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SUPER EDGE TRIMAGIC TOTAL LABELING OF GRAPHS

M. Regees[1] and C. Jayasekaran[2*]

[1]Department of Mathematics, Malankara Catholic College, Mariagiri, Kaliakavilai-629153, Tamilnadu, India.

[2]Department of Mathematics, Pioneer Kumaraswamy College, Nagercoil-629003, Tamilnadu, India.

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ABSTRACT

An edge magic total labeling of a (p, q) graph is a bijection f: $V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p+q\}$ such that for each edge $uv \in E(G)$, the value of $f(u)+f(uv)+f(v)$ is a constant k. If there exists two constants k_1 and k_2 such that $f(u)+f(uv)+f(v)$ is either $k_1$or k_2, it is said to be an edge bimagic total labeling. An edge trimagic total labeling of a (p, q) graph is a bijection f: $V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p+q\}$ such that for each edge $uv \in E(G)$, the value of $f(u)+f(uv)+f(v)$ is either $k_1$or k_2 or k_3. In this paper, we prove that the corona graph $C_n \odot K_2$, double ladder $P_n \times P_3$, quadrilateral snake Q_n and alternate triangular snake $A(TS_n)$ are edge trimagic total and super edge trimagic total.

Keywords: *Function, Bijection, Labeling, Magic, Trimagic.*

AMS Subject Classification: *05C78.*

1. INTRODUCTION

We begin with simple, finite and undirected graph G = (V, E). A graph labeling is an assignment of integers to elements of graph, the vertices or edges or both subject to certain conditions. The concept of graph labeling was introduced by Rosa in 1967. In 1970 Kotzig and Rosa[6] defined, magic labeling of graph G is a bijection f: $V \cup E \rightarrow \{1, 2, \ldots, p+q\}$ such that, for each edge $uv \in E(G)$, $f(u)+f(uv)+f(v)$ is a magic constant. In 1996, Ringel and Llado called this labeling as edge magic. In 2001, Wallis introduced this as edge magic total labeling. In 2004, J. Baskar Babujee [1, 2] introduced the edge bimagic labeling of graphs.

In 2013, C. Jayasekaran, M. Regees and C. Davidraj [3] introduced the edge trimagic total labeling of graphs. An edge trimagic total labeling of a (p, q) graph G is a bijection $f:V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p+q\}$ such that for each edge $uv \in E$, the value of $f(u)+f(uv)+f(v)$ is equal to any of the distinct constant $k_1$or k_2 or k_3. A graph G is said to be edge trimagic total if it admits an edge trimagic total labeling. An edge trimagic total labeling is called super edge trimagic total labeling if G has the additional property that the vertices are labeled with the smallest positive integers.

An alternate triangular snake $A(TS_n)$ is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} alternatively to a new vertex v_i. That is every alternate edge of a path is replaced by C_3. If G is of order n, the Corona of G with H, $G \odot H$ is the graph obtained by taking one copy of G and n copies of H and joining the i^{th} vertex of G with an edge to every vertex in the i^{th} copy of H. A Quadrilateral snake Q_n is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i, u_{i+1} to new vertices v_i, w_i respectively and joining v_i and w_i. That is, every edge of a path is replaced by a cycle C_4. A ladder L_n is a graph $P_n \times P_2$ with $V(L_n) = \{u_i, v_i / 1 \le i \le n\}$ and $E(L_n) = \{u_iu_{i+1}, v_iv_{i+1} / 1 \le i \le n-1\} \cup \{u_iv_i / 1 \le i \le n\}$. A double ladder L_n is a graph $P_n \times P_3$ with $V(L_n) = \{u_i, v_i, w_i / 1 \le i \le n\}$ and $E(L_n) = \{u_iu_{i+1}, v_iv_{i+1}, w_iw_{i+1} / 1 \le i \le n-1\} \cup \{u_iv_i, v_iw_i / 1 \le i \le n\}$.

For further references, we use dynamic survey of graph labeling by J. A. Gallian [5]. In [3], we introduced the concept edge trimagic and super edge trimagic total labeling and proved that, some family and classes of graphs are edge trimagic total and super edge trimagic total [3, 4, 7]. In this paper, we prove that the corona graph $C_n \odot K_2$, double ladder $P_n \times P_3$, quadrilateral snake Q_n and alternate triangular snake $A(TS_n)$ are trimagic total and super edge trimagic total graphs.

Corresponding author: C. Jayasekaran[2]*

2. SUPER EDGE TRIMAGIC LABELING OF $C_n \odot K_2$, $P_n x P_3$, Q_n and A (TS_n).

In this section we prove that the corona graph $C_n \odot K_2$, double ladder $P_n \times P_3$, quadrilateral snake Q_n and alternate triangular snake $A(TS_n)$ are edge trimagic total and super edge trimagic total. And give examples for super edge trimagic total labeling for each of the above graphs.

Theorem: 2.1 The graph $C_n \odot K_2$ has an edge trimagic total labeling for positive integer n.

Proof: Let $V = \{u_i, v_i, w_i \ / \ 1 \le i \le n\}$ be the vertex set and $E = \{u_iv_i, u_iw_i, v_iw_i \ /1 \le i \le n\} \cup \{u_iu_{i+1}/1 \le i \le n-1\} \cup \{u_nu_1\}$ be the edge set of the graph $C_n \odot K_2$. Then $C_n \odot K_2$ has 3n vertices and 4n edges.

Define a bijection $f: V \cup E \rightarrow \{1, 2, \ldots, 7n\}$ such that

$f(u_i) = i$, $1 \le i \le n$; $f(v_i) = n+i$, $1 \le i \le n$; $f(w_i) = 2n+i$, $1 \le i \le n$;

$f(u_iu_{i+1}) = 7n - 2i$, $1 \le i \le n-1$; $f(u_iv_i) = 7n-2i+1$, $1 \le i \le n$; $f(u_iw_i) = 5n-2i+2$, $1 \le i \le n$;

$f(v_iw_i) = 5n-2i+1$, $1 \le i \le n$ and $f(u_nu_1) = 7n$.

Now we prove the above labeling is an edge trimagic total.

For the edges u_iu_{i+1}, $1 \le i \le n-1$;

$f(u_i)+f(u_iu_{i+1})+f(u_{i+1}) = i+7n-2i+i+1 = 7n+1 = \lambda_1$(say).

For the edges u_iv_i, $1 \le i \le n$;

$f(u_i)+f(u_iv_i)+f(v_i) = i+7n-2i+1+n+i = 8n+1 = \lambda_2$(say).

For the edges u_iw_i, $1 \le i \le n$;

$f(u_i)+f(u_iw_i)+f(w_i) = i+5n-2i+2+2n+i = 7n+2 = \lambda_3$(say).

For the edges v_iw_i, $1 \le i \le n$;

$f(v_i)+f(v_iw_i)+f(w_i) = n+i+5n-2i+1+2n+i = 8n+1 = \lambda_2$.

For the edge u_nu_1, $f(u_n)+f(u_nu_1)+f(u_1) = n+7n+1 = 8n+1 = \lambda_2$.
Hence for each edge $uv \in E$, $f(u)+f(uv)+f(v)$ yields any one of the trimagic constants $\lambda_1 = 7n+1$, $\lambda_2 = 8n+1$ and $\lambda_3 = 7n+2$.

Therefore, the graph $C_n \odot K_2$ admits an edge trimagic total labeling for all positive integer n.

Theorem: 2.2 The graph $C_n \odot K_2$ has a super edge trimagic total labeling.

Proof: We proved that the graph $C_n \odot K_2$ admits an edge trimagic total labeling. The labeling given in the proof of Theorem 2.1, the vertices get labels 1, 2, …, 3n. Since the graph $C_n \odot K_2$ has 3n vertices and the 3n vertices have labels 1, 2, …, 3n, the graph $C_n \odot K_2$ admits a super edge trimagic total labeling.

Example: 2.3 A super edge trimagic total labeling of the graph $C_6 \odot K_2$ is given in figure1.

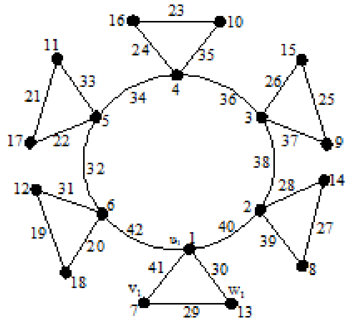

Figure 1: $C_6 \odot K_2$ with $\lambda_1 = 43$, $\lambda_2 = 49$ and $\lambda_3 = 44$.

Theorem: 2.4 The double ladder $P_n \times P_3$ admits an edge trimagic total labeling for odd n.

Proof: Let $V = \{u_i, v_i, w_i \,/\, 1 \le i \le n\}$ be the vertex set and $E = \{u_iv_i, v_iw_i \,/\, 1 \le i \le n\} \cup \{u_iu_{i+1}, v_iv_{i+1}, w_iw_{i+1} \,/ 1 \le i \le n-1\}$ be the edge set of $P_n \times P_3$. Then the doble ladder $P_n \times P_3$ has 3n vertices and 5n–3 edges.

Define a bijection $f: V \cup E \rightarrow \{1, 2, \ldots, 8n-3\}$ such that

$$f(u_i) = \begin{cases} \frac{i+1}{2}, & 1 \le i \le n \text{ and i is odd} \\ \frac{n+i+1}{2}, & 1 \le i \le n \text{ and i is even} \end{cases}$$

$$f(v_i) = \begin{cases} 2n+\frac{n+i}{2}, & 1 \le i \le n \text{ and i is odd} \\ 2n+\frac{i}{2}, & 1 \le i \le n \text{ and i is even} \end{cases}$$

$$f(w_i) = \begin{cases} n+\frac{i+1}{2}, & 1 \le i \le n \text{ and i is odd} \\ n+\frac{n+i+1}{2}, & 1 \le i \le n \text{ and i is even} \end{cases}$$

$f(u_iv_i) = 6n - i,\ 1 \le i \le n$; $f(v_iw_i) = 5n - i,\ 1 \le i \le n$; $f(u_iu_{i+1}) = 8n-i-2,\ 1 \le i \le n-1$;

$f(v_iv_{i+1}) = 4n - i,\ 1 \le i \le n-1$ and $f(w_iw_{i+1}) = 7n - i - 1,\ 1 \le i \le n-1$.

Now we prove the above labeling is an edge trimagic total.

Consider the edges u_iv_i, $1 \le i \le n$;

For odd i, $f(u_i)+f(u_iv_i)+f(v_i) = \frac{i+1}{2}+6n-i+2n+\frac{n+i}{2} = \frac{17n+1}{2} = \lambda_1$(say).

For even i, $f(u_i)+f(u_iv_i)+f(v_i) = \frac{n+i+1}{2}+6n-i+2n+\frac{i}{2} = \frac{17n+1}{2} = \lambda_1$.

Consider the edges v_iw_i, $1 \le i \le n$;

For odd i, $f(v_i)+f(v_iw_i)+f(w_i) = 2n+\frac{n+i}{2}+5n-i+n+\frac{i+1}{2} = \frac{17n+1}{2} = \lambda_1$.

For even i, $f(v_i)+f(v_iw_i)+f(w_i) = 2n+\frac{i}{2}+5n-i+n+\frac{n+i+1}{2} = \frac{17n+1}{2} = \lambda_1$.

Consider the edges u_iu_{i+1}, $1 \le i \le n-1$;

For odd i, $f(u_i)+f(u_iu_{i+1})+f(u_{i+1}) = \frac{i+1}{2}+8n-i-2+\frac{n+i+1+1}{2} = \frac{17n-1}{2} = \lambda_2$(say).

For even i, $f(u_i)+f(u_iu_{i+1})+f(u_{i+1}) = \frac{n+i+1}{2}+8n-i-2+\frac{i+1+1}{2} = \frac{17n-1}{2} = \lambda_2$.

Consider the edges v_iv_{i+1}, $1 \le i \le n-1$;

For odd i, $f(v_i)+f(v_iv_{i+1})+f(v_{i+1}) = 2n+\frac{n+i}{2}+4n-i+2n+\frac{i+1}{2} = \frac{17n+1}{2} = \lambda_1$.

For even i, $f(v_i)+f(v_iv_{i+1})+f(v_{i+1}) = 2n+\frac{i}{2}+4n-i+2n+\frac{n+i+1}{2} = \frac{17n+1}{2} = \lambda_1$.

Consider the edges w_iw_{i+1}, $1 \le i \le n-1$;

For odd i, $f(w_i)+f(w_iw_{i+1})+f(w_{i+1}) = n+\frac{i+1}{2}+7n-i-1+n+\frac{n+1+i+1}{2} = \frac{19n+1}{2} = \lambda_3$(say).

For even i, $f(w_i)+f(w_iw_{i+1})+f(w_{i+1}) = n+\frac{n+i+1}{2}+7n-i-1+n+\frac{i+1+1}{2} = \frac{19n+1}{2} = \lambda_3$.

Hence for each edge $uv \in E$, $f(u)+f(uv)+f(v)$ yields any one of the constant

$\lambda_1 = \frac{17n+1}{2}$, $\lambda_2 = \frac{17n-1}{2}$ and $\lambda_3 = \frac{19n+1}{2}$.

Therefore, the double ladder $P_n \times P_3$ admits an edge trimagic total labeling for odd n.

Theorem: 2.5 The double ladder $P_n \times P_3$ has a super edge trimagic total labeling for odd n.

Proof: We proved that the double ladder $P_n \times P_3$ admits an edge trimagic total labeling. The labeling given in the proof of Theorem 2.4, the vertices get labels 1, 2, …, 3n. Since the double ladder $P_n \times P_3$ has 3n vertices and the 3n vertices have labels 1, 2, …, 3n, the double ladder $P_n \times P_3$ admits a super edge trimagic total labeling.

Example: 2.6 A super edge trimagic total labeling of the double ladder $P_7 \times P_3$ is given in figure 2.

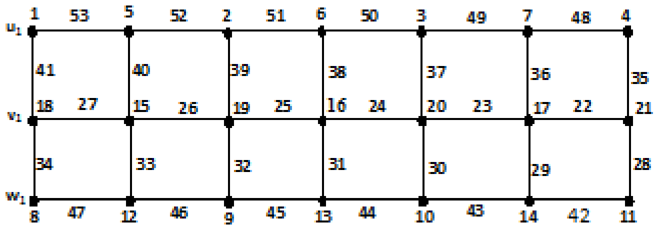

Figure 2: $P_7 \times P_3$ with $\lambda_1 = 60$, $\lambda_2 = 59$ and $\lambda_3 = 67$.

Theorem: 2.7 The Quadrilateral Snake Q_n admits an edge trimagic total labeling.

Proof: Let $V = \{u_i / 1 \le i \le n\} \cup \{v_i, w_i / 1 \le i \le n-1\}$ be the vertex set and $E = \{u_iv_i, v_iw_i, u_iu_{i+1}, u_{i+1}w_i / 1 \le i \le n-1\}$ be the edge set of the Quadrilateral Snake Q_n. Then Q_n has 3n–2 vertices and 4n– 4 edges.

Define a bijection $f: V \cup E \to \{1, 2, \ldots, 7n-6\}$ such that $f(u_i) = i$, $1 \le i \le n$; $f(v_i) = n+i$, $1 \le i \le n-1$; $f(w_i) = 2n+i-1$, $1 \le i \le n-1$; $f(u_iu_{i+1}) = 7n-2i-4$, $1 \le i \le n-1$; $f(u_iv_i) = 7n-2i-5$, $1 \le i \le n-1$; $f(u_{i+1}w_i) = 5n-2i-3$, $1 \le i \le n-1$ and $f(v_iw_i) = 5n-2i-2$, $1 \le i \le n-1$.

Now we prove the above labeling is an edge trimagic total.

For the edges u_iu_{i+1}, $1 \le i \le n-1$;

$f(u_i)+f(u_iu_{i+1})+f(u_{i+1}) = i+7n-2i-4+i+1 = 7n-3 = \lambda_1$ (say).

For the edges u_iv_i, $1 \le i \le n-1$;

$f(u_i)+f(u_iv_i)+f(v_i) = i+7n-2i-5+n+i = 8n-5 = \lambda_2$(say).

For the edges $u_{i+1}w_i$, $1 \le i \le n-1$;

$f(u_{i+1})+f(u_{i+1}w_i)+f(w_i) = i+1+5n-2i-3+2n+i-1 = 7n-3 = \lambda_1$.

For the edges v_iw_i, $1 \le i \le n-1$;

$f(v_i)+f(v_iw_i)+f(w_i) = n+i+5n-2i-2+2n+i-1 = 8n-3 = \lambda_3$(say).

Hence for each edge $uv \in E$, $f(u)+f(uv)+f(v)$ yields any one of the constants $\lambda_1 = 7n-3$, $\lambda_2 = 8n-5$ and $\lambda_3 = 8n-3$.

Therefore, the Quadrilateral snake Q_n admits an edge trimagic total labeling.

Theorem: 2.8 The Quadrilateral snake Q_n admits a super edge trimagic total labeling.

Proof: We proved that the Quadrilateral snake Q_n has an edge trimagic total labeling. The labeling given in the proof of Theorem 2.7, the vertices get labels 1, 2, …, 3n–2. Since the Quadrilateral snake Q_n has 3nvertices and the 3n–2 vertices have labels 1, 2, …, 3n–2 for all integer n, the Quadrilateral snake Q_n admits a super edge trimagic total labeling.

Example: 2.9 A super edge trimagic total labeling of Quadrilateral snake Q_6 is given in figure 3.

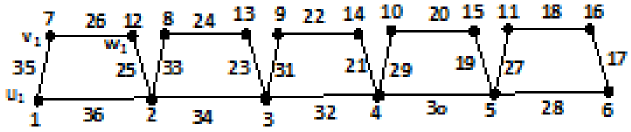

Figure 3: Q_6 with $\lambda_1 = 39$, $\lambda_2 = 43$ and $\lambda_3 = 45$.

Theorem: 2.10 The Alternate triangular snake $A(TS_n)$ admits an edge trimagic total labeling for even n.

Proof: We consider the following two cases.

Case: 1 Triangle starts from u_1.

Let $V = \{u_i / 1\le i \le n\} \cup \{v_j / 1\le j\le \frac{n}{2}\}$ be the vertex set and $E = \{v_ju_{2j-1}, v_ju_{2j} / 1\le j\le \frac{n}{2}\} \cup \{u_iu_{i+1} / 1\le i \le n-1\}$ be the edge set of the alternate triangular snake $A(TS_n)$. Since the triangle starts from u_1, the alternate triangular snake $A(TS_n)$ has $n+\frac{n}{2}$ vertices and $2n-1$ edges. Define a bijection $f: V \cup E \to \{1, 2, \ldots, 3n+\frac{n}{2}-1\}$ such that

$$f(u_i) = \begin{cases} \frac{i+1}{2}, & 1\le i \le n \text{ and i is odd} \\ \frac{n+i}{2}, & 1\le i \le n \text{ and i is even}; \end{cases}$$

$f(v_j) = n+j$, $1\le j \le \frac{n}{2}$; $f(u_iu_{i+1}) = 3n+\frac{n}{2}-i$, $1\le i\le n-1$; $f(v_ju_{2j-1}) = 2n+\frac{n}{2}-2j+1$, $1\le j \le \frac{n}{2}$;

$f(v_ju_{2j}) = 2n+\frac{n}{2}-2j+2$, $1\le j \le \frac{n}{2}$.

Now we prove the above labeling is an edge trimagic total.

Consider the edges u_iu_{i+1}, $1\le i \le n-1$;

For odd i, $f(u_i)+f(u_iu_{i+1})+f(u_{i+1}) = \frac{i+1}{2}+3n+\frac{n}{2}-i+\frac{n+i+1}{2} = 4n+1 = \lambda_1$(say).

For even i, $f(u_i)+f(u_iu_{i+1})+f(u_{i+1}) = \frac{n+i}{2}+3n+\frac{n}{2}-i+\frac{i+1+1}{2} = 4n+1 = \lambda_1$.

For the edges $u_{2j-1}v_j$, $1\le j \le \frac{n}{2}$;

$$f(u_{2j-1})+f(u_{2j-1}v_j)+f(v_j) = \frac{1+2j-1}{2}+2n+\frac{n}{2}-2j+1+n+j = \frac{7n+2}{2} = \lambda_2\text{(say)}.$$

For the edges $u_{2j}v_j$, $1\le j \le \frac{n}{2}$;

$$f(u_{2j})+f(u_{2j}v_j)+f(v_j) = \frac{n+2j}{2}+2n+\frac{n}{2}-2j+2+n+j = 4n+2 = \lambda_3\text{(say)}.$$

Hence for each edge $uv\in E$, $f(u)+f(uv)+f(v)$ yields any one of the trimagic constants $\lambda_1 = 4n+1$, $\lambda_2 = \frac{7n+2}{2}$ and $\lambda_3 = 4n+2$. Therefore, the alternate triangular snake graph $A(TS_n)$ admits an edge trimagic total labeling when the triangle starts from u_1.

Case: 2 Triangle starts from u_2.

Let $V = \{u_i / 1\le i \le n\} \cup \{v_j / 1\le j\le \frac{n}{2}-1\}$ be the vertex set and $E = \{u_{2j}v_j, u_{2j+1}v_j / 1\le j\le \frac{n}{2}-1\} \cup \{u_iu_{i+1} / 1\le i\le n-1\}$ be the edge set of the alternate triangular snake $A(TS_n)$. Since the triangle starts from u_2, the alternate triangular snake $A(TS_n)$ has $n+\frac{n}{2}-1$ vertices and $2n-3$ edges. Define a bijection $f: V\cup E \to \{1, 2, \ldots, 3n+\frac{n}{2}-4\}$ such that

$$f(u_j) = \begin{cases} \frac{i+1}{2}, & 1\le i \le n \text{ and i is odd} \\ \frac{n+i}{2}, & 1\le i \le n \text{ and i is even}; \end{cases}$$

$f(v_j) = n+j,\ 1\le j \le \frac{n}{2} - 1$; $f(u_iu_{i+1}) = 3n+\frac{n}{2} - i-3,\ 1\le i\le n-1$;

$f(u_{2j+1}v_j) = 2n+\frac{n}{2} -2j-2,\ 1\le j\le \frac{n}{2} - 1$; $f(u_{2j}v_j) = 2n+\frac{n}{2} - 2j-1,\ 1\le j \le \frac{n}{2} - 1$.

Now we prove the above labeling is an edge trimagic total.

Consider the edges $u_iu_{i+1},\ 1\le i \le n-1$;

For odd i, $f(u_i)+f(u_iu_{i+1})+f(u_{i+1}) = \frac{i+1}{2} +3n+ \frac{n}{2} - i-3+\frac{n+i+1}{2} = 4n-2 = \lambda_1$(say).

For even i, $f(u_i)+f(u_iu_{i+1})+f(u_{i+1}) = \frac{n+i}{2} +3n+ \frac{n}{2} - i-3+\frac{i+1+1}{2} = 4n-2= \lambda_1$.

For the edges $u_{2j+1}v_j,\ 1\le j \le \frac{n}{2} - 1$;

$f(u_{2j+1})+f(u_{2j+1}v_j)+f(v_j) = \frac{1+2j+1}{2} +2n+\frac{n}{2} - 2j-2+ n+j = \frac{7n-2}{2} = \lambda_2$(say).

For the edges $u_{2j}v_j,\ 1\le j \le \frac{n}{2} - 1$;

$f(u_{2j})+f(u_{2j}v_j)+f(v_j) = \frac{n+2j}{2} +2n+\frac{n}{2} - 2j-1+n+j = 4n-1 = \lambda_3$(say).

Hence for each edge $uv\in E$, $f(u)+f(uv)+f(v)$ yields any one of the trimagic constants $\lambda_1 = 4n-2$, $\lambda_2 = \frac{7n-2}{2}$ and $\lambda_3 = 4n-1$. Therefore, the alternate triangular snake graph $A(TS_n)$ admits an edge trimagic total labeling when the triangle starts from u_2.

Hence the theorem follows from case 1 and case 2.

Theorem: 2.11 The Alternate Triangular Snake $A(TS_n)$ admits a super edge trimagic total labeling for even n.

Proof: We proved that the Alternate Triangular Snake $A(TS_n)$ has an edge trimagic total labeling. The labeling given in the proof of Theorem 2.10, the vertices get labels 1, 2, …, $n+\frac{n}{2}$. Since the Alternate Triangular Snake $A(TS_n)$ has $n+\frac{n}{2}$ vertices and the vertices have labels 1, 2, …, $n+\frac{n}{2}$ for even integer n, the Alternate Triangular Snake $A(TS_n)$ admits a super edge trimagic total labeling for even n.

Example: 2.12 A super edge trimagic total labeling of the Alternate Triangular Snake $A(TS_{10})$ of the triangle starts from u_1 and triangle starts from u_2 are given in figure 4 and figure 5 respectively.

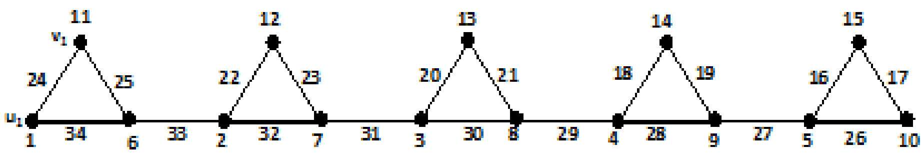

Figure 4: $A(TS_{10})$ with $\lambda_1 = 41$, $\lambda_2 = 36$ and $\lambda_3 = 42$.

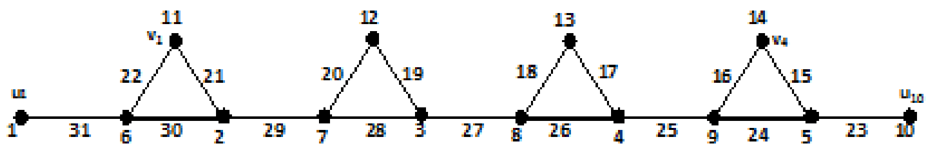

Figure 5: $A(TS_{10})$ with $\lambda_1 = 38$, $\lambda_2 = 34$ and $\lambda_3 = 39$.

Theorem: 2.13 The Alternate triangular snake $A(TS_n)$ admits an edge trimagic total labeling for odd n.

Proof: We consider the following two cases.

Case: 1 Triangle starts from u_1.

Let $V = \{u_i /1\le i \le n\}\cup\{v_j /1\le j\le \frac{n-1}{2}\}$ be the vertex set and $E = \{u_{2j-1}v_j, u_{2j}v_j /1\le j\le \frac{n-1}{2}\}\cup\{u_iu_{i+1} / 1\le i\le n-1\}$ be the edge set of the alternate triangular snake $A(TS_n)$. Since the triangle starts from u_1, the alternate triangular snake $A(TS_n)$ has $n+\frac{n-1}{2}$ vertices and $2n-2$ edges.

Define a bijection $f: V \cup E \to \{1, 2, \ldots, 3n+\frac{n-1}{2}-2\}$ such that

$$f(u_i) = \begin{cases} \frac{i+1}{2}, & 1\le i \le n \text{ and } i \text{ is odd} \\ \frac{n+i+1}{2}, & 1\le i \le n \text{ and } i \text{ is even}; \end{cases}$$

$f(v_j) = n+j,\ 1\le j \le \frac{n-1}{2};\ f(u_iu_{i+1}) = 3n+\frac{n-1}{2}-i-1,\ 1\le i\le n-1;$

$f(u_{2j-1}v_j) = 2n+\frac{n-1}{2}-2j,\ 1\le j \le \frac{n-1}{2};\ f(u_{2j}v_j) = 2n+\frac{n-1}{2}-2j+1,\ 1\le j \le \frac{n-1}{2}.$

Now we prove the above labeling is an edge trimagic total.

Consider the edges $u_iu_{i+1}, 1\le i\le n-1$;

For odd i, $f(u_i)+f(u_iu_{i+1})+f(u_{i+1}) = \frac{i+1}{2}+3n+\frac{n-1}{2}-j-1+\frac{n+i+1+1}{2} = 4n = \lambda_1$(say).

For even i, $f(u_i)+f(u_iu_{i+1})+f(u_{i+1}) = \frac{n+i+1}{2}+3n+\frac{n-1}{2}-i-1+\frac{i+1+1}{2} = 4n = \lambda_1$.

For the edges $u_{2j-1}v_j$, $1\le j \le \frac{n-1}{2}$;

$f(u_{2j-1})+f(u_{2j-1}v_j)+f(v_j) = \frac{2j-1+1}{2}+2n+\frac{n-1}{2}-2j+n+j = \frac{7n-1}{2} = \lambda_2$(say).

For the edges $u_{2j}v_j$, $1\le j \le \frac{n-1}{2}$;

$f(u_{2j})+f(u_{2j}v_j)+f(v_j) = \frac{n+2j+1}{2}+2n+\frac{n-1}{2}-2j+1+n+j = 4n+1 = \lambda_3$(say).

Hence for each edge $uv\in E$, $f(u)+f(uv)+f(v)$ yields any one of the trimagic constants $\lambda_1 = 4n$, $\lambda_2 = \frac{7n-1}{2}$ and $\lambda_3 = 4n+1$. Therefore, the alternate triangular snake graph $A(TS_n)$ admits an edge trimagic total labeling when the triangle starts from u_1.

Case: 2 Triangle starts from u_2.

Let $V = \{u_i /1\le i \le n\}\cup\{v_j/1\le j\le \frac{n-1}{2}\}$ be the vertex set and $E = \{u_{2j}v_j, u_{2j+1}v_j /1\le j\le \frac{n-1}{2}\}\cup\{u_iu_{i+1} / 1\le i\le n-1\}$ be the edge set of the alternate triangular snake $A(TS_n)$.Since the triangle starts from u_2, the alternate triangular snake $A(TS_n)$ has $n+\frac{n-1}{2}$ vertices and $2n-2$ edges. Define a bijection $f: V \cup E \to \{1, 2, \ldots, 3n+\frac{n-1}{2}-2\}$ such that

$$f(u_i) = \begin{cases} \frac{i+1}{2}, & 1\le i\le n \text{ and } i \text{ is odd} \\ \frac{n+i+1}{2}, & 1\le i\le n \text{ and } i \text{ is even}; \end{cases}$$

$f(v_j) = n+j,\ 1\le j \le \frac{n-1}{2};\ f(u_ju_{j+1}) = 3n+\frac{n-1}{2}-j-1,\ 1\le j\le n-1;$

$f(u_{2j}v_j) = 2n+\frac{n-1}{2}-2j+1,\ 1\le j \le \frac{n-1}{2};\ f(u_{2j+1}v_j) = 2n+\frac{n-1}{2}-2j,\ 1\le j\le \frac{n-1}{2};$

Now we prove the above labeling is an edge trimagic total.

Consider the edges u_iu_{i+1}, $1\leq i\leq n-1$;

For odd i, $f(u_i)+f(u_iu_{i+1})+f(u_{i+1}) = \frac{i+1}{2}+3n+\frac{n-1}{2}-i-1+\frac{n+i+1+1}{2} = 4n = \lambda_1$(say).

For even i, $f(u_i)+f(u_iu_{i+1})+f(u_{i+1}) = \frac{n+i+1}{2}+3n+\frac{n-1}{2}-i-1+\frac{i+1+1}{2} = 4n = \lambda_1$.

For the edges $u_{2j+1}v_j$, $1\leq j\leq \frac{n-1}{2}$;

$f(u_{2j+1})+f(u_{2j+1}v_j)+f(v_j) = \frac{2j+1+1}{2}+2n+\frac{n-1}{2}-2j+n+j = \frac{7n+1}{2} = \lambda_2$(say).

For the edges $u_{2j}v_j$, $1\leq j\leq \frac{n-1}{2}$;

$f(u_{2j})+f(u_{2j}v_j)+f(v_j) = \frac{n+2j+1}{2}+2n+\frac{n-1}{2}-2j+1+n+j = 4n+1 = \lambda_3$(say).

Hence for each edge $uv\in E$, $f(u)+f(uv)+f(v)$ yields any one of the trimagic constants $\lambda_1 = 4n$, $\lambda_2 = \frac{7n+1}{2}$ and $\lambda_3 = 4n+1$. Therefore, the alternate triangular snake graph $A(TS_n)$ admits an edge trimagic total labeling when the triangle starts from u_2.

Hence the theorem follows from case 1 and case 2.

Theorem: 2.14 The Alternate Triangular Snake $A(TS_n)$ admits a super edge trimagic total labeling for odd n.

Proof: We proved that the Alternate Triangular Snake $A(TS_n)$ has an edge trimagic total labeling. The labeling given in the proof of Theorem 2.13, the vertices get labels 1, 2, …, $n+\frac{n-1}{2}$. Since the Alternate Triangular Snake $A(TS_n)$ has $n+\frac{n-1}{2}$ vertices and the vertices have labels 1, 2, …, $n+\frac{n-1}{2}$ for odd integer n, the Alternate Triangular Snake $A(TS_n)$ admits a super edge trimagic total labeling for odd n.

Example: 2.15 A super edge trimagic total labeling of the Alternate Triangular Snake $A(TS_9)$ of the triangle starts from u_1 and triangle starts from u_2 are given in figure 6 and figure 7, respectively.

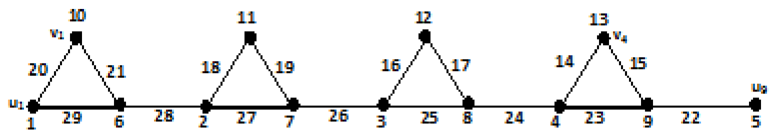

Figure 6: $A(TS_9)$ with $\lambda_1 = 36$, $\lambda_2 = 31$ and $\lambda_3 = 37$.

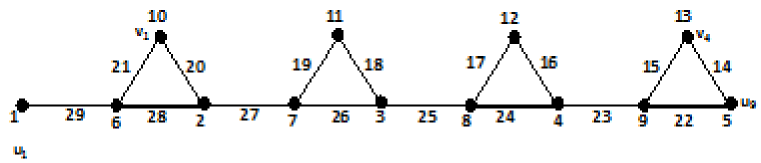

Figure 7: $A(TS_9)$ with $\lambda_1 = 36$, $\lambda_2 = 32$ and $\lambda_3 = 37$.

CONCLUSION

In this paper we proved that the corona graph $C_n\odot K_2$, double ladder P_nxP_3, quadrilateral snake Q_n, alternate triangular snake $A(TS_n)$ are edge trimagic total and super edge trimagic total graphs. There may be many interesting trimagic graphs can be constructed in future.

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