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INTUITIONISTIC FUZZY COMPLETELY REGULAR WEAKLY CONTINUOUS MAPPINGS

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ABSTRACT

In this paper we introduce intuitionistic fuzzy completely regular weakly continuous mappings and some of their properties are studied.

Key words and phrases: *Intuitionistic fuzzy topology, intuitionistic fuzzy point, intuitionistic fuzzy regular weakly closed set intuitionistic fuzzy regular weakly open set and intuitionistic fuzzy completely regular weakly continuous mappings.*

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1. INTRODUCTION

In 1965, Zadeh [16] introduced fuzzy sets and in 1968, Chang [2] introduced fuzzy topology. After the introduction of fuzzy sets and fuzzy topology, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In 1997 Coker [3] introduced the concept of intuitionistic fuzzy topological spaces. In this present paper, we introduce the concepts of intuitionistic fuzzy completely regular weakly continuous mappings in intuitionistic fuzzy topological space.

2. PRELIMINARIES

Throughout this paper, (X, τ) or X denotes the intuitionistic fuzzy topological spaces (briefly IFTS). For a subset A of X, the closure, the interior and the complement of A are denoted by cl(A), int(A) and A^c respectively. We recall some basic definitions that are used in the sequel.

Definition: 2.1 [1] Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition: 2.2 [1] Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in X\}$. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$,
(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(iii) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in X\}$,
(iv) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X\}$,
(v) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X\}$.

For the sake of simplicity, we shall use the notation $A= \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B)\rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)\rangle$. The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle / x \in X\}$ are respectively the empty set and the whole set of X.

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Definition: 2.3 [3] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,
(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,
(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition: 2.4 [3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then

(i) $int(A) = \cup\{ G / G$ is an IFOS in X and $G \subseteq A\}$,
(ii) $cl(A) = \cap\{ K / K$ is an IFCS in X and $A \subseteq K\}$,
(iii) $cl(A^c) = (int(A))^c$,
(iv) $int(A^c) = (cl(A))^c$.

Definition: 2.5 [5] An IFS A of an IFTS (X, τ) is an

(i) intuitionistic fuzzy semiclosed set (IFSCS for short) if $int(cl(A)) \subseteq A$,
(ii) intuitionistic fuzzy semiopen set (IFSOS for short) if $A \subseteq cl(int(A))$.

Definition: 2.6 [5] An IFS A of an IFTS (X, τ) is an

(i) intuitionistic fuzzy regular closed (IFRCS for short) if $cl(int(A)) = A$,
(ii) intuitionistic fuzzy regular open (IFROS for short) if $int(cl(A)) = A$,
(iii) intuitionistic fuzzy preclosed set (IFPCS for short) if $cl(int(A)) \subseteq A$,
(iv) intuitionistic fuzzy preopen set (IFPOS for short) if $A \subseteq int(cl(A))$.

Note that every IFOS in (X, τ) is an IFPOS in X.

Definition: 2.7 [15] An IFS A of an intuitionistic fuzzy topological space (X, τ) is called an intuitionistic fuzzy regular semi open (IFRSOS for short) if there is a regular open set U such that $U \subseteq A \subseteq cl(U)$.

Definition: 2.8 [5] If A is an IFS in intuitionistic fuzzy topological space(X, τ) then

(i) $scl(A) = \cap\{F: A \subseteq F$, F is intuitionistic fuzzy semi closed$\}$,
(ii) $pcl(A) = \cap\{F: A \subseteq F$, F is intuitionistic fuzzy pre closed$\}$.

Definition: 2.9 An IFS A of an intuitionistic fuzzy topological space (X, τ) is called:

(i) Intuitionistic fuzzy g-closed (IFGCS for short) if $cl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy open.[8]
(ii) Intuitionistic fuzzy g-open (IFGOS for short) if its complement A^c is intuitionistic fuzzy g-closed.[8]
(iii) Intuitionistic fuzzy rg-closed (IFRGCS for short) if $cl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy regular open.[10]
(iv) Intuitionistic fuzzy rg-open (IFRGOS for short) if its complement A^c is intuitionistic fuzzy rg-closed.[10]
(v) Intuitionistic fuzzy w-closed (IFWCS for short) if $cl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy semi open.[12]
(vi) Intuitionistic fuzzy w-open (IFWOS for short) if its complement A^c is intuitionistic fuzzy w-closed.[12]
(vii) Intuitionistic fuzzy gpr-closed (IFGPROS for short) if $pcl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy regular open.[14]
(viii) Intuitionistic fuzzy gpr-open (IFGPROS for short) if its complement A^c is intuitionistic fuzzy gpr-closed. [14]

Remark: 2.10 Every intuitionistic fuzzy closed set is intuitionistic fuzzy g-closed but its converse may not be true. [8]

Remark: 2.11 Every intuitionistic fuzzy g-closed set is intuitionistic fuzzy rg-closed but its converse may not be true. [10]

Remark: 2.12 Every intuitionistic fuzzy w-closed (resp. Intuitionistic fuzzy w-open) set is intuitionistic fuzzy g-closed (intuitionistic fuzzyg-open) but its conversemay not be true. [12]

Remark: 2.13 Every intuitionistic fuzzy g-closed (resp. Intuitionistic fuzzy g-open) set is intuitionistic fuzzy gpr-closed (intuitionisticfuzzy gpr-open) but its converse may not be true. [14]

Definition: 2.14 [13] An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is called an intuitionisticfuzzy rw-closed (IFRWCS for short) if $cl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy regular semi open in X.

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Every IFCS, IFGCS, IFWCS is an IFRWCS and every IFRWCS is an IFRGCS and IFGPRCS but the converses may not be true in general.

Definition: 2.15 [13] An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is called intuitionistic fuzzy rw-open (IFRWOS for short) if and only if its complement A^c is intuitionistic fuzzy rw-closed.

Definition: 2.16 [5] Let (X, τ) and (Y, σ) be two intuitionistic fuzzy topological spaces and let f: X → Y be a function. Then f is said to be an intuitionistic fuzzy continuous if the pre image of each intuitionistic fuzzy closed set in Y is an intuitionistic fuzzy closed set in X.

Definition: 2.17 Let (X, τ) and (Y, σ) be two intuitionistic fuzzy topological spaces and let f: X→Y be a function. Then f is said to be an

(i) intutionistic fuzzy g-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy g-closed in X. [9]
(ii) intutionistic fuzzy w-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy w-closed in X. [12]
(iii) intutionistic fuzzy rg-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy rg-closed in X. [11]
(iv) intutionistic fuzzy gpr-continuous if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy gpr-closed in X. [14]

Remark: 2.18 Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy gcontinuous, but the converse may not be true [9].

Remark: 2.19 Every intuitionistic fuzzy w-continuous mapping is intuitionistic fuzzy gcontinuous, but the converse may not be true [12].

Remark: 2.20 Every intuitionistic fuzzy g-continuous mapping is intuitionistic fuzzy rgcontinuous, but the converse may not be true [8].

Remark: 2.21 Every intuitionistic fuzzy g-continuous mapping is intuitionistic fuzzy gpr-continuous, but the converse may not be true [14].

Definition: 2.22 [4] Let α, β ∈ [0, 1] and α+ β≤ 1. An intuitionistic fuzzy point (IFP for short) $p_{(\alpha, \beta)}$ of X is an IFS of X defined by

$$p_{(\alpha, \beta)}(y) = \begin{cases} (\alpha,\beta) & if \quad y = x \\ (0,1) & if \quad y \neq x \end{cases}$$

Definition: 2.23 [7] Let $p_{(\alpha, \beta)}$ be an IFP of an IFTS (X, τ). An IFS A of X is called an intuitionistic fuzzy neighborhood (IFN for short) of $p_{(\alpha, \beta)}$ if there exists an IFOS B in X such that $p_{(\alpha, \beta)} \in B \subseteq A$.

Definition: 2.24 [6] An IFS A is said to be intuitionistic fuzzy dense (IFD for short) in another IFS B in an IFT (X, τ) if cl(A) = B.

Definition: 2.25 [5] Two IFSs A and B are said to be q-coincident ($A_q B$ in short) if and only if there exists and element x ∈ X such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

3. INTUITIONISTIC FUZZY COMPLETELY REGULAR WEAKLY CONTINUOUS MAPPINGS

In this section, we introduced intuitionistic fuzzy completely regular weakly continuous mappings and studied some of its properties.

Definition: 3.1 A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy completely regular weakly continuous (IFcRW continuous in short) mapping if $f^{-1}(B)$ is an IFRCS in (X, τ) for every IFRWCS B of (Y, σ).

For the sake of simplicity, we shall use the notation A= ⟨x, (μ, μ), (ν, ν)⟩ instead of A= $\langle x,(a/\mu_a, b/\mu_b), (a/\nu_a, b/\nu_b)\rangle$ in all the examples used in this paper. Similarly we shall use the notation B= ⟨x, (μ, μ), (ν, ν)⟩ instead of B= $\langle x,(u/\mu_u, v/\mu_v), (u/\nu_u, v/\nu_v)\rangle$ in the following examples.

Example: 3.2 Let X = {a, b}, Y = {u, v} and G_1= ⟨x, (0.3, 0.4), (0.7, 0.6)⟩, G_2 = ⟨y, (0.4, 0.4), (0.6, 0.6)⟩. Then τ = $\{0_\sim, G_1, 1_\sim\}$ and σ = $\{0_\sim, G_2, 1_\sim\}$ are IFT on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u

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and f(b) = v. Then f is an IFcRW continuous mapping, since B = ⟨y, (0.7, 0.6), (0.3, 0.4)⟩ is an IFRWCS in Y then f^{-1}(B) = ⟨x, (0.7, 0.6), (0.3, 0.4)⟩ is IFRCS in X.

Theorem: 3.3 Every IFcRW continuous mapping is an IF continuous mapping but not conversely.

Proof: Assume that f: (X, τ) → (Y, σ) is an IFcRW continuous mapping. Let B be an IFCS in Y. Then B is an IFRWCS in Y. Since f is an IFcRW continuous mapping, f^{-1}(B) is an IFRCS in X. This implies f^{-1}(B) is an IFCS in X. Hence the mapping f is an IF continuous mapping.

Example: 3.4 Let X = {a, b}, Y = {u, v} and G_1= ⟨x, (0.2, 0.3), (0.7, 0.5)⟩, and G_2 = ⟨y, (0.2, 0.3), (0.7, 0.5) ⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF continuous mapping. But f is not an IFcRW continuous mapping, since B = ⟨y, (0.8, 0.5), (0.2, 0.5) ⟩ is an IFRWCS in Y but f^{-1}(B) = ⟨x, (0.8, 0.5), (0.2, 0.5)⟩ is not an IFRCS in X.

Theorem: 3.5 Every IFcRW continuous mapping is an IFG continuous mapping but not conversely.

Proof: Assume that f: (X, τ) → (Y, σ) is an IFcRW continuous mapping. Let B be an IFCS in Y. This implies B is an IFRWCS in Y. Since f is an IFcRWG continuous mapping, f^{-1}(B) is an IFRCS in X. This implies f^{-1}(B) is an IFGCS in X. Hence f is an IFG continuous mapping.

Example: 3.6 Let X = {a, b}, Y = {u, v} and G_1= ⟨ x, (0.3, 0.4), (0.7, 0.6)⟩, and G_2 = ⟨y, (0.4, 0.4), (0.6, 0.6) ⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$ } are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFG continuous mapping. But f is not an IFcRW continuous mapping, since B =⟨y, (0.9, 0.3), (0.1, 0.7) ⟩ is an IFRWCS in Y but f^{-1}(B) = ⟨x, (0.9, 0.3), (0.1, 0.7)⟩ is not an IFRCS in X.

Theorem: 3.7 Every IFcRW continuous mapping is an IFW continuous mapping but not conversely.

Proof: Assume that f: (X, τ) → (Y, σ) is an IFcRW continuous mapping. Let B be an IFCS in Y. This implies B is an IFRWCS in Y. Since f is an IFcRW continuous mapping, f^{-1}(B) is an IFRCS in X. This implies f^{-1}(B) is an IFWCS in X. Hence f is an IFW continuous mapping.

Example: 3.8 Let X = {a, b}, Y = {u, v} and G_1 =⟨x, (0.5, 0.3), (0.2, 0.6)⟩ and G_2 = ⟨y, (0.5, 0.3), (0.2, 0.6)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b)=v. Then f is an IFW continuous mapping. But f is not an IFcRW continuous mapping, since B = ⟨y, (0.2, 0.4), (0.8, 0.5) ⟩ is an IFRWCS in Y but f^{-1}(B) = ⟨x, (0.2, 0.4), (0.8, 0.5)⟩ is not an IFRCS in X.

Theorem: 3.9 Every IFcRW continuous mapping is an IFRG continuous mapping but not conversely.

Proof: Assume that f: (X, τ) → (Y, σ) is an IFcRW continuous mapping. Let B be an IFCS in Y. This implies B is an IFRWCS in Y. Since f is an IFcRW continuous mapping, f^{-1}(B) is an IFRCS in X. This implies f^{-1}(B) is an IFRGCS in X. Hence f is an IFRG continuous mapping.

Example: 3.10 Let X = {a, b}, Y = {u, v} and G_1= ⟨x, (0.5, 0.7), (0.5, 0.3)⟩ and G_2 = ⟨y, (0.2, 0.3), (0.7, 0.7) ⟩. Then we define τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFRG continuous mapping. But f is not an IFcRW continuous mapping since B = ⟨y, (0.8, 0.1), (0.2, 0.8)⟩ is an IFRWCS in Y but f^{-1}(B) = ⟨x, (0.8, 0.1), (0.2, 0.8) ⟩ is not an IFRCS in X.

Theorem: 3.11 Every IFcRW continuous mapping is an IFGPR continuous mapping but not conversely.

Proof: Assume that f: (X, τ) → (Y, σ) is an IFcRW continuous mapping. Let B be an IFCS in Y. This implies B is an IFRWCS in Y. Since f is an IFcRW continuous mapping, f^{-1}(B) is an IFRCS in X. This implies f^{-1}(B) is an IFGPRCS in X. Hence f is an IFGPR continuous mapping.

Example: 3.12 Let X = {a, b}, Y= {u, v} and G_1 = ⟨x, (0.5, 0.3), (0.5, 0.7)⟩ and G_2 = ⟨y, (0.3, 0.2), (0.7, 0.8)⟩. Then we define τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFGPR continuous mapping. But f is not an IFcRW continuous mapping since B = ⟨y, (0.8, 0.3), (0.2, 0.2)⟩ is an IFRWCS in Y but f^{-1}(B) = ⟨x, (0.8, 0.3), (0.2, 0.2)⟩ is not an IFRCS in X.

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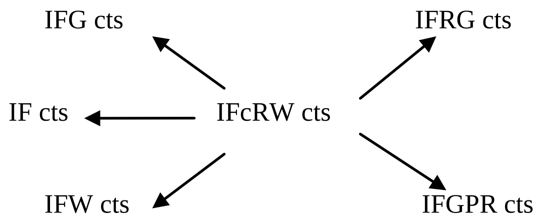
The relation between various types of intuitionistic fuzzy continuity is given in the following diagram. In this diagram 'cts' means continuous.



The reverse implications are not true in general in the above diagram.

Definition: 3.13 A mapping f: $(X, \tau)\to (Y, \sigma)$ is called an intuitionistic fuzzy regular weakly irresolute (IFRW irresolute in short) mapping if $f^{-1}(B)$ is an IFRWCS in (X, τ) for every IFRWCS B of (Y, σ).

Example: 3.14 Let X = {a, b}, Y = {u, v} and $G_1= \langle x, (0.3, 0.4), (0.7, 0.6)\rangle$, and $G_2 = \langle y, (0.4, 0.4), (0.6, 0.6)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFcRW irresolute mapping, since $B =\langle y, (0.9, 0.3), (0.1, 0.7) \rangle$ is an IFRWCS in Y then $f^{-1}(B) = \langle x, (0.9, 0.3), (0.1, 0.7)\rangle$ is an IFRWCS in X.

Theorem: 3.15 Every IFcRW continuous mapping is an IFRW irresolute mapping.

Proof: Assume that f: X → Y is an IFcRW continuous mapping. Let B be an IFRWCS in Y. Since f is an IFcRW continuous mapping, $f^{-1}(B)$ is an IFRCS in X. This implies $f^{-1}(B)$ is an IFRWCS in X. Hence f is an IFRW irresolute mapping.

Theorem: 3.16 A mapping f: X →Y is an IFcRW continuous mapping if and only if the inverse image of each IFRWOS in Y is an IFROS in X.

Proof: Necessity: Let A be an IFRWOS in Y. This implies A^c is an IFRWCS in Y. Since f is an IFcRW continuous, $f^{-1}(A^c)$ is IFRCS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFROS in X.

Sufficiency: Let A be an IFRWCS in Y. This implies A^c is an IFRWOS in Y. By hypothesis $f^{-1}(A^c)$ is an IFROS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFRCS in X. Hence f is an IFcRW continuous mapping.

Theorem: 3.17 Let $p_{(\alpha, \beta)}$ be an IFP in an IFTS (X, τ). A mapping f : X → Y is an IFcRW continuous mapping if for every IFcRWOS A in Y with $f(p_{(\alpha, \beta)}) \in A$, there exists an IFROS B in X with $p_{(\alpha, \beta)} \in$ B such that $f^{-1}(A)$ is IFD in B.

Proof: Let A be an IFRWOS in Y and let $f(p_{(\alpha, \beta)}) \in A$. Then there exists an IFROS B in X with $p_{(\alpha, \beta)} \in B$ such that $cl(f^{-1}(A)) = B$. Since B is an IFROS, $cl(f^{-1}(A))$ is also an IFROS in X. Therefore, $int(cl(cl(f^{-1}(A)))) = cl(f^{-1}(A))$. That is $int(cl(f^{-1}(A))) = cl(f^{-1}(A))$. This implies $f^{-1}(A)$ is also an IFROS in X.

Theorem: 3.18 If a mapping f: X →Y is an IFcRW continuous mapping, then for every IFP $p_{(\alpha, \beta)} \in X$ and for every IFN A of $f(p_{(\alpha, \beta)})$, there exists an IFROS $B \subseteq X$ such that $p_{(\alpha, \beta)} \in B \subseteq f^{-1}(A)$.

Proof: Let $p_{(\alpha, \beta)} \in X$ and let A be an IFN of $f(p_{(\alpha, \beta)})$. Then there exists an IFOS C in Y such that $f(p_{(\alpha, \beta)}) \in C \subseteq A$. Since every IFOS is an IFRWOS, C is an IFRWOS in Y. Hence by hypothesis, $f^{-1}(C)$ is an IFROS in X and $p_{(\alpha, \beta)} \in f^{-1}(C)$. Now, let $f^{-1}(C) = B$. Therefore $p_{(\alpha, \beta)} \in B \subseteq f^{-1}(A)$.

Theorem: 3.19 If a mapping f: X →Y is an IFcRW continuous mapping, then for every IFP $p_{(\alpha, \beta)} \in X$ and for every IFN A of $f(p_{(\alpha, \beta)})$, there exists an IFROS $B \subseteq X$ such that $p_{(\alpha, \beta)} \in B$ and $f(B) \subseteq A$.

Proof: Let $p_{(\alpha, \beta)} \in X$ and let A be an IFN of $f(p_{(\alpha, \beta)})$. Then there exists an IFOS C in Y such that $f(p_{(\alpha, \beta)}) \in C \subseteq A$. Since every IFOS is an IFRWOS, C is an IFRWOS in Y. Hence by hypothesis, $f^{-1}(C)$ is an IFROS in X and $p_{(\alpha, \beta)} \in f^{-1}(C)$. Now let $f^{-1}(C) = B$. Therefore $p_{(\alpha, \beta)} \in B \subseteq f^{-1}(A)$. Thus $f(B) \subseteq f(f^{-1}(A)) \subseteq A$, which implies $f(B) \subseteq A$.

Theorem: 3.20 A mapping f: X → Y is an IFcRW continuous mapping then $int(cl(f^{-1}(int(B)))) \subseteq f^{-1}(B)$ for every IFS B in Y.

Proof: Let B ⊆ Y be an IFS. Then, int(B) is an IFOS in Y and hence an IFRWOS in Y. By hypothesis, $f^{-1}(int(B))$ is an IFROS in X. Hence $int(cl(f^{-1}(int(B)))) = f^{-1}(int(B)) \subseteq f^{-1}(B)$.

Theorem: 3.21 A mapping f: X → Y is an IFcRW continuous mapping then the following are equivalent:

(i) for any IFRWOS A in Y and for any IFP $p_{(\alpha, \beta)} \in X$, if $f(p_{(\alpha, \beta)})\ _qA$ then $p_{(\alpha, \beta)\ q}\ int(f^{-1}(A))$.

(ii) for any IFRWOS A in Y and for any $p_{(\alpha, \beta)} \in X$, if $f(p_{(\alpha, \beta)})\ _qA$ then there exists an IFOS B such that $p_{(\alpha, \beta)\ q}B$ and $f(B) \subseteq A$.

Proof:

(i) → (ii): Let A⊆Y be an IFRWOS and let $p_{(\alpha, \beta)} \in X$. Let $f(p_{(\alpha, \beta)})\ _q A$. Then (i) implies that $p_{(\alpha, \beta)q}\ int(f^{-1}(A))$, where $int(f^{-1}(A))$ is an IFOS in X. Let $B = int(f^{-1}(A))$. Since $int(f^{-1}(A)) \subseteq f^{-1}(A), B \subseteq f^{-1}(A)$. Then, $f(B) \subseteq f(f^{-1}(A)) \subseteq A$.

(ii) → (i): Let A⊆Y be an IFRWOS and let $p_{(\alpha, \beta)} \in X$. Suppose $f(p_{(\alpha, \beta)})\ _qA$, then by (ii) there exists an IFOS B in X such that $p_{(\alpha, \beta)\ q}B$ and f(B)⊆A. Now $B \subseteq f^{-1}(f(B)) \subseteq f^{-1}(A)$. That is $B = int(B) \subseteq int(f^{-1}(A))$. Therefore, $p_{(\alpha, \beta)\ q}B$ implies $p_{(\alpha, \beta)\ q}\ int(f^{-1}(A))$.

Theorem: 3.22 For any two IFcRW continuous mappings $f_1$and f_2 : (X, τ)→(Y, σ), the function (f_1, f_2): (X, τ)→(Y x Y, σ x σ) is also an IFcRW continuous mapping where $(f_1, f_2)(x) = (f_1(x), f_2(x))$ for every x ∈ X.

Proof: Let AxB be an IFRWOS in YxY. Then $(f_1, f_2)^{-1}(AxB)(x) = (AxB)\ (f_1(x),f_2(x)) = \langle(x,\ \min(\mu_A(f_1(x)), \mu_B(f_2(x))), \max(\nu_A(f_1(x)), \nu_B(f_2(x)))\rangle = \langle(x, \min(f_1^{-1}(\mu_A(x)), f_2^{-1}(\mu_B(x))), \max(f_1^{-1}(\nu_A(x)), f_2^{-1}(\nu_B(x)))\rangle = f_1^{-1}(A) \cap f_2^{-1}(B)(x)$. Since $f_1$and f_2 are IFcRW continuous mappings, $f_1^{-1}(A)$ and $f_2^{-1}(B)$ are IFROSs in X. Since intersection of IFROSs is an IFROS, $f_1^{-1}(A) \cap f_2^{-1}(B)$ is an IFROS in X. Hence (f_1, f_2) is an IFcRW continuous mappings.

Theorem: 3.23 Let f: X → Y be a mapping. Then the following are equivalent.

(i) f is an IFcRW continuous mapping.

(ii) $f^{-1}(B)$ is an IFROS in X for every IFRWOS B in Y.

(iii) for every IFP $p_{(\alpha, \beta)} \in X$ and for every IFRWOS B in Y such that $f(p_{(\alpha, \beta)}) \in B$ there exists an IFROS A in X such that $p_{(\alpha, \beta)} \in A$ and $f(A) \subseteq B$.

Proof:

(i) → (ii): is obviously.

(ii) → (iii): Let $p_{(\alpha, \beta)} \in X$ and B⊆ Y such that $f(p_{(\alpha, \beta)}) \in B$. This implies $p_{(\alpha, \beta)} \in f^{-1}(B)$. Since B is an IFRWOS B in Y, by hypothesis $f^{-1}(B)$ is an IFROS in X. Let $A = f^{-1}(B)$. Then $p_{(\alpha, \beta)} \subseteq f^{-1}(f(p_{(\alpha, \beta)})) \subseteq f^{-1}(B) = A$. Therefore $p_{(\alpha, \beta)} \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$. This implies $f(A) \subseteq B$.

(iii) → (i): Let B ⊆ Y be an IFRWOS. Let $p_{(\alpha, \beta)} \in X$ and $f(p_{(\alpha, \beta)}) \in B$. By hypothesis, there exists an IFROS C in X such that $p_{(\alpha, \beta)} \in C$ and $f(C) \subseteq B$. This implies $C \subseteq f^{-1}(f(C)) \subseteq f^{-1}(B)$. Therefore, $p_{(\alpha, \beta)} \in C \subseteq f^{-1}(B)$. That is $f^{-1}(B) = \cup_{p(\alpha, \beta) \in f^{-1}(B)}\ p_{(\alpha, \beta)} \subseteq \cup_{p(\alpha, \beta) \in f^{-1}(B)} C \subseteq f^{-1}(B)$. This implies $f^{-1}(B) = \cup_{p(\alpha, \beta) \in f^{-1}(B)} C$. Since union of IFROSs is IFROS, $f^{-1}(B)$ is an IFROS in X. Hence f is an IFcRW continuous mapping.

4. CONCLUSION

In this paper we have introduced intuitionistic fuzzy completely regular weakly continuous mapping and studied some of its basic properties. Also we have studied the relationship between intuitionistic fuzzy completely regular weakly continuous mappings and some of the intuitionistic fuzzy continuous mappings that already exist.

REFERENCES

[1] Atanassov, K. Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems, 20, 1985, 87–96.
[2] Chang, C. L. Fuzzy Topological Spaces, J. Math. Anal. Appl. Vol. 24, 1968, 182–190.
[3] Çoker, D. An Introduction to Intuitionistic Fuzzy Topological Spaces, Fuzzy Sets and Systems, Vol. 88, 1997, 1–89.
[4] Çoker D., M. Demirci. On Intuitionistic Fuzzy Points. Notes on Intuitionistic Fuzzy Sets, Vol. 1, 1995, No. 2, 79–84.
[5] Gurcay, H., D. Çoker, A. Es. Haydar. On Fuzzy Continuity in Intuitionistic Fuzzy Topological Spaces. The Journal of FuzzyMathematics, Vol. 5, 1997, No. 2, 365–378.
[6] R. Santhi and D. Jayanthi, Intuitionistic fuzzy almost generalized semi-pre continuous mappings, Tamkang Journal of Mathematics, Vol. 42, 2011, No.2, 175-191.
[7] Seok Jong Lee and Eun Pyo Lee, The category of intuitionistic fuzzy topological spaces, Bull. Korean Math. Soc. 37, No. 1, 2000, pp. 63-76.

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[8] Thakur S. S., R. Chaturvedi. Generalized closed set in intuitionistic fuzzy topology. The Journal of Fuzzy Mathematics, Vol. 16, 2008, No. 3, 559–572.

[9] Thakur, S. S., R. Chaturvedi. Generalized Continuity in intuitionistic fuzzy topological spaces. Notes on Intutionistic Fuzzy Sets. Vol. 12, 2006, No. 1, 38–44.

[10] Thakur, S. S., R. Chaturvedi. Regular generalized closed sets in intuitionistic fuzzy topology, Studii Si Cercetari Stiintifice Seria Mathematica, Vol.16, 2010, 257–272.

[11] Thakur, S. S., R. Chaturvedi. Intutionisticfuzzy rg-continuous mappings, Journal of Indian Academy of Mathematics, Vol. 29, 2007, No. 2, 467–473.

[12] Thakur, S. S., Jyoti Pandey Bajpai, Intuitionistic Fuzzy w-closed sets and intuitionistic fuzzy w-continuity, International Journal of Contemporary Advanced Mathematics, Vol. 1, 2010, No. 1, 1–15.

[13] Thakur, S. S., Jyoti Pandey Bajpai, Intuitionistic Fuzzy rw-closed sets and intuitionistic fuzzy rw-continuity, Fifteenth Int. Conf. on IFSs, Burgas, NIFS 17(2011), 2, 82-96.

[14] Thakur, S. S., Jyoti Pandey Bajpai, On Intuitionistic fuzzy Gpr-closed sets, Fuzzy Inf. Eng, (2012)4: 425-444.

[15] Thakur, S. S., M. Thakur. Regular semi open sets in Intuitionistic fuzzy topology. Vikram Mathematical Journal, Vol. 28, 2008 (in press).

[16] Zadeh, L. A. Fuzzy Sets, Information and Control, Vol. 18, 1965, 338–353.

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