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BIANCHI TYPE VIII COSMOLOGICAL MODEL WITH PERFECT FLUID IN $f(R,T)$ THEORY OF GRAVITATION

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ABSTRACT

In the present paper Bianchi type VIII space time with perfect fluid in $f(R,T)$ theory of gravitation (Harko et al, Phys. Rev. D.84, 024020, 2011) is presented. Exact solution of the field equations is obtained. It is observed that the universe is anisotropic with early acceleration and late deceleration.

***Keywords**: Bianchi type VIII cosmological model, Perfect fluid, $f(R,T)$ gravity.*

1. INTRODUCTION

Cosmological observations such as cosmic microwave background radiation, type Ia supernova [1, 2] shows that the universe is accelerating. There are various interesting possibilities invoked in order to explain acceleration of the universe. Among these $f(R)$ modified gravity models have attracted a lot of attentions [3-10]. In Einstein field equations of general relativity, which are derived from an action principle by Hilbert, by adopting a linear function of the scalar curvature R, in the gravitational lagrangian density. There is no reason why the gravitational action should be linear in the Ricci scalar R. It has been suggested that cosmic acceleration can be achieved by replacing Einstein-Hilbert action of general relativity with a general function $f(R)$. Harko *et al* [11] proposed another extension of standard general relativity $f(R,T)$. This theory of gravitation is generalizes $f(R)$ in which an arbitrary function of Ricci scalar is coupled with the stress energy tensor T.

Very recently Adhav [12] obtained exact solution of the field equations in respect of LRS Bianchi type I space time filled with perfect fluid in the frame work of $f(R,T)$. D.R.K. Reddy and R.Santhi Kumar [13] explores LRS Bianchi type II universe in $f(R,T)$ theory of gravity. FRW viscous fluid cosmological model in $f(R,T)$ gravity studied by Naidu *et al* [14].

Motivated by the above investigations we have investigated Bianchi type VIII cosmological model in $f(R,T)$ gravity.

2. METRIC AND FIELD EQUATIONS.

The action for the modified theory of gravity is

$$s=\frac{1}{16\pi}\int (f(R,T)+L_m)\sqrt{-g}\,d^4x \tag{1}$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar R and T is the trace energy momentum tensor of matter T_{ij}, L_m is the matter lagrangian density. The field equations of $f(R,T)$ gravity model as (Harko *et al* 2011)

$$f_R(R,T)R_{ij}=\frac{1}{2}f(R,T)g_{ij}+\left(g_{ij}\nabla^i\nabla_j-\nabla_i\nabla_j\right)f_R(R,T)=8\pi T_{ij}-f_T(R,T)T_{ij}-f_T(R,T)\theta_{ij} \tag{2}$$

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where $T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\partial\sqrt{-g}}{\partial g^{ij}}L_m, \theta_{ij} = -2T_{ij} - pg_{ij}$ (3)

Here $f_R = \frac{\partial f(R,T)}{\partial R}$, $f_T = \frac{\partial f(R,T)}{\partial T}$, ∇_j is covariant derivative.

Assume the function $f(R,T)$ given Harko *et al*

$$f(R,T) = R + 2f(T) \tag{4}$$

where $f(T)$ is an arbitrary function of trace of the stress energy tensor of matter. The matter lagrangian can be taken as $L_m = -p$. Also we choose

$$f(T) = \mu T \tag{5}$$

Therefore using equation (2), (3), (4) we obtain

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + \left(2pf'(T) + f(T)\right)g_{ij} \tag{6}$$

where the prime indicate the differentiation with respect to argument. We consider a spatially homogeneous Bianchi type VIII metric of the form

$$ds^2 = -dt^2 + R^2[d\theta^2 + \cosh^2(\theta)d\phi^2] + S^2[d\psi + \sinh(\theta)d\phi]^2 \tag{7}$$

where (θ,ϕ,ψ) are the Eulerian angles. R & S are functions of t only.

The energy momentum tensor for perfect fluid distribution is given by

$$T_{ij} = (\rho + p)u_i u_j + pg_{ij} \tag{8}$$

where ρ is the energy density, p is pressure. Here we consider the case when the perfect fluid obeys the following equation of the state

$$p = \gamma\rho \quad ,0 \le \gamma \le 1 \tag{9}$$

In commoving coordinate system we have

$$T_1^1 = T_2^2 = T_3^3 = p, T_4^4 = \rho \tag{10}$$

$$T = T_1^1 + T_2^2 + T_3^3 + T_4^4 = 3p - \rho \tag{11}$$

The field equation for the metric (7) takes the form

$$\frac{R_{44}}{R} + \frac{S_{44}}{S} + \frac{R_4 S_4}{RS} + \frac{S^2}{4R^4} = 8\pi p + 7\mu p - \mu\rho \tag{12}$$

$$2\frac{R_{44}}{R} + \frac{R_4^2}{R^2} - \frac{1}{R^2} - \frac{3S^2}{4R^4} = 8\pi p + 7\mu p - \mu\rho \tag{13}$$

$$2\frac{R_4 S_4}{RS} + \frac{R_4^2}{R^2} - \frac{1}{R^2} - \frac{S^2}{4R^4} = -8\pi\rho + 5\mu p - 3\mu\rho \tag{14}$$

where subscript '4' denotes differentiation with respect to t. using the transformation $R = e^{\alpha}, S = e^{\beta}, dt = R^2 S d\tau$ equations (12)-(14) reduce to

$$\ddot{\alpha}+\ddot{\beta}-\dot{\alpha}^2-2\dot{\alpha}\dot{\beta}+\frac{1}{4}e^{4\beta}=(8\pi p+7\mu p-\mu\rho)e^{4\alpha+2\beta} \tag{15}$$

$$2\ddot{\alpha}-\dot{\alpha}^2-2\dot{\alpha}\dot{\beta}-e^{2\alpha+2\beta}-\frac{3}{4}e^{4\beta}=(8\pi p+7\mu p-\mu\rho)e^{4\alpha+2\beta} \tag{16}$$

$$2\dot{\alpha}\dot{\beta}-e^{2\alpha+2\beta}-\frac{1}{4}e^{4\beta}=(-8\pi\rho+5\mu p-3\mu\rho)e^{4\alpha+2\beta} \tag{17}$$

where overhead dot indicate differentiation with respect to τ .

Using equations (15) & (16) we get

$$\ddot{\alpha}-e^{2\alpha+2\beta}-\ddot{\beta}-e^{4\beta}=0 \tag{18}$$

Assuming the relation between the metric potentials

$$\alpha=n\beta \tag{19}$$

where n is arbitrary constant. We obtain

$$\beta=\frac{1}{n-1}e^{2(n+1)\beta}+\frac{1}{n-1}e^{4\beta} \tag{20}$$

Taking $\beta=\log(u)$ & $n=0$ equation (20) reduces to

$$\frac{du}{d\tau}=u^3\left[\frac{c^2}{u^4}-\frac{1}{u^2}-\frac{1}{2}\right]^{1/2} \tag{21}$$

where c is integration constant.

On further integration equation (21) leads to

$$R=1 \tag{22}$$

$$S=\left[\frac{1}{2}+\frac{\sqrt{3}}{4}\cosh(2\tau)\right]^{-1/2} \tag{23}$$

3. PHYSICAL AND GEOMETRICAL PARAMETER

Density and Pressure are obtained as

$$\rho=\frac{\mu-6\pi-(1+4\pi)\sqrt{3}\cosh(2\tau)}{[5\mu^2-(8\pi+3\mu)(8\pi+7\mu)]\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\cosh(2\tau)\right)} \tag{24}$$

$$p=\frac{\mu}{8\pi+7\mu}\rho-\frac{1.25+\frac{\sqrt{3}}{2}\cosh(2\tau)}{(8\pi+7\mu)\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\cosh(2\tau)\right)} \tag{25}$$

Volume $V=\left[\frac{1}{2}+\frac{\sqrt{3}}{2}\cosh(2\tau)\right]^{-1/2}$ (26)

Expansion scalar $\theta = \dfrac{-\sqrt{3}\sinh(2\tau)}{3+3\sqrt{3}\cosh(2\tau)}$ (27)

Shear scalar $\sigma^2 = \dfrac{3\sinh^2(2\tau)}{\left(3+3\sqrt{3}\cosh(2\tau)\right)^2}$ (28)

Deceleration parameter $q = \dfrac{6\,\mathrm{cosech}(2\tau)}{\sqrt{3}\sinh(2\tau)-\cosh(2\tau)}$ (29)

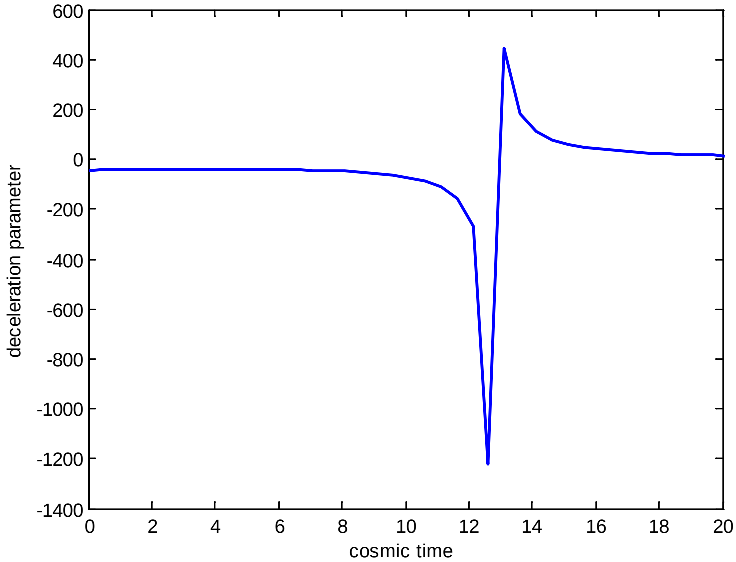

Fig. 1. Deceleration parameter versus time

It is from equations (27) and (28) observed that the ratio of shear scalar to expansion scalar is not zero. The value of expansion scalar tends to constant at large time. This means that the universe is anisotropic. The deceleration parameter at early epoch is negative and at late time positive i.e. early acceleration and late deceleration.

4. CONCLUSION

We have presented Bianchi type VIII universe in the framework of modified $f(R,T)$ theory of gravitation in the presence of perfect fluid. We found that the universe is anisotropic. Early acceleration and late time deceleration nature of the universe is also observed.

5. REFERENCES.

[1] Riess *et.al*: Astron J.116 (1998), 100.

[2] Perlmutter *et.al:* Astrophys.J.517, (1999), 565.

[3] Nojiri S., Odintsov S.D.: Phys.Rev.D.74, (2006), 086005.

[4] Carroll S.M., Duvvuri V., Trodden M., Turner M.S.: Phys.Rev.D.70, (2004), 043528.

[5] Capozziello S., Cardone V.F., Corloni S., Troisi A.: Int.J.Mod.Phs.D.12, (2003), 1969.

[6] Multamaki T., Vilja I.: Phys.Rev.D.73, (2006), 024018.

[7] Starobinski A.A.: JETP Lett. 86, (2007), 157.

[8] Bertoami O., Boehamer C.G., Harko T., Lobo F.S.N.: Phys.Rev.D.75, (2007), 104016.

[9].Nozari K., Azizi T.:Phys. Lett.B.680, (2009), 205.

[10] Sotiriou T.P., Farooni V.: Rev.Mod.Phys.82, (2010), 457.

[11] Harko T., Lobo F.S.N., Nojiri S.,Odintsov S.D:arXiv.1104.2669v2 [gr-qc] (2011).

[12] Adhav K.S.: Astrophys Space.Sci., 339, (2012), 365.

[13] DRK Reddy, R.Santhikumar: Global Journal of Science frontier research physics and space science Vol.13, issue2 ver.1.0, (2013).

[14] R.L.Naidu, K.Dasu Naidu, T. Ramprasad, DRK Reddy: Journal of Science frontier research physics and space science Vol.13, issue 3 ver.1.0, (2013).

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