International Journal of Mathematical Archive-4(12), 2013, 223-229
Available online through www.ijma.info ISSN 2229 – 5046

B(1, 2) $^{+}$-πgα-REGULAR AND NORMAL SPACES IN SIMPLE EXPANSION

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(Received on: 24-11-13; Revised & Accepted on: 17-12-13)

ABSTRACT

In this paper, we generalize the concept of simple expansion due to Levine to the bitopological setting. Also, we present some new classes of set and spaces namely, B(1, 2) $^{+}$-πgα-closed set, B (1, 2) $^{+}$-πgα-Regular and Normal Spaces. Here, we define the notion of (1, 2)-πgα-closed sets in the extended topology as a subset A of a space X is called B(1, 2) $^{+}$-πgα-closed set if B(1, 2) $^{+}$-αcl (A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1, 2}$-π-open in X. This class of sets and spaces not only depend on (1, 2)*-topology, but also on its simple expansion. Further, we discuss their properties and some of their characterizations are obtained in the light of B (1, 2) $^{+}$-πgα-closed sets.*

Keywords: *B(1, 2) $^{+}$-πgα-closed set, B(1, 2) $^{+}$-πgα-irresolute, B(1, 2) $^{+}$-πgα-regular space, B(1, 2) $^{+}$-πgα-normal space.*

1. INTRODUCTION

In 1963 Levine [5] started the study of generalized open sets with the introduction of semi-open sets. Since then many topologists have utilized the concepts to the various notions of subsets, weak axioms, weak regularity, weak normality and weaker and stronger forms of covering axioms in the literature. The concept of g-regular and g-normal spaces in topological spaces were introduced and studied by Munshi [6]. Further, Noiri and Popa [7] investigated the concepts introduced by Munshi [6]. M. E. Abd El. Monsef *et al* [1] have developed and studied Bg-closed sets, gB-continuity and gB-irresolute map. In 2010 Vadivel [10] have defined the notion of B-Generalized Regular and B-Generalized Normal spaces in topological spaces. Using τ_p^{+}g-closed and $\tau^{+}_{\beta g}$-closed sets, Nirmala Irudayam and I. Arockiarani [8, 9] introduced τ_p^{+}g-regular, normal spaces and $\tau^{+}_{\beta g}$ –regular, normal spaces in simple extension. The notion of (1, 2)*-πgα-closed set was introduced by I. Arockiarani and K. Mohana [2]. In this paper we apply B(1, 2) $^{+}$-πgα-closed sets as a tool to extend the two classes of spaces called B(1, 2) $^{+}$-πgα-regular and B(1, 2) $^{+}$-πgα-Normal spaces in bitopological spaces. Also we characterize the basic properties along with weaker forms of regularity and normality.

2. PRELIMINARIES

We list some definitions which are useful in the following sections. Throughout, this paper (X, τ_1, τ_2), (Y, σ_1, σ_2), (Z, η_1, η_2) [briefly X, Y, Z] represent non-empty bitopological spaces on which no separation axiom is defined, unless otherwise mentioned.

Definition: 2.1 [4] A subset S of a bitopological space X is said to be $\tau_{1, 2}$-open if S=A $\cup$ B where $A \in \tau_1$ and $B \in \tau_2$. A subset S of X is said to be (i) $\tau_{1, 2}$-closed if the complement of S is $\tau_{1, 2}$-open. (ii) $\tau_{1, 2}$-clopen if S is both $\tau_{1, 2}$-open and $\tau_{1, 2}$-closed.

Definition: 2.2 [4] Let S be a subset of the bitopological space X. Then the $\tau_{1, 2}$-interior of S denoted by $\tau_{1, 2}$-int(S) is defined by $\cup${G: G $\subseteq$ S and G is $\tau_{1, 2}$-open set} and the $\tau_{1, 2}$-closure of S denoted by $\tau_{1, 2}$-cl(S) is defined by $\cap${F: S $\subseteq$ F and F is $\tau_{1, 2}$- closed set}.

The family of all $\tau_{1, 2}$-closed sets of X will be denoted by $\tau_{1, 2}$-C(X).

The set $\tau_{1, 2}$-C(X, x) = {V $\in$ $\tau_{1, 2}$-C(X) / x $\in$ V}, for x $\in$ X.

Remark: 2.3 [4] $\tau_{1, 2}$-open sets need not form a topology.

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Definition: 2.4 [4] A subset A of a bitopological space X is called
(i) (1, 2)*-regular open if A= $\tau_{1,2}$-int ($\tau_{1,2}$-cl (A)).
(ii) (1, 2)*-α -open if $A \subseteq \tau_{1,2}$-int ($\tau_{1,2}$-cl ($\tau_{1,2}$-int (A))).

The complement of the sets mentioned from (i) and (ii) are called their respective closed sets.

Definition: 2.5 [2] The finite union of (1, 2)*-regular open sets is said to be $\tau_{1,2}$-π-open. The complement of $\tau_{1,2}$-π-open is said to be $\tau_{1,2}$-π-closed.

Definition: 2.6 [2] A subset A of a bitopological space X is called (1, 2)*- πgα - closed if (1, 2)*-αcl (A) $\subseteq$ U whenever $A \subseteq U$ and U is $\tau_{1,2}$-π-open. The complement of the above set is called (1, 2)*- πgα-open set.

Definition: 2.7[3] A function $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called (1, 2)*-πgα-irresolute if $f^{-1}(V)$ is (1, 2)*-πgα-closed in X, for every (1, 2)*-πgα-closed set V of Y.

3. B (1, 2) $^{+}$-πgα-CLOSED SETS

Definition: 3.1 Let B be a subset of a bitopological space X then $\tau_{1,2}^{+}(B)$ is defined by $\tau_{1,2}^{+}(B) = \{O \cup (O^1 \cap B): O \in \tau_{1,2}$ and called its simple expansion of $\tau_{1,2}$, $O^1 \in \tau_{1,2}\}$ by B , where $B \notin \tau_{1,2}(O)$

Definition: 3.2 A subset of X belonging to $\tau_{1,2}$ (B) is denoted by B $\tau_{1,2}$-open set, the complement of B $\tau_{1,2}$-open set is denoted by B $\tau_{1,2}$-closed set.

Defintion: 3.3 The B$\tau_{1,2}$-interior of S, denoted by B $\tau_{1,2}$-int(S) is defined by ∪ {G: G $\subseteq$ S and G is $\tau_{1,2}$-open in $\tau_{1,2}$ (B)}. The B $\tau_{1,2}$-closure of S, denoted by ∩ {F: S $\subseteq$ F and F is $\tau_{1,2}$-closed in $\tau_{1,2}$ (B)}.

Definition: 3.4 A subset A of a bitoplogical space X is called B (1, 2) $^{+}$-α-open if A $\subseteq$ B $\tau_{1,2}$ –int (B $\tau_{1,2}$-cl (B $\tau_{1,2}$-int (A))). The complement of B (1, 2) $^{+}$-α-open set is called B (1, 2) $^{+}$-α-closed set.

Definition: 3.5
(i) A subset A of a space X is B (1, 2) $^{+}$-α-regular if A is both B (1, 2) $^{+}$-α-open and B (1, 2) $^{+}$-α-closed.
(ii) The B (1, 2) $^{+}$-α-closure of S, denoted by B (1, 2) $^{+}$-αcl(S) is defined by ∩ {F: S $\subseteq$ F and F is (1, 2) $^{+}$-α-closed in $\tau_{1,2}$ (B)}.

The family of all B (1, 2) $^{+}$-α-open [B $\tau_{1,2}$ -open] sets is denoted by B (1, 2) $^{+}$-αO(X) [B $\tau_{1,2}$ -O(X)] and the family of all B (1, 2) $^{+}$-α-closed [B $\tau_{1,2}$ -closed] sets is denoted by B (1, 2) $^{+}$-αC(X) [B $\tau_{1,2}$-C(X)].

Definition: 3.6 A subset A of a bitopological space X is said to be
(i) B (1, 2) $^{+}$-g-closed set if B $\tau_{1,2}$-cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1,2}$-open in X.
(ii) B (1, 2) $^{+}$-πg-closed set if B $\tau_{1,2}$-cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1,2}$-π-open in X.
(iii) B (1, 2) $^{+}$-πgα-closed set if B (1, 2) $^{+}$-αcl (A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1,2}$-π-open in X.

Example: 3.7 Consider X = {a, b, c}, τ_1 = {ϕ, X, {a}}, τ_2 = {ϕ, X, {b}}. Let B={c}. Here the B (1, 2) $^{+}$-πgα-closed sets are {ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}}.

Theorem: 3.8 The union of two B (1, 2) $^{+}$-πgα-closed set is B (1, 2) $^{+}$-πgα-closed.

Proof: Let A and B be B (1, 2) $^{+}$-πgα-closed sets. Let U be $\tau_{1,2}$ -π-open set such that $A \cup B \subseteq U$. This implies $A \subseteq U$ and B $\subseteq$ U. Since A is B (1, 2) $^{+}$-πgα-closed set, we have B (1, 2) $^{+}$-αcl (A) $\subseteq$ U. Also B is B (1, 2) $^{+}$-πgα-closed set, we have B (1, 2) $^{+}$-αcl (B) $\subseteq$ U. That is, B (1, 2) $^{+}$-αcl (A) ∪ B (1, 2) $^{+}$-αcl (B) $\subseteq$ U. This implies B (1, 2) $^{+}$-αcl ($A \cup B$) $\subseteq$ U whenever $A \cup B \subseteq U$ where U is $\tau_{1,2}$ -π-open.

Theorem: 3.9
(i) Every closed set in $\tau_{1,2}$ (B) is B (1, 2) $^{+}$-πgα-closed set.
(ii)Every B (1, 2) $^{+}$-g-closed set in $\tau_{1,2}$ (B) is B (1, 2) $^{+}$-πgα-closed set.
(iii) Every B (1, 2) $^{+}$-πg-closed set in $\tau_{1,2}$ (B) is B (1, 2) $^{+}$-πgα-closed set.

Proof: The proof is obvious.

The converse of the above theorem is not true in general. The following examples shows this result.

Example: 3.10 Consider X = {a, b, c}, τ_1 = {ϕ, X, {a}}, τ_2 = {ϕ, X, {a, b}}. Let B={c}. Here the B (1, 2) $^{+}$-πgα-closed sets are {ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}}. Then {a, c} is B (1, 2) $^{+}$-πgα-closed set, but not B $\tau_{1, 2}$-closed set.

Example: 3.11 In example 3. 10, the B (1, 2) $^{+}$-g-closed sets are {ϕ, X, {b}, {c}, {a, b}, {a, c}, {b, c}}. Then {a} is B (1, 2) $^{+}$-πgα-closed set but not B (1, 2) $^{+}$-g-closed set.

Example: 3.12 Consider X = {a, b, c, d}, τ_1 = {ϕ, X, {d}, {c, d}, {a, d}, {a, c, d}},τ_2 = {ϕ, X, {a, c}}. Let B = {a}. Here the B(1, 2) $^{+}$-πgα-closed sets are {ϕ, X, {b}, {c}, {b, c}, {a, b}, {b, d}, {a, b, c}, {b, c, d}, {a, b, d}} and B (1, 2) $^{+}$-πg-closed sets are {ϕ, X, {b}, {b, c}, {a, b}, {b, d}, {a, b, c}, {b, c, d}, {a, b, d}} . Then {c} is B(1, 2) $^{+}$-πgα-closed set but not B (1, 2) $^{+}$-πg-closed set.

4. B (1, 2) $^{+}$-πgα-REGULAR SPACES

In this section, we introduce B(1, 2)$^{+}$-πgα-regular space in bitopological spaces and also we obtain some characterizations of B (1, 2) $^{+}$-πgα-regular spaces.

Definition: 4.1 A subset A of a space X is said to be B (1, 2) $^{+}$-πgα-regular if for every B(1, 2)$^{+}$-πgα-closed set F and a point $x \notin F$, there exist disjoint B (1, 2) $^{+}$-α-open sets U and V such that $F \subseteq U$ and $x \in V$.

Theorem: 4.2 For a bitopological spaces X, the following are equivalent:
1. X is B(1, 2) $^{+}$-πgα-regular.
2. Every B(1, 2) $^{+}$-πgα-open set U is a union of B (1, 2) $^{+}$-α-regular sets.
3. Every B(1, 2) $^{+}$-πgα-closed set A is an intersection of B (1, 2) $^{+}$-πgα-regular sets.

Proof:
[1] => [2]: Let U be a B (1, 2) $^{+}$-πgα-open set and let $x \in U$. If A = X-U, then A is B(1, 2) $^{+}$-πgα-closed. By assumption there exist disjoint B (1, 2) $^{+}$-α-open subsets W_1 and W_2 of X such that $x \in W_1$ and $A \subseteq W_2$. If V = B (1, 2) $^{+}$-αcl (W_1) then V is B (1, 2) $^{+}$-α-closed and $V \cap A \subset V \cap W_2 = \phi$. It follows that $x \in V \subseteq U$. Thus U is a union of B (1, 2) $^{+}$-α-regular sets.

[2] => [3]: This is obvious.
[3] => [1]: Let A be B (1, 2) $^{+}$-πgα-closed and let $x \notin A$. By assumption there exists a B (1, 2) $^{+}$-α-regular set V such that $A \subseteq V$ and $x \notin V$. If U = X/V, then U is B (1, 2) $^{+}$-α-open set containing x and U∩V = ϕ. Thus, X is B(1, 2) $^{+}$-πgα-regular.

Definition: 4.3 A bitopological space X is called a
(1) B (1, 2) $^{+}$-πgα-open-T_0 if for each pair of distinct points, there exists an B (1, 2) $^{+}$-πgα-open set containing one point but not the other.
(2) B (1, 2) $^{+}$-πgα-open-T_1 if for any two distinct points x, y in X, there exists a pair of B (1, 2) $^{+}$-πgα -open sets, one containing x but not y and the other containing y but not x.
(3) B (1, 2) $^{+}$-α-open-T_2 if for each pair of distinct points x and y in X, there exist disjoint B (1, 2) $^{+}$-α -open sets U and V in X such that $x \in U$ and $y \in V$.
(4) (B, B (1, 2) $^{+}$-πgα-open-R_0)-space if B (1, 2) $^{+}$-πgαcl ({x}) $\subseteq$ U whenever U is B (1, 2) $^{+}$-πgα-open and $x \in U$.

Theorem: 4.4 Every B (1, 2) $^{+}$-πgα-regular space X is both B(1, 2) $^{+}$-α-T_2 and (B, B (1, 2) $^{+}$-πgα-open-R_0)-space.

Proof: Let X be B (1, 2) $^{+}$-πgα-regular space and let x, y $\in$ X such that $x \neq y$. By theorem 4. 2. {x} is either B(1, 2) $^{+}$-α-open (or) B(1, 2) $^{+}$-α-closed. Since every space is B (1, 2) $^{+}$-α-T_2. If {x} is B (1, 2) $^{+}$-α-open, hence B (1, 2) $^{+}$-πgα-open. Thus {x} and X – {x} are separating B (1, 2) $^{+}$-α-open sets. If {x} is B (1, 2) $^{+}$-α-closed, then X – {x} are B (1, 2) $^{+}$-α-open and hence by theorem 4. 2 X is the union of B(1, 2) $^{+}$-α-regular sets. Hence there is a B (1, 2) $^{+}$-α-regular set V $\subseteq$ X – {x} containing y. This proves that X is B(1, 2) $^{+}$-α-T_2. By Theorem 4. 2. it follows immediately that X is also (B, B (1, 2) $^{+}$-πgα-open-R_0)-space.

Definition: 4.5 The intersection of all B (1, 2) $^{+}$-πgα-closed sets containing A is called B (1, 2) $^{+}$-πgα-closure of A and is denoted by B (1, 2) $^{+}$-πgα-cl (A).

Theorem: 4.6 The following are equivalent for a space X:
1. X is B (1, 2) $^{+}$-πgα-regular.
2. B (1, 2) $^{+}$-πgα-cl (A) = $\bigcap$ {F: $A \subseteq F$ and F is B (1, 2) $^{+}$-πgα-closed in X} for each subset A of X.
3. B (1, 2) $^{+}$-πgα-cl (A) = A for each B (1, 2) $^{+}$-πgα-closed A.

Proof:
(1) => (2): For any subset A of X, we have $A \subset \bigcap\{F: A \subseteq F$ and F is B (1, 2) $^+$-πgα-closed in X$\} \subseteq$ B (1, 2) $^+$-πgα-cl (A). Suppose $x \in X \setminus \bigcap\{F: A \subseteq F$ and F is B (1, 2) $^+$-πgα-closed in X$\}$, then there exists a B (1, 2) $^+$-πgα-closed set F such that $x \in X\setminus F$, $A \subseteq F$. Since X is B (1, 2) $^+$-πgα-regular, there exists disjoint B (1, 2) $^+$-α-open sets U and V such that $x \in U$ and $F \subseteq A$. Therefore, we have $x \in U \subset$ B (1, 2) $^+$-αcl(U) $\subset X\setminus V \subset X\setminus F \subset X\setminus A$.

Hence, B (1, 2) $^+$-αcl(U) $\cap A = \phi$. Therefore, $x \in X \setminus$ B (1, 2) $^+$-πgα-cl (A). Hence $\bigcap\{F: A \subseteq F$ and F is B (1, 2) $^+$-πgα-closed in X$\}$=B (1, 2) $^+$-πgα-cl (A).

(2) => (3): We always have $A \subset$ B (1, 2) $^+$-πgα-cl (A) to be true.

Now to prove B (1, 2) $^+$-πgα-cl (A) $\subset$ A. B (1, 2) $^+$-πgα-cl (A) is the smallest B (1, 2) $^+$-πgα-closed set and A is a B (1, 2) $^+$-πgα-closed set contained in F.

Therefore, B (1, 2) $^+$-πgα-cl (A) $\subset$ A => A = B (1, 2) $^+$-πgα-cl (A).

(3) => (1): Let F be any B (1, 2) $^+$-πgα-closed set and $x \in X\setminus F$, then $x \in$ B (1, 2) $^+$-πgα-cl (F). Then there exists an B (1, 2) $^+$-α-open set U such that B (1, 2) $^+$-α cl(U) $\cap F = \phi$. Therefore, we have $F \subset X\setminus$B (1, 2) $^+$-α cl(U). Also U and X\B (1, 2) $^+$-α cl(U) are disjoint hence X is B (1, 2) $^+$-πgα-regular. Hence the proof.

Theorem: 4.7 Let X be a bitopological space. Then the following statements are equivalent.

(1) X is B (1, 2) $^+$-πgα-regular.
(2) For each point $x \in X$ and for each B (1, 2)$^+$-πgα-open nbhd W of x, there exists a B (1, 2) $^+$-α-open set U of X such that B (1, 2) $^+$-αcl(U) $\subseteq$ W.
(3) For each point $x \in X$ and for each B(1, 2) $^+$-πgα-closed set F not containing x, there exists a B (1, 2) $^+$-α-open set V of X such that B (1, 2) $^+$-αcl(V)$\cap F = \phi$.

Proof:
(1) => (2): Let W be any B (1, 2) $^+$-πgα-open nbhd of x. Then there exists an B (1, 2) $^+$-πgα-open set G such that $x \in G \subseteq W$. Since G^c is B(1, 2) $^+$-πgα-closed and $x \notin G^c$, by hypothesis, there exist B (1, 2) $^+$-α-open sets U and V such that $G^c \subseteq U$, $x \in V$ and $U \cap V = \phi$ and so $V \subseteq U^c$. Now B (1, 2) $^+$-αcl (V) $\subseteq$ B (1, 2) $^+$-αcl (U^c) = U^c and $G^c \subseteq U$ implies $U^c \subseteq G \subseteq W$. Thus B (1, 2) $^+$-αcl (U) $\subseteq$ W.

(2) => (1): Let F be any B (1, 2) $^+$-πgα-closed set and $x \notin F$. Then $x \in F^c$ and F^c is a B (1, 2) $^+$-πgα-open and so F^c is an B (1, 2) $^+$-πgα-nbhd of x. By hypothesis, there exists a B (1, 2) $^+$-α-open set V of x such that $x \in V$ and B (1, 2) $^+$-αcl(V) $\subseteq F^c$, which implies $F \subseteq$ (B (1, 2) $^+$-αcl(V))^c. Then (B (1, 2) $^+$-αcl (V))^c is B (1, 2) $^+$-α-open set containing F and $V \cap$ (B (1, 2) $^+$-αcl(V))$^c = \phi$. Therefore X is B(1, 2) $^+$-πgα-regular.

(2)=> (3): Let $x \in X$ and F be a B(1, 2) $^+$-πgα-closed set such that $x \notin F$. Then F^c is a B (1, 2) $^+$-πgα-nbhd of x and by hypothesis, there exists a B (1,2) $^+$-α-open set V of X, such that B (1,2) $^+$-αcl(V) $\subseteq F^c$ and hence B (1,2) $^+$-α- cl(V) $\cap F = \phi$.

(3)=> (2): Let $x \in X$ and W be a B(1, 2) $^+$-πgα-nbhd of x. Then there exists a B (1, 2) $^+$-πgα-open set G such that $x \in G \subseteq W$. Since G^c is B (1, 2) $^+$-πgα-closed and $x \notin G^c$, by hypothesis there exists a B (1, 2) $^+$-α-open set U of x such that B (1, 2) $^+$-αcl(U) $\cap G^c = \phi$. Therefore, B(1, 2) $^+$-αcl(U) $\subseteq G \subseteq W$.

Theorem: 4.8 A bitopological space X is B (1, 2) $^+$-πgα-regular if and only if for each B (1, 2) $^+$-πgα-closed set F of X and each $x \in F^c$, there exist B (1, 2) $^+$-α-open sets U and V of X such that $x \in U$ and $F \subseteq V$ and B (1, 2) $^+$-αcl(U) $\cap$B (1, 2) $^+$-αcl(V) = ϕ.

Proof: Let F be a B (1, 2) $^+$-πgα-closed set of X and $x \notin F$. Then there exist B (1, 2) $^+$-α-open sets U_x and V suchthat $x \in U_x$, $F \subseteq V$ and $U_x \cap V = \phi$. Which implies that $U_x \cap$ B (1, 2) $^+$-αcl(V) = ϕ. Since X is B (1, 2) $^+$-πgα-regular, there exist B (1, 2) $^+$-α-open sets G and H of X such that $x \in G$, B (1, 2) $^+$-αcl(V) $\subseteq$ H and $G \cap H = \phi$. This implies B (1, 2) $^+$-αcl(G) $\cap H = \phi$. Now put $U = U_x \cap G$, then U and V are B (1, 2) $^+$-α-open sets of X such that $x \in U$ and $F \subseteq V$ and B (1, 2) $^+$-αcl(U) $\cap$ B (1, 2) $^+$-αcl(V) = ϕ.

Converse is obvious.

Definition: 4.9 A space X is said to be B (1, 2) $^{+}$-α-symmetric if for any distinct points x and y of X, $x \in$ B (1, 2) $^{+}$-αcl{y} implies that $y \in$ B (1, 2) $^{+}$-αcl{x}.

Definition: 4.10 A space X is called B (1, 2) $^{+}$-α-Urysohn if for every pair of points x, y $\in$ X, $x \neq y$ there exist $U \in$ B (1, 2) $^{+}$-αO(x), $V \in$ B (1, 2) $^{+}$-αO(y) such that B(1, 2) $^{+}$-αcl(U)∩B(1, 2) $^{+}$-αcl(U) = ϕ.

Theorem: 4.11 A space X is B(1, 2)$^{+}$-α-symmetric if and only if {x} is B (1, 2) $^{+}$-πgα-closed in X for each $x \in$ X.

Proof: Sufficiency part: Suppose $x \in$ B(1, 2)$^{+}$-α cl{y}. But $y \notin$ B(1, 2)$^{+}$-α cl{x}, then $y \in$ B (1, 2) $^{+}$-α cl{x}^{c}. Thus B (1, 2) $^{+}$-α cl{y} $\in$ B (1, 2)$^{+}$-α cl{x}, then x $\in$ B(1, 2)$^{+}$-α cl{x}, which is a contradiction.

Necessity Part: Suppose {x} $\subseteq$ O $\in \tau_{1,2}$. But {x} $\not\subset$ O then B(1, 2)$^{+}$-α cl{x} $\cap$ O^{c} = ϕ. Let $y \in$ B (1, 2)$^{+}$-α cl{x} $\cap$ O^{C} therefore $x \in$ B(1, 2)$^{+}$-α cl{y} $\subseteq$ O^{c} and $x \notin$ O, which is a contradiction. Hence the proof.

Theorem: 4.12 If a space X is B (1, 2) $^{+}$-πgα-regular and B (1, 2) $^{+}$-α-symmetric, then it is B (1, 2) $^{+}$-α-Urysohn.

Proof: Let x and y be any distinct points of X. Since X is B (1, 2) $^{+}$-α-symmetric, {x} and {y} are B (1, 2) $^{+}$-πgα-closed in X. By theorem 4. 8, there exists B (1, 2) $^{+}$-α-open sets U and V such that $x \in$ U and $y \in$ V and B (1, 2) $^{+}$-αcl (U) ∩ B (1, 2) $^{+}$-αcl (V) = ϕ. This implies X is B (1, 2) $^{+}$-α-Urysohn.

Theorem: 4.13 If a space X is B (1, 2) $^{+}$-πgα-regular and B (1, 2) $^{+}$-α-symmetric, then it is B (1, 2) $^{+}$-α-open-T_2.

Proof: Let x and y be any distinct points of X. Since X is B (1, 2) $^{+}$-α-symmetric, {x} and {y} are B (1, 2) $^{+}$-πgα-closed in X. Since X is B (1, 2) $^{+}$-πgα-regular, there exists disjoint B (1, 2) $^{+}$-α-open sets U and V such that {x} $\subseteq$ U and {y} $\subseteq$ V and U ∩ V= ϕ. This implies X is B (1, 2) $^{+}$-α-open-T_2.

Lemma: 4.14 If Y is a B (1, 2) $^{+}$-πgα-closed set of a space X and A is a B (1, 2) $^{+}$-πgα-closed set of the subspace Y, then A is B (1, 2) $^{+}$-πgα-closed set of X.

Proof: Let A $\subseteq$ O. Suppose O is B (1, 2) $^{+}$-α-open in X, then A $\subseteq$ Y∩O. Hence B (1, 2) $^{+}$-αcl(A) $\subseteq$ Y∩O. It follows that Y∩B (1, 2) $^{+}$-αcl (A) $\subseteq$ Y∩O and Y $\subseteq$ O $\cup$ [B (1, 2) $^{+}$-αcl (A)]^{c}. Therefore, B (1, 2) $^{+}$-αcl (A) $\subseteq$ B (1, 2) $^{+}$-αcl(Y) $\subseteq$ O $\cup$ [B (1, 2) $^{+}$-αcl (A)]^{c} and B (1, 2) $^{+}$-αcl (A) $\subseteq$ O. This shows that, A is B (1, 2) $^{+}$-πgα-closed set in X.

Theorem: 4.15 If X is a B (1, 2) $^{+}$-πgα-regular space and Y is a B (1, 2) $^{+}$-πgα-closed subset of X, then the subspace Y is B (1, 2) $^{+}$-πgα-regular.

Proof: Let A be any B (1, 2) $^{+}$-πgα-closed set of Y, $y \in$ Y \ A, by lemma 4. 14, A is B (1, 2) $^{+}$-πgα-closed set in X. Since X is B (1, 2) $^{+}$-πgα-regular, there exists disjoint B (1, 2) $^{+}$-α-open sets U and V of X such that $y \in$ U and A $\subseteq$ U. Therefore U∩Y and V∩Y are disjoint B (1, 2) $^{+}$-α-open sets of the subspace Y, such that $y \in$ U∩Y and A $\subseteq$ V∩Y. This implies that the subspace Y is B (1, 2) $^{+}$-πgα-regular.

Definition: 4.16 A map f : X→ Y is called M - B (1, 2) $^{+}$-α-open (resp. M-B (1, 2) $^{+}$-α-closed) if f(V) is B (1, 2) $^{+}$-α-open(resp. B (1, 2) $^{+}$-α-closed)set in Y for every B (1, 2) $^{+}$-α-open(resp. B (1, 2) $^{+}$-α-closed) set V of X.

Definition: 4.17 A map f: X → Y is called B (1, 2) $^{+}$-πgα-continuous (resp. B (1, 2) $^{+}$-πgα-irresolute) if f^{-1}(V) is B (1, 2) $^{+}$-πgα-openset in X, for every B $\tau_{1,2}$- (resp. B (1, 2) $^{+}$-πgα-) open set V of Y.

Theorem: 4.18 If f : X → Y is bijectiveB $\tau_{1,2}$-open and B (1, 2) $^{+}$-πgα-continuous, then it is B (1, 2) $^{+}$-πgα-irresolute.

Proof: Let A be a B (1, 2) $^{+}$-πgα-closed set in Y. Let f^{-1}(A) $\subseteq$ O, where O is B $\tau_{1,2}$-open in X. Therefore, A $\subset$ f (O), holds. Since f (O) is B $\tau_{1,2}$-open and A is B (1, 2) $^{+}$-πgα-closed in Y. B (1, 2) $^{+}$-αcl (A) $\subseteq$ f (O) holds. Hence f^{-1}(B (1, 2) $^{+}$-αcl (A)) $\subseteq$ O. Since f is B (1, 2) $^{+}$-πgα-continuous, B (1, 2) $^{+}$-αcl (A) is B (1, 2) $^{+}$-α-closed in Y, f^{-1}(B (1, 2) $^{+}$-αcl (A)) $\subseteq$ O. So B (1, 2) $^{+}$-αcl (f^{-1}(A)) $\subseteq$ O. Therefore, f^{-1}(A) is B (1, 2) $^{+}$-πgα-closed in X. Hence, f is B (1, 2) $^{+}$-πgα-irresolute.

Theorem: 4.19 If f: X → Y is an B $\tau_{1,2}$-open, B (1, 2) $^{+}$-πgα-continuous bijection and X is B (1, 2) $^{+}$-πgα-regular, then Y is B (1, 2) $^{+}$-πgα-regular.

Proof: Let F be any B (1, 2) $^{+}$-πgα-closed set in Y and $y \in$ Y \ F. Since f is B $\tau_{1,2}$-open, B (1, 2) $^{+}$-πgα-continuous bijective then f is B (1, 2) $^{+}$-πgα-irresolute. Hence f^{-1}(F) is B (1, 2) $^{+}$-πgα-closed in X, for every B (1, 2) $^{+}$-πgα-closed set F in Y. Let f(x) = y, we have $x \in$ X\f^{-1}(F). Since X is B (1, 2) $^{+}$-πgα-regular, there exist disjoint B $\tau_{1,2}$-open sets U and V such that $x \in U$, $f^{-1}(F) \subset V$. Since f is B $\tau_{1,2}$-open and bijective, $y \in f(U)$, $F \subset f(V)$. Hence $f(U) \cap f(V) = \phi$. Therefore, Y is B (1, 2) $^{+}$-πgα-regular.

Theorem: 4.20 If X is B (1, 2) $^{+}$-πgα-regular space and f: X → Y is bijective, B (1, 2) $^{+}$-πgα-irresolute and M- B (1, 2) $^{+}$-α-open, then Y is B (1, 2) $^{+}$-πgα-regular space.

Proof: Let $y \in Y$ and F be any B (1, 2) $^{+}$-πgα-closed subset of Y with $y \notin F$. Since f is B (1, 2) $^{+}$-πgα-irresolute, $f^{-1}(F)$ is B (1, 2) $^{+}$-πgα-closed set in X. Since f is bijective. Let f(x) = y, then $x \neq f^{-1}(y)$. By hypothesis, thereexist B (1, 2) $^{+}$-α-open sets U and V such that $x \in U$ and $f^{-1}(F) \subseteq V$. Since f is M- B (1, 2) $^{+}$-α-open and bijective. We have $y \in f(U)$ and $F \subseteq f(V)$ and $f(U) \cap f(V) = f(U \cap V) = \phi$. Hence Y is B (1, 2) $^{+}$-πgα-regular space.

Theorem: 4.21 If f: X → Y is B (1, 2) $^{+}$-πgα-irresolute and M- B (1, 2) $^{+}$-α-closed and A is a B (1, 2) $^{+}$-πgα-closed subset of X, then f (A) is B (1, 2) $^{+}$-πgα-closed.

Theorem: 4.22 Iff: X →Y is B (1, 2) $^{+}$-πgα-irresolute, M- B (1, 2) $^{+}$-α-closed and injective and Y is B (1, 2) $^{+}$-πgα-regular, then X is B (1, 2) $^{+}$-πgα-regular.

Proof: Let F be any B (1, 2) $^{+}$-πgα-closed set of X and $x \notin F$. Since f is (1, 2) *-πgα-irresolute, M-B (1, 2) $^{+}$-α-closed, by Theorem 4. 21., f (F) is B (1, 2) $^{+}$-πgα-closed inY and $f(x) \notin f(F)$. Since Y is B (1, 2) $^{+}$-πgα-regular and so there exist disjoint B (1, 2) $^{+}$-α-open sets U and V in Y such that $f(x) \in U$ and $f(F) \subseteq V$. By hypothesis, $f^{-1}(U)$ and $f^{-1}(V) \in$ B (1, 2) $^{+}$-αO(X) such that $x \in f^{-1}(U)$ and $F \subseteq f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \phi$. Therefore X is B (1, 2) $^{+}$-πgα-regular.

5. B(1, 2) $^{+}$-πgα-NORMAL SPACES

Definition: 5.1 A bitopological space X is called a B (1, 2) $^{+}$-πgα-normal, if for every pair of disjoint B (1, 2) $^{+}$-πgα-closed subsets A and B , there exist disjoint B (1, 2) $^{+}$-α-open sets U and V of X such that $A \subseteq U$ and $B \subseteq V$.

Theorem: 5.2 Let X be a bitopological space. Then the following statements are equivalent:

(1) X is B (1, 2) $^{+}$-πgα-normal.

(2) For each B (1, 2) $^{+}$-πgα-closed set F and for each B (1, 2) $^{+}$-πgα-open set U containing F, there exists a B (1, 2) $^{+}$-α-open set V containing F such that B (1, 2) $^{+}$-αcl(V) $\subseteq$ U.

(3) For each pair of disjoint B (1, 2) $^{+}$-πgα-closed set A and B in X, there exists a B (1, 2) $^{+}$-α-open set U containing A such that B (1, 2) $^{+}$-αcl (U)∩B = ϕ.

(4) For each pair of disjoint B (1, 2) $^{+}$-πgα-closed set A and B in X, there exists a B (1, 2) $^{+}$-α- open sets U and V such that $A \subseteq U$, $B \subseteq V$ and B (1, 2) $^{+}$-α-cl(A) ∩ B (1, 2) $^{+}$-α-cl(B) = ϕ.

Proof:

(1) → (2): Let F be a B (1, 2) $^{+}$-πgα-closed set and U be a B (1, 2) $^{+}$-πgα-open set such that $F \subseteq U$. Then $F \cap U^c = \phi$. By assumption, there exist B (1, 2) $^{+}$-α-open sets V and W such that $F \subseteq U$, $U^c \subseteq W$ and $V \cap W = \phi$, which implies B (1, 2) $^{+}$-α-cl(V) ∩ W = ϕ. Now B (1,2) $^{+}$-α-cl (V) $\cap U^c \subseteq$ B (1, 2) $^{+}$-α-cl (V) ∩W = ϕ and so B (1, 2) $^{+}$-α-cl(V) $\subseteq$ U.

(2) → (3): Let A and B be disjoint B (1, 2) $^{+}$-πgα-closed sets of X. Since $A \cap B = \phi$, $A \subseteq B^c$ and B^c is B (1, 2) $^{+}$-πgα-open. By assumption, there exists B (1, 2) $^{+}$-α-open set U containing A such that B (1, 2) $^{+}$-αcl(U) $\subseteq B^c$ and so B (1, 2) $^{+}$-αcl(U)∩B = ϕ.

(3) → (4): Let A and B be B (1, 2) $^{+}$-πgα-closed sets of X. Then by assumption, there exists B (1, 2) $^{+}$-α-open set U containing A such that B (1, 2) $^{+}$-αcl(U)∩B = ϕ. Since B (1, 2) $^{+}$-αcl(A) is B (1, 2) $^{+}$-α-closed, it is B (1, 2) $^{+}$-πgα-closed and so B and B (1, 2) $^{+}$-αcl(A) are disjoint B (1, 2) $^{+}$-πgα-closed sets in X. Therefore again by assumption, there exists a B (1, 2) $^{+}$-α-open set V containing B such that B (1, 2) $^{+}$-αcl(A) ∩ B (1, 2) $^{+}$-αcl(B) = ϕ.

(4) → (1): Let A and B be any disjoint B (1, 2) $^{+}$-πgα-closed sets of X. By assumption, there exist B (1, 2) $^{+}$-α-open sets U and V such that $A \subseteq U$, $B \subseteq V$ and B (1, 2) $^{+}$-αcl(A) ∩B (1, 2) $^{+}$-αcl(B) = ϕ, we have $U \cap V = \phi$ and thus X is B (1, 2) $^{+}$-πgα-normal.

Theorem: 5.3 If f: X → Y is bijective, B (1, 2) $^{+}$-πgα-irresolute and M- B (1, 2) $^{+}$-α-open mapping and X is B (1, 2) $^{+}$-πgα-normal space, then Y is B (1, 2) $^{+}$-πgα-normal space.

Proof: Let A and B be any disjoint B(1, 2) $^{+}$-πgα-closed subset of Y. Since f is B (1, 2)$^{+}$-πgα-irresolute, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint B (1, 2) $^{+}$-πgα-closed set of X. As X is B (1, 2) $^{+}$-πgα-normal space, there exist disjoint B (1, 2) $^{+}$-α-open sets U and V such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$. Since f is M- B (1, 2) $^{+}$-α-open and bijective. We have f(U) and f(V) are B (1, 2) $^{+}$-α-open sets in Y such that $A \subseteq f(U)$ and $B \subseteq f(V)$ and $f(U) \cap f(V) = \phi$. Hence Y is B (1, 2) $^{+}$-πgα-normal space.

Theorem: 5.4 If f: X →Y is B (1, 2) $^{+}$-πgα-irresolute, M- B (1, 2) $^{+}$-α-closed and B (1, 2) $^{+}$-α-irresolute injective and Y is B (1, 2) $^{+}$-πgα-normal, then X is B (1, 2) $^{+}$-πgα-normal.

Proof: Let A and B be any two disjoint B (1, 2) $^{+}$-πgα-closed sets of X. Since the map f is B (1, 2) $^{+}$-πgα-irresolute, M- B (1, 2) $^{+}$-α-closed, by Theorem 4. 12., f (A) and f (B) are disjoint B (1, 2) $^{+}$-πgα-closed sets of Y. Since Y is B (1, 2) $^{+}$-πgα-normal, there exist disjoint B (1, 2) $^{+}$-α-open sets U and V in Y such that $f(A) \subseteq U$ and $f(B) \subseteq V$. That is, $A \subseteq f^{-1}(U)$ and $B \subseteq f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \phi$. Since f is B (1, 2) $^{+}$-α-irresolute, $f^{-1}(U)$ and $f^{-1}(V)$ are B (1, 2) $^{+}$-α-open sets in X, we have X is B (1, 2) $^{+}$-πgα-normal space.

Theorem: 5.5 Suppose that $B \subseteq A \subseteq X$, B is B (1, 2) $^{+}$-πgα-closed set relative to A and that A is B (1, 2) $^{+}$-α-open and B (1, 2) $^{+}$-πgα-closed in X. Then B is B (1, 2) $^{+}$-πgα-closed in X.

Theorem: 5.6 If X is B (1, 2) $^{+}$-πgα-normal space and Y is an B (1, 2) $^{+}$-α-open and B (1, 2) $^{+}$-πgα-closed subset of X, then the subspace Y is B (1, 2) $^{+}$-πgα-normal.

Proof: Let A and B be any two disjoint B (1, 2) $^{+}$-πgα-closed sets of Y. By theorem 5. 5 and A and B are B (1, 2) $^{+}$-πgα-closed in X. Since X is B (1, 2) $^{+}$-πgα-normal, there exists disjoint B (1, 2) $^{+}$-α-open sets U and V of X such that $A \subseteq U$ and $B \subseteq V$. Since Y is B (1, 2) $^{+}$-α-open, $U \cap Y$ and $V \cap Y$ are disjoint B (1, 2) $^{+}$-α-open sets of the subspace Y. Hence the subspace Y is B (1, 2) $^{+}$-πgα-normal.

Theorem: 5.7 If f: X → Y is bijective, B (1, 2) $^{+}$-πgα-continuous and M- B (1, 2) $^{+}$-α-open mapping and X is B (1, 2) $^{+}$-πgα-normal space, then Y is B (1, 2) $^{+}$-πgα-normal space.

Proof: Let A and B be any disjoint B (1, 2) $^{+}$-πgα-closed subset of Y. The map f is B (1, 2) $^{+}$-πgα-irresolute by theorem 4. 18 and so $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint B (1, 2) $^{+}$-πgα-closed set of X. Since X is B (1, 2) $^{+}$-πgα-normal space, there exist disjoint B (1, 2) $^{+}$-α-open sets U and V such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$. Since f is M- B (1, 2) $^{+}$-α-open and bijective. We have f(U) and f(V) are B (1, 2) $^{+}$-α-open sets in Y such that $A \subseteq f(U)$ and $B \subseteq f(V)$ and $f(U) \cap f(V) = \phi$. Hence Y is B (1, 2) $^{+}$-πgα-normal space.

ACKNOWLEDGEMENT

The authors gratefully acknowledge the University Grants Commission, India for the financial support.

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Source of support: University Grants Commission, India, Conflict of interest: None Declared