M))$ *is* $3,4,\infty$. By theorem 1 of sec1.2 of Bollobas (1979), a graph is bipartite if and only if it does not contain an odd cycle. Hence $\mathrm{gr}(\Gamma_I(M)) \neq 3$.

If $\mathrm{gr}(\Gamma_I(M)) = \infty$, then by theorem:13,$\Gamma_I(M)$ is a graph on two vertices and therefore bipartite. Suppose $\mathrm{gr}(\Gamma_I(M)) = 4$ and $\Gamma\left(\frac{M}{I}\right)$is bipartite. Let W_1, W_2be the two vertex classes of $\Gamma\left(\frac{M}{I}\right)$. Define $V_j = \{x+i/i \in I, x+I \in W_j\}$for j = 1, 2. Then $V_1 \cap V_2 = \varphi$ and the vertex set of $\Gamma_I(M)$ is $V_1 \cup V_2$.

Let x and y be adjacent vertices of $\Gamma_I(M)$. Without loss of generality, say $x \in V_1$ By theorem: 13, $\Gamma_I(M)$has no connected columns. Thus x+I ≠y+I. Hence x+I-y+I is an edge in $\Gamma\left(\frac{M}{I}\right)$(By theorem: 6, Since x+I $\in W_1$,y+I$\in W_2$. Therefore y$\in V_2$. Hence all edges of $\Gamma_I(M)$ join vertices from $V_1$to those of V_2. Thus $\Gamma_I(M)$is bipartite.

Theorem: 15 Let I be a reflexive ideal of M and let S be a clique in $\Gamma_I(M)$ such that $x^2 = 0$ for all x ∈ S. Then $S \cup I$ is a reflexive ideal of M.

Proof: Suppose that x, y $\in S \cup I$. consider the following three cases

Case: (i) If x, y $\in I$ then $x\alpha y \in S \cup I, \alpha \in \Gamma$

Case: (ii) If x, y ∈ S with $x\alpha y \notin I$ then for all c ∈ S $c\Gamma(x\alpha y) \in I$ and hence S ∪ $\{x\alpha y\}$ is a clique. Now since S is a clique, $x\alpha y$ ∈ S

Case: (iii) If x ∈ I and y ∈ S then $x\alpha y \notin I$ and hence for any c ∈ S $c\Gamma(x\alpha y) \in I$. Therefore $x\alpha y \in S$. Now let $x \in S \cup I$ and $r \in M$.Suppose that r, x∉ I, $\alpha \in \Gamma$.If rΓx ⊆ I then rΓx ⊆ S ∪ I. If rΓx ⊈ I. Since for any c ∈ S, (rΓx) Γc ⊆ I. We have rΓx ∈ S

Theorem: 16 Let I be a nonzero reflexive ideal of M and a ∈ $\Gamma_I(M)$ adjacent to every vertex of $\Gamma_I(M)$. Then (I: a) is a maximal element of the set {(I: x)/ x ∈ M}.Moreover (I: a) is a completely prime ideal.

Proof: Let V = V($\Gamma_I(M)$). Choose 0 ≠ x ∈ I.It is easy to see that a≠ a+x ∈ $\Gamma_I(M)$. Thus $a\Gamma(a+x) \in I$ and hence $a^2 \in I$.

Therefore V ∪ I = (I: a) and so for any x ∈ M,(I:x) is contained in V ∪ I=(I: a). Thus the first assertion holds.

Now we prove that (I: a) is a completely prime ideal. Let $x\alpha y \in (I{:}\,a)$ and x, y ∉(I:a). Therefore $x\alpha y\Gamma a \in I$. If $y\Gamma a \not\subseteq I$ then $x \in (I{:}\,y\Gamma a)$. We know that (I: a) ⊆ $(I{:}\,y\Gamma a)$. And therefore (I: a) = $(I{:}\,y\Gamma a)$. Hence $x \in (I{:}\,a)$ which is a contradiction.

Theorem: 17 Let I be a non-zero reflexive ideal of M. Then the followings are hold.

a) If P_1 and P_2 are completely prime ideals of M and I = $P_1 \cap P_2 \neq \varphi$
Then $\Gamma_I(M)$ is a complete bipartite graph

b) If I ≠ 0 is a reflexive ideal of M for which I = $\sqrt{I}$ then $\Gamma_I(M)$ is a complete bipartite graph if and only if there exists prime ideals P_1 and P_2 of M such that I = $P_1 \cap P_2$

Proof:
a) Let a, b ∈ M – I with $a\alpha b \in I$. Then $a\alpha b \in P_1$ and $a\alpha b \in P_2$. Since P_1 and P_2 are completely prime, we have $a \in P_1$ or $b \in P_1$ and $a \in P_2$ (or)$b \in P_2$. Therefore suppose $a \in \frac{P_1}{P_2}$ and $b \in \frac{P_2}{P_1}$. Thus $\Gamma_I(M)$ is a complete bipartite graph with parts $\frac{P_1}{P_2}$ *and* $\frac{P_2}{P_1}$

b) Suppose that the parts of $\Gamma_I(M)$ are V_1 and V_2. Set $P_1 = V_1 \cup I$ and $P_2 = V_2 \cup I$. It is clear that I = $P_1 \cap P_2$. We now prove that P_1 is a reflexive ideal of M

To show this let $a, b \in P_1$

Case: (i) If a, b ∈ I, $\gamma \in \Gamma$ then $a\gamma b \in$ I andso $a\gamma b \in P_1$

Case: (ii) If a, b ∈ V_1, $\gamma \in \Gamma$ then there is c ∈ V_2 such that $c\gamma a \in$ I and $c\gamma b \in$ I. So c $\Gamma(a\gamma b) \in$ I. If $a\gamma b \in$ I then $a\gamma b \in P_1$. Otherwise $a\gamma b \subset V_1$ => $a\gamma b \in P_1$

Case: (iii) If a∈V_1 and b ∈ I then $a\gamma b \notin$ I. So there is c∈ V_2 such that c $\Gamma(a\gamma b) \in$ I => $a\gamma b \in V_1$ and so $a\gamma b \in P_1$.
Now let $r \in M$ and a ∈ P_1

Case: (1) If a ∈ I then $r\gamma a \in I$ andso $r\gamma a \in P_1$

Case: (2) If a ∈ V_1 then there exists c ∈ V_2 such that $c\gamma a \in I$. So, c $\Gamma(r\gamma a) \in$ I. If $r\gamma a \in$ I then $r\gamma a \in P_1$. And so $r\gamma a \notin I$ then $r\gamma a \in V_1 \Rightarrow r\gamma a \in P_1 \Rightarrow P_1 \trianglelefteq$ M. We now prove P_1 is prime. For proving this let $a\gamma b \in P_1$ and a, b ∉ P_1. Since $P_1 = V_1 \cup I$ $a\gamma b \in I$ *or* $a\gamma b \in V_1$ andso in any case there exists c ∈ V_2 such that c $\Gamma(r\gamma a) \in$ I. Thus $a\Gamma(c\gamma b) \in$ I. If $c\gamma b \in$ I then by the definition of $\Gamma_I(M)$ we have $b \in V_1$ which is a contradiction. Hence $c\gamma b \notin I$ and $c\gamma b \in V_1$. Therefore $c^2\gamma$ b ∈ I.Since I = $\sqrt{I}$, $c^2 \notin I$. Hence $c^2 \in V_2$ so b ∈ V_1 which is a contradiction. Therefore P_1 is a completely prime ideal of M.

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