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COMBINED EFFECTS OF COUPLE STRESS AND MHD ON SQUEEZE FILM LUBRICATION BETWEEN TWO PARALLEL PLATES

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ABSTRACT

This paper presents theoretically the combined effects of couple stress and magnetic field between two parallel plates of which upper plate has roughness surface and lower has smooth material. The lubricating fluid is linearly viscous, isothermal, incompressible, electrically conducting fluid in the presence of uniform transverse magnetic field. This model incorporates the appropriate no-slip boundary conditions. The finite difference based multigrid method is applied to solve the modified Reynolds equation. The bearing characteristics such as pressure distribution and load carrying capacity are obtained for different values of couple stress parameter and Hartmann number. The results reveal that the bearing characteristics such as pressure distribution and load carrying capacity increases for increasing couple stress parameter and Hartmann number compared to classical Newtonian case.

Keywords: *Couple stress fluid, MHD, Multigrid method, Reynolds equation, Squeeze film.*

1. INTRODUCTION

In the recent years, the development in the field of lubrication has enhanced the quest of the lubricating effectiveness of non- Newtonian fluids in modern industries and technology based real world. To satisfy the needs of modern mechanics, the use of complex fluids as lubricants is getting more and more important. It is a known fact that the classical continuum mechanics of fluids neglect the size of fluid particles to describe the flow of fluids. Currently there exist several theories to describe the flow of such complex fluids. All the theories imply the polar nature of the continuum, exhibiting the asymmetric stress tensor. The simplest generalization theory that describes the classical behaviour of couple stress fluids was given by Stokes [16]. Many researchers and investigators have used this model to study the characteristic performance of various types of bearings, such as Naduvinamani [12], [13] and Lin [7], [8], [9], [10]. The study of squeeze film lubrication between two rough surfaces is quite important to clarify the contact situation between two mating surfaces during squeeze effect in conjunction with surface topography.

The basic concept behind hydromagnetics is that magnetic field which can induce currents in a moving conductive fluid, in turn creates forces on the fluid and also enhances the magnetic field itself. Examples of such fluids include plasmas, liquid metals and salt water or electrolytes. Several investigations have been focused on hydromagnetic lubrication. Theoretically and experimentally many investigations have taken place on MHD lubrication. The applications of MHD are identified in many areas like geophysics, astrophysics, MHD generators, MHD pumps, fusion reactors, crystal growth, MRI scanning, magnetic drug targeting and metallurgical applications.

The MHD lubrication in an externally pressurized thrust bearing has been investigated both theoretically and experimentally by Maki. Limited studies of MHD lubrication are available in the literature which include –MHD slider bearings (Anwar and Rodkiewicz [1], Bujurke [2], [3], Das [4], MHD Journal bearings by Kamiyama [6], Malik and Singh [11], MHD squeeze film bearings by Shukla [15], Hamza [5] has shown the effects of MHD on a fluid film squeezed between rotating surfaces). Recently Kudenatti [14] have investigated the Numerical solution of the MHD Reynolds equation for squeeze film lubrication between porous and rough rectangular plates.

The aim of this paper is to study the combined effects of couple stress fluid and magnetic field on hydrodynamic squeeze film lubrication between two parallel plates of which upper plate has roughness surface and lower is smooth in nature.

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This paper is classified into different sections. The necessary basic equations with appropriate boundary conditions are given and subsequently the modified MHD Reynolds equation is derived in section 2. In section 3, finite difference based multigrid method is applied to solve the modified Reynolds equation for pressure in the fluid film. In section 4 the bearing characteristics like pressure distribution and load carrying capacity are analysed by varying couple-stress parameter, Hartmann number and aspect ratio. In the final section the important findings and their usefulness are discussed which helps in designing bearings.

2. FORMULATION OF THE PROBLEM

This model consists of flow of viscous isothermal and incompressible electrically conducting couple stress fluid between two rectangular plates of which the upper plate has a rough surface. The physical configuration of the problem is shown in figure 1. The upper rough plate approaches the lower smooth plate with a constant velocity $\frac{dH}{dt}$. A uniform transverse magnetic field M_0 is applied in the *z*-direction. The upper and lower plates are separated by film thickness *H*, where, the total film thickness is made up of two parts as

$$H = h_0 + h_s(x, y, \xi) \tag{1}$$

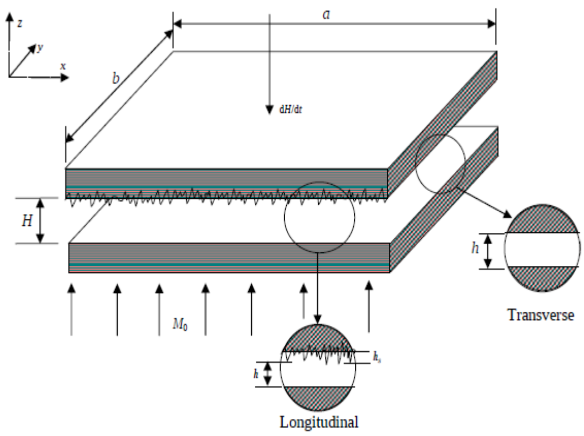


Figure: 1 The physical configuration of squeeze film between rough and smooth rectangular plate in the presence of magnetic field.

where h_0 is the height of the nominal smooth part of the film region, h_s is part due to the surface asperities measured from the nominal level which is a randomly varying quantity of zero mean and ξ is the index parameter determining a definite roughness structure.

In addition to the usual assumptions of lubrication theory, we assume fluid inertia to be negligible and except the Lorentz force, the body forces are also neglected. Under these assumptions, the governing equations in Cartesian co-ordinates system are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{2}$$

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial z^2} - \eta \frac{\partial^4 u}{\partial z^4} - \sigma M_0^2 u, \tag{3}$$

$$\frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial z^2} - \eta \frac{\partial^4 v}{\partial z^4} - \sigma M_0^2 v, \tag{4}$$

$$\frac{\partial p}{\partial z} = 0. \tag{5}$$

where *u*, *v* and *w* are the velocity components in *x*, *y* and *z* directions respectively, *p* is the pressure, σ is electrical conductivity of the fluid, M_0 is the impressed magnetic field. μ is viscosity of the fluid and η couple stress parameter responsible for polar additive in the non- polar lubricant. The material length of the fluid is given by the quantity η/μ which has the dimension of length-squared. The above equation reduces to the classical Newtonian fluid when $\eta = 0$.

The relevant boundary conditions for the velocity components are:

$$u = 0, \quad v = 0, \quad w = 0, \quad \frac{\partial^2 u}{\partial z^2} = 0, \quad \frac{\partial^2 v}{\partial z^2} = 0 \qquad \text{at} \quad z = 0. \tag{6a}$$

$$u = 0, \quad v = 0, \quad w = \frac{dH}{dt}, \quad \frac{\partial^2 u}{\partial z^2} = 0, \quad \frac{\partial^2 v}{\partial z^2} = 0 \qquad \text{at} \quad z = H. \tag{6b}$$

Making use of the solution of (3) and (4) for u and v using appropriate boundary conditions in (6) then substituting in the continuity equation and integrating with respect to *z* from 0 to *H* using the conditions $w = 0$ at $z = 0$ and $w = \frac{dH}{dt}$ at $z = H$, we get the modified Reynolds equation for unknown pressure distribution in the fluid film region

$$\frac{\partial}{\partial x}\left(F(H, K_3, K_4)\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(F(H, K_3, K_4)\frac{\partial p}{\partial y}\right) = \frac{\mu K_2}{K_1}(K_3^2 - K_4^2)\frac{dH}{dt} \tag{7}$$

where

$$F(H, K_3, K_4) = H(K_3^2 - K_4^2) - \frac{2K_3^2}{K_4}\tanh\left(\frac{K_4 H}{2}\right) + \frac{2K_4^2}{K_3}\tanh\left(\frac{K_3 H}{2}\right),$$

$$K_3 = \sqrt{\frac{K_1}{2} + \frac{1}{2}\sqrt{K_1^2 - 4K_2}}, \quad K_4 = \sqrt{\frac{K_1}{2} - \frac{1}{2}\sqrt{K_1^2 - 4K_2}}, \tag{8}$$

$$K_1 = \tau^2, \quad K_2 = \frac{M^2}{h_0^2}\tau^2, \quad M = \sqrt{\frac{\sigma}{\mu}}\, M_0 h_0, \tag{9}$$

The couple- stress parameter $\tau = \sqrt{\frac{\mu}{\eta}}$ is responsible for the effect of couple stress parameter. The parmeter *M* gives the Hartmann (or Magnetic) number and is responsible for magnetic field on squeeze film lubrication.

In order to solve the modified Reynolds equation for the pressure, the following boundary conditions are used

$$p = 0, \quad \text{at} \quad x = 0, \ a \quad \text{and} \quad y = 0, \ b. \tag{10}$$

where *a* and *b* are finite dimensions of plates in *x* and *y* directions respectively.

Following non-dimensional parameters and variables are introduced.

$$\bar{x} = \frac{x}{a}, \quad \bar{y} = \frac{y}{b}, \quad \bar{H} = \frac{H}{h_0}, \quad \bar{p} = \frac{-h_0^3(p)}{a^2\mu\frac{\partial h}{\partial t}}, \quad \bar{\tau} = \frac{\tau}{h_0}, \quad \alpha = \frac{b}{a}. \tag{11}$$

Here *p* is the non-dimensional fluid film pressure. α is the aspect ratio, after dropping the over head bars, the modified Reynolds equation is given by

$$\frac{\partial}{\partial x}\left(\left(F(H, K_3, K_4)\right)\frac{\partial p}{\partial x}\right) + \frac{1}{\alpha^2}\frac{\partial}{\partial y}\left(\frac{1}{\left(1/F(H, K_3, K_4)\right)}\frac{\partial p}{\partial y}\right) = -M^2(K_3^2 - K_4^2) \tag{12}$$

where

$$K_3 = \frac{\tau}{\sqrt{2}}\left(1 + \sqrt{1 - \frac{4M^2}{\tau^2}}\right)^{1/2}, \quad K_4 = \frac{\tau}{\sqrt{2}}\left(1 - \sqrt{1 - \frac{4M^2}{\tau^2}}\right)^{1/2}$$

and boundary conditions for pressure field are given by

$$p = 0, \quad \text{at} \quad x = 0, \ 1 \ \text{and}\ y = 0, \ 1. \tag{13}$$

3. NUMERICAL SOLUTION BY MULTIGRID METHOD

The modified Reynolds equation (12) is of elliptic in nature, which is too complicated to be solved analytically, hence, we solve it numerically using finite difference based multigrid method. The terms in equation (12) have been approximated using standard second order finite difference scheme. The number of grids in each direction is taken to be 257×257. Thus, there are 257×257 number of unknowns and hence equations in the problem. We resort to the convergent accelerator multigrid method for the solution of the discretized Reynolds equation (12). This method provides us a simple way to compute the pressure distribution. In the multigrid method, few Gauss-Seidel iterations are applied for smoothing the errors; half weighting restriction operator is used for transferring the calculated residual to the coarser grid level. Repeat this procedure till we reach the coarsest level with just single grid, and solve it exactly. Next, bilinear interpolation operator is used to prolongate the solution obtained at the coarsest level to the next finer grid level. Repeat this till the original finest level is reached. The convergent solution for the pressure is obtained when the pressures at two consecutive finest levels are almost same upto 10^{-6}. Full numerical results using 257×257 grid level have been compared with those obtained with 513×513 grid level, the pressure distribution between them are graphically indistinguishable, thus, former grid level is adopted for further computation.

4. RESULTS AND DISCUSSIONS

Multigrid method is applied to solve the modified Reynolds equation to study various physical parameters such as pre - ssure distribution, couple stress parameter, load carrying capacity, Hartmann number etc.,between two parallel plates in the presence of transverse of magnetic field. The characteristics of squeeze film bearings are obtained as functions of couple-stress fluid, aspect ratio and Hartmann number M. Multigrid solution to the Reynolds equation (12) exists when the quantities K_3 and K_4 are real and condition for K_3 and K_4 to become real is $\tau > 2M$.

Figure 2a: Variation of pressure distribution p for $\tau = 15$, $M = 2$ and $\alpha = 1.0$.

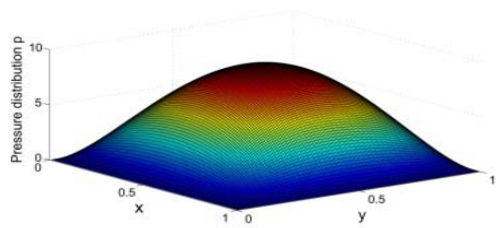


Figure 2b: Variation of pressure distribution p for $\tau = 15$, $M = 4$ and $\alpha = 1.0$.

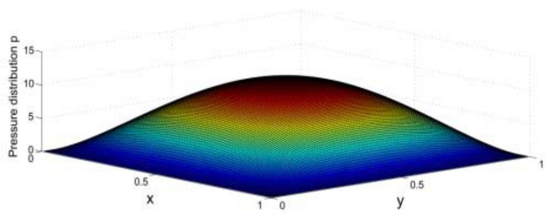


Figure 2c: Variation of pressure distribution p for $\tau = 15$, $M = 6$ and $\alpha = 1.0$.

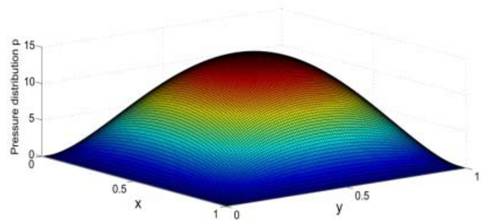


Figure 2d: Variation of pressure distribution p for $M = 2, \tau = 5$ and $\alpha = 1.0$.

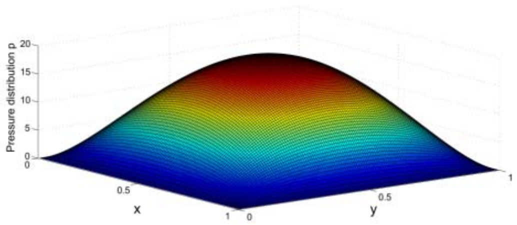


Figure 2e: Variation of pressure distribution p for $M = 2, \tau = 10$ and $\alpha = 1.0$.

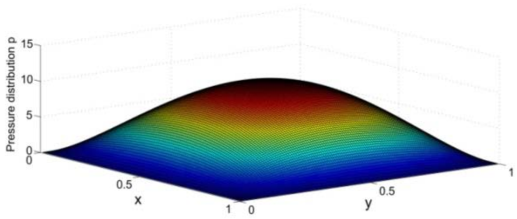
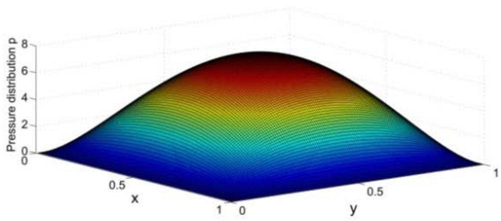


Figure 2f: Variation of pressure distribution p for $M = 2,\ \tau \to \infty$ and $\alpha = 1.0$.



Pressure distribution: The variation of pressure distribution p with rectangular co-ordinates x and y are shown in figures (2a-2f). Keeping $\boldsymbol{\tau}$ constant $(\tau = 15)$ for different values of Hartmann number M = 2 to 6, the pressure distribution is depicted in figures (2a-2c). It is found that as Hartmann number M *increases* the pressure rise in the fluid film also increases. Hence Magnetic field enhances the pressure distribution in the fluid film. This is due to the application of uniform magnetic field which reduces the velocity of the lubricant. As a result large amount of fluid is collected in the film region and which results in the pressure rise. Hence, increase of application of magnetic field leads to reduce the velocity of the fluid consequently pressure rise increases.

The effect of couple stress fluid with the variation of pressure distribution p with rectangular co-ordinates x and y are depicted in figures (2d-2f), i.e. for different values of couple-stress parameter $\boldsymbol{\tau}$ keeping other parameters constant. It is noticed that, for smaller values of couple- stress parameter, the built up pressure in fluid film region is higher than that of larger one. The built up pressure increases as an effect of couple stress. As $\boldsymbol{\tau}$ increases its value, the pressure rise starts to decreases in which the fluid loses its non-Newtonian characteristics. At large value of $\boldsymbol{\tau}$ the fluid becomes Newtonian at $(\tau = \infty)$ and hence there is no variation of pressure rise.

Non-dimensional load carrying capacity: Load carrying capacity is one of salient feature of hydrodynamic characteristics. This can be obtained once the fluid film pressure is calculated. The load carrying capacity W of the bearing surface per unit area in a non-dimensional form is

$$W = \int_0^1 \int_0^1 p(x, y)\, dxdy \,.$$

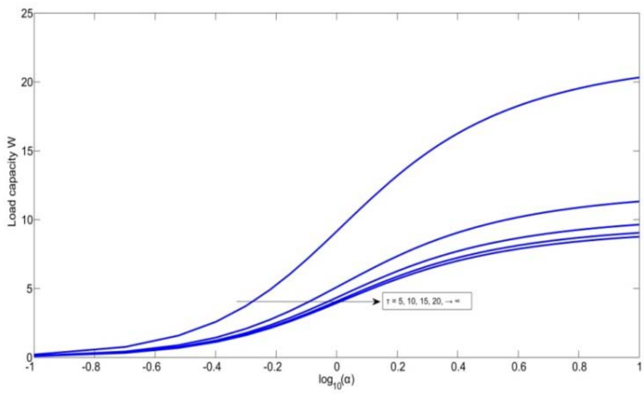


Figure 3: Variation of non-dimensional load carrying capacity W as a function of aspect ratio α for different values couple-stress parameter τ**.**

Figure 3 shows that the load capacity W as a function of aspect ratio α for different values couple stress parameters $\boldsymbol{\tau}$ keeping other parameters constant. Couple–stress fluid increases the load capacity compared to the corresponding Newtonian case. The effect of couple stress fluid is to increase the pressure rise in the fluid film region and therefore load carrying capacity also increases. As $\tau \to \infty$, load carrying capacity also decreases. However, as the aspect ration α increases from 0.1 to 10, the load carrying capacity increases and this trend is observed for all values of $\boldsymbol{\tau}$ **.**

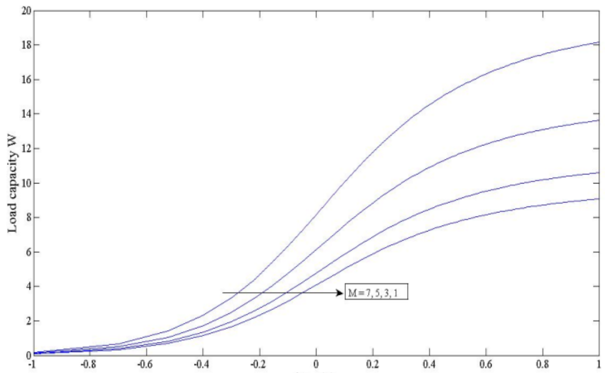


Figure 4: Variation of non-dimensional load carrying capacity W as a function of aspect ratio α for different values of Hartmann number M.

Figure 4 represents the load capacity W as a function of aspect ratio α for different values of Hartmann number M keeping other parameters constant. As the Hartmann number increases the load carrying capacity also increases. Hence application of uniform transverse magnetic field enhances the non-dimensional load carrying capacity W. However, as the aspect ration α increases from 0.1 to 10, the load carrying capacity increases for increasing Hartmann number.

CONCLUSIONS

The combined effects of couple stress fluid and magnetic field between two parallel plates of which the upper plate has roughness structure and the lower plate has smooth surface are studied using finite difference based multigrid method.

Our investigations revealed that:

1. The bearing characteristics like pressure distribution and load carrying capacity increases for increasing Hartmann number (M).
2. The effect of couple stress (τ) is to increase the built up pressure in the film region.
3. Also, the load carrying capacity increases as the aspect ratio (λ) increases.

It is expected that these results help the Mechanical engineers to choose the suitable parameters for given magnetic field to enhance the life of the bearings.

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