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COVER PEBBLING NUMBER FOR CUBE OF A PATH

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ABSTRACT

Given a configuration of pebbles on the vertices of a connected graph G, a pebbling move (or pebbling step) is defined as the removal of two pebbles from a vertex and placing one pebble on an adjacent vertex. The cover pebbling number $\mu(G)$ of a graph, is the least positive integer m such that however the m pebbles are placed on the vertices of G, we can eventually put a pebble on every vertex. In this paper we compute the cover pebbling number of the cube of a path.

Key words: *Graphs, Pebbling, Cover pebbling, Cube of a path.*

1. INTRODUCTION

Pebbling, one of the latest evolutions in graph theory proposed by Lagaris and Saks has been the topic of vast investigation with significant observations, Having Chung [1] as the forerunner to familiarize pebbling into writings, many other authors too have developed this topic. Hurlbert published a survey of pebbling results in [4]. Given a connected graph

G = (V, X), where V is the set of all vertices and X is the set of all edges, we distribute certain number of pebbles on the vertices in some configuration. Precisely, a configuration on a graph G is a function from V(G) to $N \cup \{0\}$ representing a placement of pebbles on G. The size of the configuration is the total number of pebbles placed on the vertices. A pebbling move is the removal of two pebbles from one vertex and the addition of one pebble to an adjacent vertex. In pebbling a target vertex is selected and the aim is to move a pebble to the target vertex. The minimum number of pebbles, such that regardless of their initial placement and regardless of the target vertex, we can pebble target vertex is called the pebbling number of G. In cover pebbling, the aim is to cover all the vertices with pebbles i.e., to move a pebble to every vertex of the graph simultaneously. The minimum number of pebbles required such that, regardless of their initial placement on G, there is a sequence of pebbling moves, at the end of which, every vertex has at least one pebble on it, is called the cover pebbling number of G. In [2].crul *et al* determine the cover pebbling number for path complete graphs, Fuses.

In this paper, we determine the cover pebbling number for cube of the path.

Theorem: 1.1[7] For a path on n vertices, $\pi(P_n) = 2^{n-1}$

Theorem: 1.2[2] For a path on n vertices $\mu(P_n) = 2^n - 1$

Theorem: 1.3[3] The cover pebbling number of the n-cube is $\mu(Q_n) = 3^n$.

Theorem: 1.4[2] For a complete graph K_t, $\mu(K_t) = 2t - 1$.

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2.1 Definition [7]

Let G= (V(G), E(G)) be a connected graph. The p^{th} power of G denoted by G^p is the graph obtained from G by adding the edge uv to G whenever $2 \le d(u, v) \le p$ in G, that is, $G^p = \{V(G), E(G) \cup uv: 2 \le d(u, v) \le p$ in G$\}$. If p = 1, we defined $G^1 = G$. We know that if p is large enough, that is $p \ge n-1$ then $G^p = K_n$.
We start with the cover pebbling number for cube of the path P_4.

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Theorem: 1 The cover pebbling number for P_4^3 is $\mu(P_4^3) = 7$.

Proof: If we place six pebbles on v_1, then we use 6 pebbles to pebble v_2, v_3, v_4. So v_1 remains uncovered.Thus $\mu(P_4^3) \geq 7$.

Now consider the distribution of seven pebbles on the vertices of P_4^3. If either $P(v_i) \geq 3, P(v_j) \geq 3$, $P(v_k) = 0$ for $i \neq j \neq k$ and $i, j, k \in \{1,2,3,4\}$ (or) $P(v_i) \geq 1$ $P(v_j) \geq 5$, $P(v_k) = 0$ for $i \neq j \neq k$ and $i, j, k \in \{1,2,3,4\}$ then we are done. Otherwise $P(v_i) = 7, P(v_j) = 0$ for $i \neq j$ and $i, j \in \{1,2,3,4\}$and so we are done . Thus $\mu(P_4^3) \leq 7$.

Theorem: 2 The cover pebbling number for P_5^3 is $\mu(P_5^3) = 11$.

Proof: Consider the configuration such that $p(v_1) = 10$ and $p(v_i) = 0$ for all $v_i \in V(P_5^3) - \{v_1\}$ then we use 10 pebbles to pebble v_2, v_3, v_4, v_5. So v_1 remains uncovered. Thus $\mu(P_5^3) \geq 11$

Now consider the distribution of eleven pebbles on the vertices of P_5^3.

If either $p(v_i) \geq 3$, $p(v_j) \geq 5$, $P(v_k) = 0$ for $i \neq j \neq k$ and $i, j, k \in \{1,2,3,4,5\}$ (or) $p(v_i) \geq 3$, $p(v_j) \geq 3$ $p(v_k) \geq 1$, $P(v_s) = 0$ for $i \neq j \neq k \neq s$ and $i, j, k, s \in \{1,2,3,4,5\}$then we are done. otherwise $P(v_i) = 11, P(v_j) = 0$ for $i \neq j$ and $i, j \in \{1,2,3,4,5\}$. So we can move one pebble to all vertices of P_5^3 and we are done.Thus $\mu(P_5^3) \leq 11$.

Theorem: 3 The cover pebbling number for P_6^3 is 15.

Proof: Consider the configuration such that $p(v_1) = 14$ and $p(v_i) = 0$ for all $v_i \in V(P_6^3) - \{v_1\}$ then we use 14 pebbles to pebble v_2, v_3, v_4, v_5, v_6. So v_1 remains uncovered. Thus $(P_6^3) \geq 15/\mu(P_6^3) \geq 15$.

Now consider the distribution of fifteen pebbles on the vertices of P_6^3.

If either $p(v_i) = p(v_j) = p(v_k) \geq 3$, $p(v_s) = 0$ for $i \neq j \neq k \neq s$ and $i, j, k, s \in \{1,2,3,4,5,6\}$ (or) $p(v_i) \geq 1$ $p(v_j) \geq 7$ for $i \neq j$ i, j $\in \{1,2,3,4,5,6\}$ then we are done. otherwise $p(v_i) = 15$, $p(v_j) = 0$ for $i \neq j$ i,j $\in \{1,2,3,4,5,6\}$.

So we can move one pebble to all vertices of P_6^3 and we are done. Thus $(P_6^3) \geq 15/\mu(P_6^3) \geq 15$

Theorem: 4 $\mu(P_n^3) = \begin{cases} 3\mu(P_{s+1}) - 2 \text{ when } n = 3s + 1, s = 1,2,3 \ldots\ldots. \\ 2\mu(P_{s+1}) + \mu(P_s) - 2, \text{when } n = 3s, s = 2,3,4 \ldots\ldots\ldots. \\ \mu(P_{s+1}) + 2\mu(P_s) - 2, \text{when } n = 3s - 1, s = 2,3, \ldots\ldots.. \end{cases}$

Proof: In P_n^3, clearly v_i ,3 < i < n-3, is adjacent to six vertices $v_{i-3}, v_{i-2}, v_{i-1,} v_{i+1}, v_{i+2}, v_{i+3}$ and v_i, n-3 $\leq$ i $\leq$ n, is adjacent to three vertices $v_{i-3}, v_{i-2}, v_{i-1,}$. Therefore to pebble the vertex v_i we use the path ....,v_{i-3} , v_i, v_{i+3} , ...So to pebble a vertex of P_n^3, we use any one of the following paths ....v_{n-6}, v_{n-3}, v_n (or)..........$v_{n-7}, v_{n-4}, v_{n-1}$ (or) ....$v_{n-8}, v_{n-5}, v_{n-2}$..

Case: (i) when $n = 3s + 1$,
$3\mu(P_{s+1}) - 3$ pebbles are placed on v_1, we use $\mu(P_{s+1}) - 1$ pebbles to pebble the vertices of the path $v_n, v_{n-3}, v_{n-6} \ldots\ldots\ldots\ldots\ldots.., v_4$, we use $\mu(P_{s+1}) - 1$ pebbles to pebble the vertices of the path $v_{n-1}, v_{n-4}, v_{n-7} \ldots\ldots\ldots\ldots\ldots.., v_3$ and we use $\mu(P_{s+1}) - 1$ pebbles to pebble the vertices of the the path $v_{n-2}, v_{n-5}, v_{n-8} \ldots.. v_2$. Then no pebbles will remain to cover v_1. Hence$\mu(P_n^3) \geq 3\mu(P_{s+1}) - 2$ when $n = 3s + 1, s = 1,2,3 \ldots\ldots$

We now use induction to show that $\mu(P_n^3) \leq 3\mu(P_{s+1}) - 2$ when $n = 3s + 1$, $s = 1,2,3 \ldots\ldots$

The assertion is clear for s =1 by theorem 1.Therefore we assume it is true for all $P_{3s'+1}^3$, when $1 \leq s' < s$.

Consider an arbitrary configuration of P_{3s+1}^3 having $3\mu(P_{s+1}) - 2$ pebbles. Clearly we can cover $v_{3s-1}, v_{3s}, v_{3s+1}$ in a finite number of movers with 32^s pebbles or less,since $\pi(v_{3s-1}) = \pi(v_{3s}) = \pi(v_{3s+1}) = 2^s$(these three vertices in the path P_{s+1}) . Thus, we need to cover $P_{3s-2}^3 = P_{3(s-1)+1}^3$ with the remaining $3\mu(P_{s+1}) - 2$ - $32^s = 3\mu(P_s) - 2$ *pebbles* . This number of pebbles is enough to cover P_{3s-2}^3 *by* hypothesis. Thus ,$3\mu(P_{s+1}) - 2 \leq \mu(P_n^3) \leq 3\mu(P_{s+1}) - 2$ when $n = 3s + 1, s = 1,2,3 \ldots\ldots$

A similar inductive proof works also for $\mu(P_n^3)$ when n = 3s-1,3s and yields the following results.

Case: (ii) $\mu(P_n^3) = 2\mu(P_{s+1}) + \mu(P_s) - 2, \text{when n} = 3\text{s}, \text{s} = 2,3,4 \dots\dots\dots$

Case: (iii) $\mu(P_n^3) = \mu(P_{s+1}) + 2\mu(P_s) - 2, \text{when n} = 3\text{s} - 1, \text{s} = 2,3,4 \dots\dots\dots$

Hence, $\mu(P_n^3) = \begin{cases} 3\mu(P_{s+1}) - 2 \text{ when n} = 3\text{s} + 1, s = 1,2,3 \dots\dots. \\ 2\mu(P_{s+1}) + \mu(P_s) - 2, \text{when n} = 3\text{s}, \text{s} = 2,3,4 \dots\dots\dots. \\ \mu(P_{s+1}) + 2\mu(P_s) - 2, \text{when n} = 3\text{s} - 1, \text{s} = 2,3, \dots\dots.. \end{cases}$

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