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CHEMICAL REACTION AND HEAT GENERATION ON MOVING ISOTHERMAL VERTICAL SURFACE THROUGH POROUS MEDIUM WITH UNIFORM MASS FLUX AND TRANSPIRATION

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ABSTRACT

Steady two-dimensional MHD heat and mass transfer free convection flow of an incompressible, viscous, electrically conducting and chemically reacting fluid via a porous medium along a continuously moving isothermal vertical surface in the presence of the magnetic field with heat source and transpiration is considered. The boundary layer equations are transformed into ordinary differential equations. The fluids considered in the study are air and water. Representative results for velocity, temperature, concentration, skin friction and rate of heat transfer are obtained for several values of pertinent parameters which are of physical and engineering interest. The numerical results of the velocity distribution of a chemically reacting fluid are compared with the corresponding flow problems for an ordinary fluid.

Key Words: *Chemical reaction; heat generation; porous medium; transpiration.*

2000 MSC: *80A20, 80A32, 76S05, 76W05.*

INTRODUCTION

The influence of magnetic field on viscous incompressible flow of electrically conducting fluid has its importance in many applications such as extrusion of plastics in the manufacture of Rayon and Nylon, purification of crude oil, pulp, paper industry, textile industry and in different geophysical cases etc. Also, the transpiration cooling is considered to be very effective process to protect certain structural elements such as combustion chamber walls, exhaust nozzle walls or gas turbine blades of turbo jets and rocket engines, from the influence of hot gases. In many process industries, the cooling of threads or sheets of some polymer materials is of importance in the production line. The rate of cooling can be controlled effectively to achieve final products of desired characteristics by drawing threads, etc. in the presence of an electrically conducting fluid subject to a magnetic field.

Flow in the boundary layer on moving solid surfaces was investigated by Sakaidis [1]. Tsou *et. al* [2] analyzed the hydrodynamic stability of the flow within the framework of small-perturbation stability theory. Vajravelu [3] studied the exact solution for hydrodynamic boundary layer flow and heat transfer over a continuous, moving, horizontal flat surface with uniform suction and internal heat generation/absorption. In all these studies, the authors have taken the continuous moving surface to be oriented in the horizontal direction. Again, Vajravelu [4] extended the problem of [3] to vertical surface. The heating as well as cooling effect of moving isothermal vertical plate were analyzed. Muthucumaraswamy and Kulandaivel [5] have discussed the MHD effects on moving isothermal vertical surface with uniform mass flux. Saxena and Dubey [6] investigated the MHD effects on moving isothermal vertical surface through porous medium with uniform mass flux and transpiration.

Chemical reactions usually accompany a large amount of exothermic and endothermic reactions. These characteristics can be easily seen in a lot of industrial processes. Recently, it has been realized that it is not always permissible to neglect the convection effects in porous constructed chemical reactors [7]. The present trend in the field of chemical reaction analysis is to give a mathematical model for the system to predict the reactor performance. A large amount of research work has been reported in this field and some of them are given in [8-11].

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In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications viz., polymer production, manufacturing of ceramics or glassware and food processing. Chambre and Young [12] have presented a first order chemical reaction in the neighborhood of a horizontal plate. Radiation and chemical reaction effects on an unsteady MHD convection flow past a vertical moving porous plate embedded in a porous medium with viscous dissipation was analyzed by Babu *et al* [13].

However, the theoretical solution for hydromagnetic chemically reacting convective flow on continuously moving isothermal vertical surface via a porous medium with mass flux and transpiration in the present of heat source is not studied in the literature. The present work is an attempt to shed some light on these issues. The study deals with heat and mass transfer effects on chemically reacting flow past an impulsively started vertical surface through porous medium in the presence of transverse magnetic field. The plate temperature is raised uniformly and the mass is diffused from the plate to the fluid at a uniform rate. Representative results for the velocity, temperature and concentration are displayed graphically showing the effect of various governing parameters entering into the problem. Also, we have prepared tables of the values of skin friction and rate of heat transfer (Nusselt number) displaying the effect of several material parameters. To the best of our knowledge this problem has not been studied before and the results reported here are new.

NOMENCLATURE

A	:	thermal diffusion parameter
C	:	concentration
C_p	:	specific heat at constant pressure
D	:	concentration diffusivity
g	:	acceleration due to gravity
G_r	:	Grashof number (Thermal Grashof number)
G_c	:	Modified Grashof number (Mass Grashof number)
K_0	:	permeability of the porous medium
K	:	porosity parameter
k	:	thermal conductivity of the fluid
K_r^*	:	coefficient of chemical reaction
K_r	:	chemical reaction parameter
m	:	mass flux per unit area
M	:	Magnetic field parameter (Hartmann number)
Nu	:	Nusselt number
P_r	:	Prandtl number
Q	:	coefficient of heat source
S	:	heat source parameter
T	:	fluid temperature
U	:	dimensionless velocity
S_c	:	Schmidt number
v_0	:	suction velocity
D_1	:	thermal diffusivity

GREEK SYMBOLS

β	:	coefficient of volume expansion
β^*	:	coefficient of concentration expansion
υ	:	kinematic viscosity
μ	:	dynamic viscosity
θ	:	dimensionless temperature
ϕ	:	dimensionless concentration
ρ	:	density of the fluid
τ	:	skin friction coefficient

SUPER/SUB SCRIPTS

$'$	:	dimensional properties
w	:	condition on the wall
∞	:	free stream conditions

MATHEMATICAL ANALYSIS

Consider a polymer or metal sheet extruded continuously from a die, or a long fiber or filament traveling between a feed roller and a take-up roller, are typical examples of moving continuous surface. It is well known that on a moving surface of finite length, the boundary layer grows in the direction opposite to the direction of the motion whereas on a moving continuous surface, such as a long continuous polymer sheet or fiber extruded from a slot and taken up by a wind-up roller at a finite distance away, the boundary layer on the sheet or fiber originates at the slot and grows in the direction of motion of the body.

We consider here a two dimensional steady incompressible flow of a viscous chemically reacting fluid in a continuous vertical surface through porous medium, issuing from a slot and moving with a uniform velocity u_w, in a fluid at rest, in the presence of a transverse magnetic field of strength B_0. Let the x-axis be taken along the direction of motion of the sheet in the upward direction and the y-axis is taken normal to it with velocity components u and v directed along their axes respectively. The surface temperature is raised uniformly and concentration level near the surface is also raised at a uniform rate. If σ is the electrical conductivity of the fluid, then the governing equations can be written in a Cartesian frame of reference, as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T'-T'_\infty)+g\beta^*(C'-C'_\infty)+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u-\frac{\nu}{K_0}u \tag{2}$$

$$\rho C_p\left(u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}\right)=k\frac{\partial^2 T'}{\partial y^2}+Q(T'-T'_\infty) \tag{3}$$

$$u\frac{\partial C'}{\partial x}+v\frac{\partial C'}{\partial y}=D\frac{\partial^2 C'}{\partial y^2}+D_1\frac{\partial^2 T'}{\partial y^2}-K_r^*(C'-C'_\infty) \tag{4}$$

The initial and boundary conditions are

$$\left.\begin{array}{lll} u=u_w\,, v=v_0=\text{const.}<0\,, T'=T'_w\,, \dfrac{\partial C'}{\partial y}=-\dfrac{m}{D} & \text{at} & y=0 \\ u\to 0\,, \quad T'\to T'_\infty\,, \quad C'\to C'_\infty & \text{as} & y\to\infty \end{array}\right\} \tag{5}$$

Making use of the assumptions that the velocity, temperature and concentration fields are independent of the distance parallel to the surface and the Boussinesq's approximation, equations (1) to (4) and the boundary conditions (5) can be written as

$$-v_0\frac{du}{dy}=g\beta(T'-T'_\infty)+g\beta^\star(C'-C'_\infty)+\nu\frac{d^2u}{dy^2}-\frac{\sigma B_0^2}{\rho}u-\frac{\nu}{K_0}u \tag{6}$$

$$-\rho C_p v_0\frac{dT'}{dy}=k\frac{d^2T'}{dy^2}+Q(T'-T'_\infty) \tag{7}$$

$$-v_0\frac{dC'}{dy}=D\frac{d^2C'}{dy^2}+D_1\frac{d^2T'}{dy^2}-K_r^*(C'-C'_\infty) \tag{8}$$

and the corresponding initial and boundary conditions are

$$\left.\begin{array}{lll} u=u_w\,, \quad T'=T'_w\,, \quad \dfrac{dC'}{dy}=-\dfrac{m}{D} & \text{at} & y=0 \\ u\to 0\,, \quad T'\to T'_\infty\,, \quad C'\to C'_\infty & \text{as} & y\to\infty \end{array}\right\} \tag{9}$$

introducing the following non-dimensional quantities

$$\left.\begin{array}{l} Y=\dfrac{yv_0}{\nu}\,,\ U=\dfrac{u}{u_w}\,,\ \theta=\dfrac{T'-T'_\infty}{T'_w-T'_\infty},\ \phi=\dfrac{C'-C'_\infty}{\left({m\nu}/{Dv_0}\right)},\ G_r=\dfrac{\nu g\beta\,(T'_w-T'_\infty)}{u_w v_0^2},\ G_c=\dfrac{\nu g\beta^*\left({m\nu}/{Dv_0}\right)}{u_w v_0^2}, \\ P_r=\dfrac{\mu C_p}{k},\quad M=\dfrac{\sigma B_0^2\nu}{\rho v_0^2}\,,\ S_c=\dfrac{\nu}{D}\,,\ A=\dfrac{D_1}{\nu}\dfrac{(T'_w-T'_\infty)}{\left({m\nu}/{Dv_0}\right)},\ K=\dfrac{K_0v_0^2}{\nu^2}\,,\ S=\dfrac{Q\nu}{\rho C_p v_0^2}\,,\ K_r=\dfrac{K_r^*\nu}{v_0^2} \end{array}\right\} \tag{10}$$

Equations (6) to (8) are reduced to the following dimensionless form

$$\frac{d^2U}{dY^2}+\frac{dU}{dY}-NU=-G_r\theta-G_c\phi \tag{11}$$

$$\frac{d^2\theta}{dY^2}+P_r\frac{d\theta}{dY}+SP_r\theta=0 \tag{12}$$

$$\frac{d^2\phi}{dY^2}+S_c\frac{d\phi}{dY}-K_rS_c\phi=-AS_c\frac{d^2\theta}{dY^2} \tag{13}$$

where $N=M+\frac{1}{K}$

and the corresponding initial and boundary conditions in non-dimensional form are

$$\left.\begin{array}{llll} U = 1\,, & \theta = 1\,, & \frac{d\phi}{dY} = -1 & \text{at } Y = 0 \\ U \to 0\,, & \theta \to 0\,, & \phi \to 0 & \text{as } \; Y \to \infty \end{array}\right\} \tag{14}$$

solving equations (11) to (13) with the boundary conditions (14), we get

$$U = a_1 e^{-b_1 Y} + a_2 e^{-b_2 Y} + a_3 e^{-b_3 Y} \tag{15}$$

$$\theta = e^{-b_2 Y} \tag{16}$$

$$\phi = a_4 e^{-b_2 Y} + a_5 e^{-b_3 Y} \tag{17}$$

where

$$a_1 = 1 - a_2 - a_3\,,\; a_2 = -\frac{G_r + G_c a_4}{b_2^2 - b_2 - N}\;,\; a_3 = -\frac{G_c a_5}{b_3^2 - b_3 - N}\;, a_4 = -\frac{A S_c b_2^2}{b_2^2 - S_c b_2 - K_r S_c}\;,$$

$$a_5 = \frac{1}{b_3}\left[1 + \frac{A S_c b_2^3}{b_2^2 - S_c b_2 - K_r S_c}\right],\; b_1 = \frac{1+\sqrt{1+4N}}{2}\,, b_2 = \frac{P_r + \sqrt{P_r^2 - 4SP_r}}{2}\;, b_3 = \frac{S_c + \sqrt{S_c^2 + 4K_r S_c}}{2}$$

The dimensionless skin friction at the surface is given by

$$\tau = \left(\frac{dU}{dY}\right)_{Y=0} = -a_1 b_1 - a_2 b_2 - a_3 b_3$$

In non-dimensional form, the rate of heat transfer (Nusselt number) is given by

$$Nu = -\left(\frac{\partial\theta}{\partial Y}\right)_{Y=0} = b_2$$

RESULTS AND DISCUSSION

In the preceding sections, we have obtained the mathematical expressions for *U*, θ and ϕ. The solutions are in terms of exponential functions. In order to get physical insight into the problem, the effects of the different flow parameters like chemical reaction parameter, porosity parameter, magnetic parameter (Hartmann number), heat source parameter, thermal diffusion parameter, Grashof number, modified Grashof number and Schmidt number on the velocity, temperature, concentration, skin friction and rate of heat transfer are presented and discussed with the help of tables and graphs. The fluids taken in this study are air (P_r=0.71) and water (P_r=7.0) and all the graphs are plotted against *Y*.

Fig.1 depicts the velocity profiles in air and water for different values of magnetic parameter *M*. In air and water, it is obvious that the existence of the magnetic field decreases the velocity which shows that the velocity decreases in the presence of magnetic field, as compared to its absence. This agrees with the expectations, since the magnetic field exerts a retarding force on the free convective flow.

The velocity profiles for different values of Schmidt number S_c are plotted in Fig. 2. The trend show that an increase in S_c, leads to a fall in the values of velocity, both in air and water. Also for any fixed value of S_c, the velocity increases in air as compared to that in water.

The influence of the porosity of the boundary on the velocity of the fluid medium has been shown in Fig. 3. It is seen that as porosity parameter *K* increases, the velocity increases in air and water both. It is also observed that for different values of *K*, the velocity graphs in water are lower than the respective velocity graphs in air.

Fig. 4 illustrates the effect of chemical reaction over velocity of the fluid. From Fig. 4, it is noticed that the velocity at the start of the boundary layer increases slowly till it attains the maximum value, after that they are decreasing and this trend is seen for all the values of the chemical reaction parameter K_r. Furthermore, decreasing velocity in air and water is seen due to increase in K_r. It is also observed that the velocity increases for the ordinary fluid as compared with the chemically reacting fluid.

The variations of the temperature profiles along the coordinate *Y* are depicted in Fig. 5 for several values of *S*. It is noticed that as *S* increases, the temperature increases. Also, the numerical results indicate that the effect of increasing values of Prandtl number P_r results in a decreasing thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of P_r are equivalent to increase in the thermal conductivity of the fluid, and thus heat is able to diffuse away from the heated surface more rapidly than for higher values of P_r.

The effect of concentration profiles for several values of thermal diffusion parameter A and chemical reaction parameter K_r is displayed in Figs. 6 and 7. Influence of A on the concentration is presented in Fig. 6. It is seen that an increase in A contributes to increase the concentration of the fluid medium both in air and water. The effect of K_r is dominant in the concentration field. From Fig. 7, it is observed that the concentration of the fluid medium decreases with an increase of K_r.

Table-1 shows the numerical values of skin friction for different values of flow and material parameters. It is observed that the effect of skin friction is more in water as compared to that in air. Furthermore, the skin friction increases due to an increase in G_r, G_c, A, K and it decreases due to an increase in K_r and S_c. The results also reveal that the skin friction is more in the absence of magnetic field as compared to its presence. Also, strong heat source results in an increasing skin friction in air while the reverse effect is observed in water. In table-2, the numerical values of Nusselt number for different values of P_r and S are displayed. It is observed that the Nusselt number increases due to increase in P_r and decreases due to increase in S.

CONCLUSIONS

We have examined the governing equations for a two dimensional steady MHD heat and mass transfer free convection flow of an incompressible, viscous, chemically reacting fluid past an impulsively started vertical surface through porous medium in the presence of transverse magnetic field with heat source whose plate temperature is raised uniformly and the mass is diffused from the plate to the fluid at a uniform rate. The fluids taken for the study are air and water. The following results of physical interest on the velocity, temperature and concentration of the flow field were obtained.

1. The velocity distribution is lower in water as compared with air.
2. A growing chemical reaction parameter or magnetic parameter or Schmidt number reduces the velocity of the flow field.
3. An increase in the velocity is seen due to increasing values of porosity parameter.
4. Growing source parameter leads to increase the temperature at all points.
5. The effect of increasing chemical reaction parameter is to decrease the concentration of the flow field.
6. The skin friction is more in water (P_r=7.0) as compared to that in air (P_r=0.71).

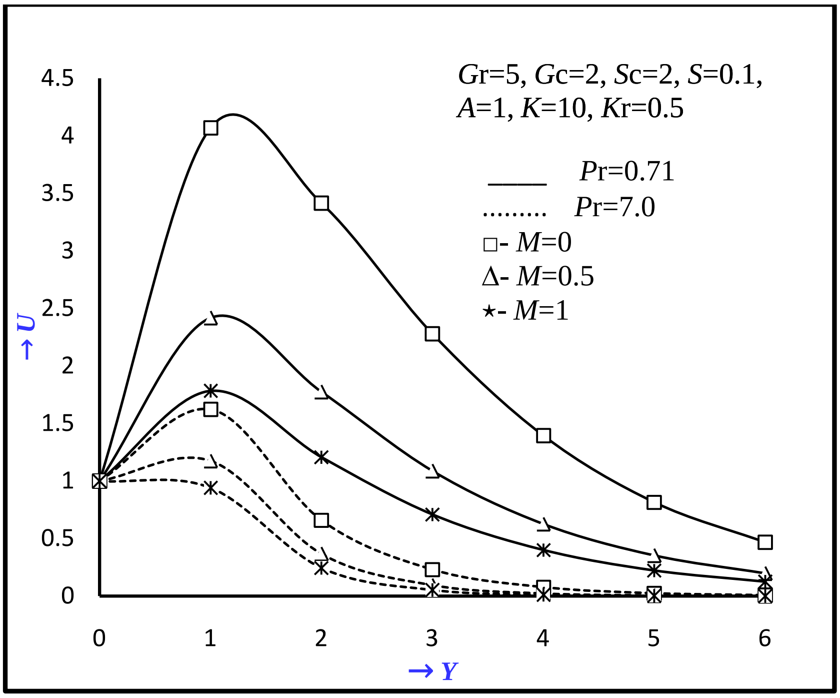

Fig. 1: Velocity profiles for different M in air and water

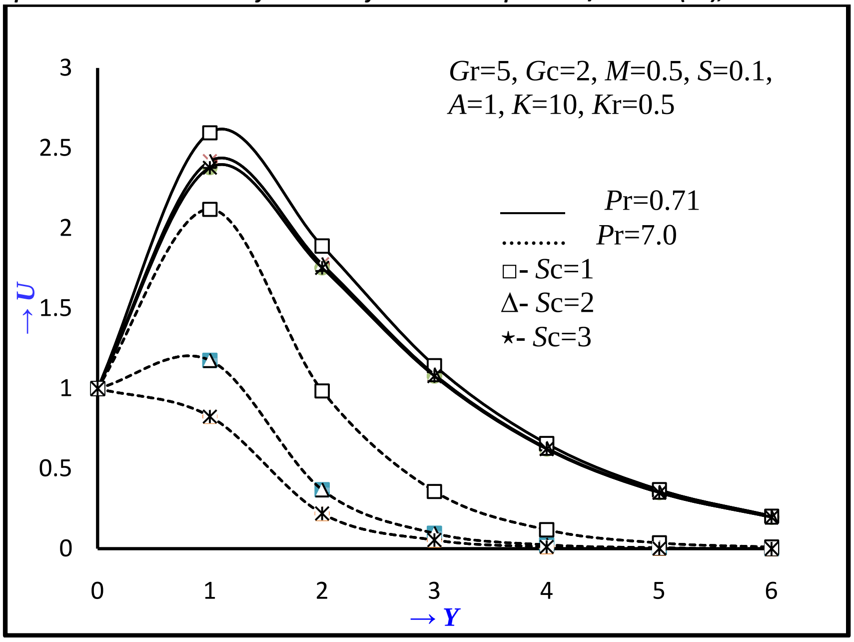

Fig. 2: Velocity profiles for different S_c in air and water

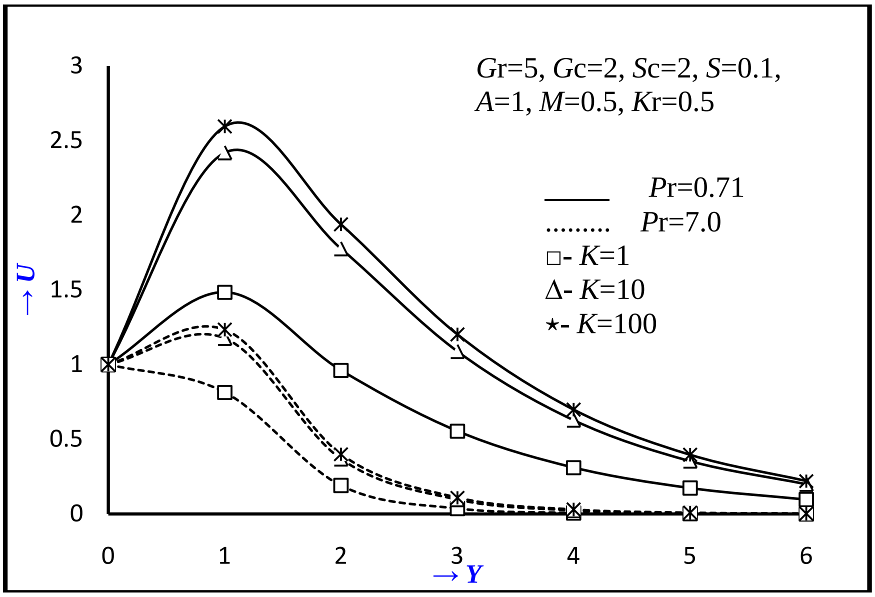

Fig. 3: Velocity profiles for different *K* in air and water

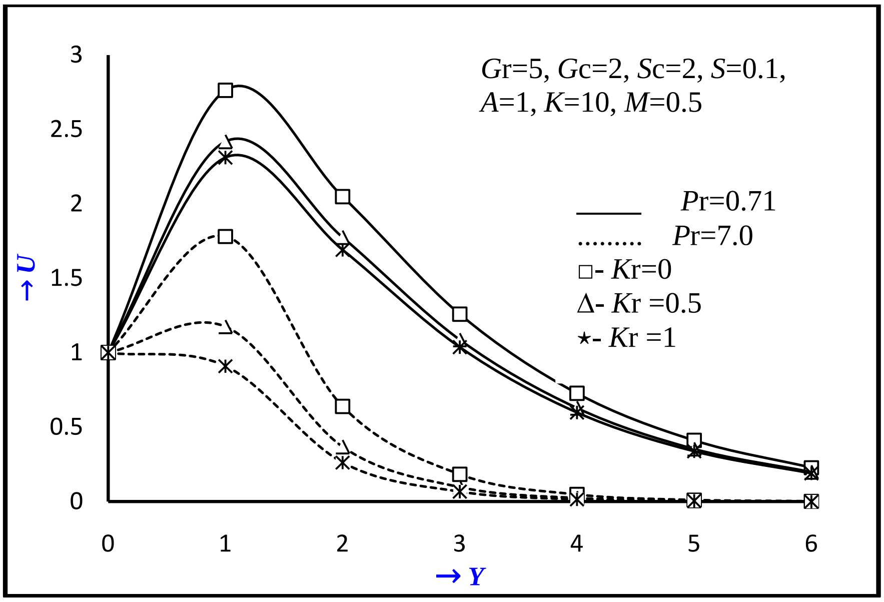

Fig. 4: Velocity profiles for different K_r in air and water

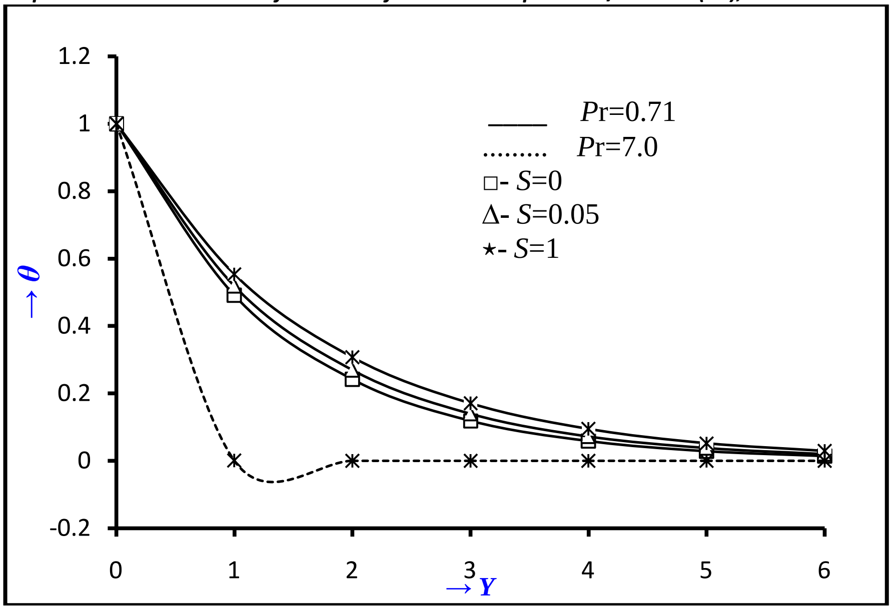

Fig. 5: Temperature profiles for different S in air and water

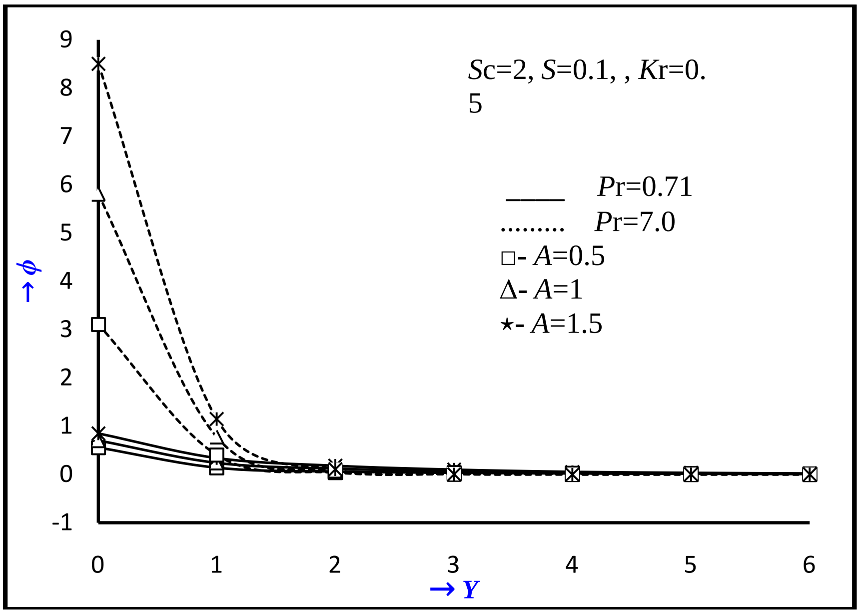

Fig. 6: Concentration profiles for different A in air and water

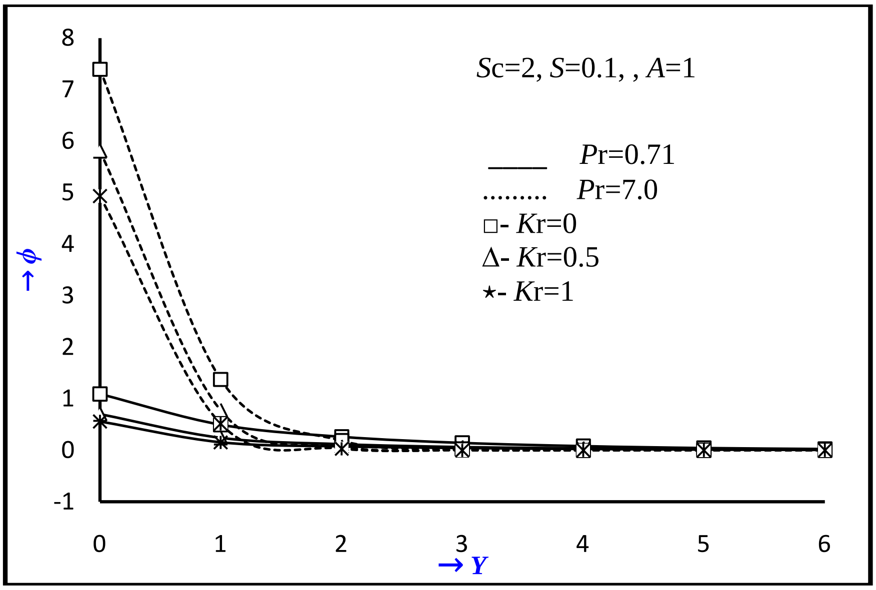

Fig. 7: Concentration profiles for different K_r in air and water

Table – 1: for values of skin friction

K_r	K	M	G_r	G_c	S_c	S	A	τ at $P_r = 0.71$	τ at $P_r = 7.0$
0.5	1	0.5	5	2	2	0.05	1	2.3559	3.4760
0.5	1	0.5	6	2	2	0.05	1	3.0322	3.6047
0.5	1	0.5	5	2	3	0.05	1	2.2316	2.8225
0.5	1	0	5	2	2	0.05	1	3.2160	4.0429
1.0	1	0.5	5	2	2	0.05	1	2.1278	2.5260
0.5	10	0.5	5	2	2	0.05	1	4.2700	4.6348
0.5	1	0.5	5	3	2	0.05	1	2.7547	5.8038
0.5	1	0.5	5	2	2	0.1	1	2.4532	3.4530
0.5	1	0.5	5	2	2	0.05	2	2.8976	7.8757

Table – 2: for values of Nusselt number

P_r	S	Nu
0.71	0.05	0.6559
0.71	0.1	0.5896
7.0	0.05	6.9496

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