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wgα-closed and wαg-closed in Ideal Topological Spaces

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ABSTRACT

In this paper some properties of wgα-I-closed sets and wαg-I- closed sets are studied.

Keywords: *gα-I-closed, αg-I- closed, wgα-I-closed, wαg-I- closed.*

1. INTRODUCTION AND PRELIMINARIES

An ideal I on a topological space (X,τ) is a non-empty collection of subsets of X which satisfies the following properties. (1) A∈I and B⊆A implies B ∈ I, (2) A∈I and B∈I implies A∪B∈I. An ideal topological space is a topological space (X, τ) with an ideal I on X and is denoted by (X, τ, I). For a subset A ⊆ X, A*(I,τ) = {x ∈ X: A ∩ U ∉I for every U∈τ (X, x)} is called the local function of A with respect to I and τ [7]. We simply write A* in case there is no chance for confusion. A kuratowski closure operator cl*(.) for a topology τ*(I,τ) called the *- topology, finer than τ is defined by cl*(A) = A ∪ A* [11]. If A ⊆ X, cl(A) and int(A) will respectively, denote the closure and interior of A in (X, τ).

Definition: 1.1 A subset A of a topological space (X, τ) is called
1. α-closed [10], if cl (int(cl(A))) ⊆ A
2. αg-closed [5], if α cl (A) ⊆ U whenever A ⊆ U and U is open in (X, τ)
3. gα-closed [5], if αcl(A) ⊆ U whenever A ⊆ U and U is α-open in (X, τ)
4. wgα-closed [6], if αcl (int(A)) ⊆ U whenever A ⊆ U and U is α-open in(X, τ).
5. wαg-closed [6], if αcl (int(A)) ⊆ U whenever A ⊆ U and U is open in (X, τ).
6. g-closed [8], if cl (A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
7. gs-closed [1], if scl (A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
8. sg-closed [3], if scl (int(A)) ⊆ U whenever A ⊆ U and U is semi open in (X, τ)
9. β-closed [10], if int(cl (int(A))) ⊆ A

The complements of the above mentioned closed sets are called their respective open sets.

Definition: 1.3 A subset A of an ideal topological spaces (X,τ, I) is said to be
1. α - I – closed [4], if cl (int*(cl(A))) ⊆ A
2. gα - I – closed [9], if α I cl(A) ⊆ U whenever A ⊆ U and U is α - open in X.
3. αg - I – closed [9], if α I cl(A) ⊆ U whenever A ⊆ U and U is open in X.

The complements of the above mentioned closed sets are called their respective open sets.

2. wgαI-closed and wαgI-closed sets

Definition: 2.1 A subset A of an Ideal topological space (X, τ, I) is said to be
1) wgαI-closed set, if αIcl(Int (A)) ⊆U whenever A ⊆U and U is α-open in X.
2) wαgI-closed set, if αIcl(Int (A)) ⊆U whenever A ⊆U and U is open in X.

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Proposition: 2.2 Every α-I-closed set is wgαI-closed set but not conversely.

Proof: Assume that a subset A of (X, τ, I) is α-I-closed set. Let U be an α-open set containing A. Then αIcl(A) ⊆U as A is α-I-closed. So αIcl(Int (A)) ⊆αIcl(A) ⊆U. This implies that αIcl(Int (A)) ⊆U. Hence A is wgαI-closed.

Example: 2.3 Let X = {a, b, c}, τ = {ϕ, {a}, X}} and I = {ϕ, {c}}. Then A = {c} is a wgαI-closed but not α-I-closed.

Proposition: 2.4 Every gα-closed set is wgαI-closed set but not conversely.

Proof: Let A be a subset of (X, τ) which is gα-closed and Let U be an α-open set containing A. Since A is gα-closed ,αcl (A) ⊆ U, αIcl(A) ⊆ αcl(A) ⊆U. This implies that αIcl(Int (A)) ⊆ αIcl (A) ⊆U. Hence A is wgαI-closed.

Example: 2.5 Let X = {a, b, c}, τ = {ϕ, {a, b}, X}} and I = {ϕ,{a}}. Then A = {b} is a wgαI-closed but not gα-closed.

Proposition: 2.6 Every gα-I-closed set is wgαI-closed set but not conversely.

Proof: Assume that a subset A of (X, τ, I) is gα-I-closed set. Let U be an α-open set containing A. Therefore αIcl(Int (A)) ⊆ αIcl(A) ⊆ U, therefore A is wgα-I-closed.

Example: 2.7 Let X = {a, b, c}, τ = {ϕ, {a, b}, X}} and I = {ϕ,{a}}. Then A = {b} is a WgαI-closed but not gα-I-closed.

Remark: 2.8 suppose I= {ϕ}, then the notion of wgαI-closed and wαgI-closed sets coinside with wgα-closed and wαg-closed set.

Remark: 2.9 The following examples show that the concepts of g-closed and wgαI-closed sets are independent.

Example: 2.10 Let X = {a, b, c}, τ = {ϕ, {a,b}, X}} and I = {ϕ,{a}}. Then A = {b} is a wgαI-closed but not g-closed set.

Example: 2.11 Let X = {a, b, c}, τ = {ϕ, {a}, X}} and I = {ϕ,{c}}. Then A = {a, b} is a g-closed but not wgα-I-closed.

Remark: 2.12 The following examples show that the concepts of sg-closed and wgαI-closed sets are independent.

Example: 2.13 Let X = {a, b, c}, τ = {ϕ, {a,b}, X}} and I = {ϕ,{a}}. Then A = {b} is a wgαI-closed but not sg-closed.

Example: 2.14 Let X = {a, b, c}, τ = {ϕ, {a},{b},{a, b}, X}} and I = {ϕ,{a}}. Then A = {b} is a sg-closed but not wgα-I-closed.

Remark: 2.15 The following examples show that the concept of gs-closed and wgαI-closed sets is independent.

Example: 2.16 Let X = {a, b, c}, τ = {ϕ, {a},{b},{a,b}, X}} and I = {ϕ,{a}}. Then A = {c} is a gs-closed but not wgα-I-closed.

Example: 2.17 Let X = {a, b, c}, τ = {ϕ, {a, b}, X}} and I = {ϕ,{a}}. Then A = {a} is a wgαI-closed but not gs-closed.

Proposition: 2.18 Every α-I-closed set is wαgI-closed set but not conversely.

Proof: Assume that a subset A of (X,τ, I) is α-I-closed set. Let U be an α-open set containing A. Then αIcl(A) ⊆U, as A is α-I-closed. So αIcl(Int (A)) ⊆αIcl(A) ⊆U. This implies that αIcl(Int (A)) ⊆U. Hence A is wαgI-closed.

Example: 2.19 Let X = {a, b, c}, τ = {ϕ, {a}, X}} and I = {ϕ, {a}, {b}, {a, b}}. Then A = {a, b} is a wαg-I-closed but not α-I-closed.

Proposition: 2.20 Every αg-closed set is wαgI-closed set but not conversely.

Proof: Assume that a subset A of (X,τ) is αg-closed set. Let U be an open set containing A. Then αcl(A) ⊆U, as A is αg-closed. Since every α-I-closed set is αclosed, αIcl(Int (A)) ⊆αIcl(A) ⊆U. Hence A is wαgI-closed.

Example: 2.21 Let X ={a, b, c}, τ = {ϕ, {a, b}, X}} and I = {ϕ,{a}}. Then A = {a} is a wαg-I-closed but not αg-closed.

Proposition: 2.22 Every αgI-closed set is wαgI-closed set but not conversely.

Proof: Assume that a subset A of (X,τ, I) is αgI-closed set. Let U be an open set containing A. Then αIcl(A) ⊆U and αIcl(Int (A)) ⊆αIcl(A) ⊆U. Hence A is wαgI-closed.

Example: 2.23 Let X = {a, b, c}, τ = {ϕ, {a}, {a, c}. X}} and I = {ϕ, {a}}. Then A = {c} is a wαg-I-closed but not αg-I-closed.

Remark: 2.24 The following examples show that the concept of β-closed and wαgI-closed sets are independent.

Example: 2.25 Let X = {a, b, c}, τ = {ϕ, {a}, {a, c}. X}} and I = {ϕ, {b}}. Then A = {a, b} is a wαg-I-closed but not β-closed.

Example: 2.26 Let X = {a, b, c}, τ = {ϕ, {b},{c},{b, c}. X}} and I = {ϕ, {c}}. Then A = {a} is β-closed but not wαg-I-closed.

Remark: 2.27 The following examples show that the concepts of sg-closed and wαgI-closed sets are independent.

Example: 2.28 Let X = {a, b, c}, τ = {ϕ, {a}, {a, c}. X}} and I = {ϕ, {a}}. Then A = {a, b} is a wαg-I-closed but not sg-closed.

Example: 2.29 Let X = {a, b, c}, τ = {ϕ, {a}, {b}, {a, b}. X}} and I = {ϕ, {a}}. Then A = {b} is sg-closed but not wαg-I-closed.

Remark: 2.30 The following examples show that the concepts of gs-closed and wαgI-closed sets are independent.

Example: 2.31 Let X = {a, b, c}, τ = {ϕ, {b}, {c}, {b, c}. X}} and I = {ϕ, {c}}. Then A = {c} is gs-closed but not wαg-I-closed.

Example: 2.32 Let X = {a, b, c}, τ = {ϕ,{a, b}. X}} and I = {ϕ, {a}}. Then A = {a} is wαg-I-closed but not gs-closed.

Proposition: 2.33 Every g-closed set is wαgI-closed set but not conversely.

Proof: Assume that a subset A of (X,τ) is g-closed set. Let U be an open set containing A. Then cl(A) ⊆U, as A is g-closed. Then αcl(A)⊆cl(A) ⊆U. Since every α-I-closed set is α-closed.αIcl(A)⊆ αcl(A)⊆cl(A) ⊆U. So αIcl(Int (A)) ⊆αIcl(A) ⊆U. Hence A is wαgI-closed.

Example: 2.34 Let X = {a, b, c}, τ = {ϕ, {a}, {a, c}. X}} and I = {ϕ, {a}}. Then A = {c} is wαg-I-closed but not in g-closed.

Proposition: 2.35 Every gαI-closed set is wαgI-closed set but not conversely.

Proof: Assume that a subset A of (X,τ,I) is gαI-closed set. Let U be α-open set containing A. From the above theorems, αIcl(Int (A)) ⊆U. Since every open set is α-open. Hence A is wαgI-closed.

Example: 2.36 Let X = {a, b, c}, τ = {ϕ,{a},X}} and I = {ϕ,{a},{b},{a, b}}. Then A = {a,c} is wαg-I-closed but not in gαI-closed.

Proposition: 2.37 Every wgαI-closed set is wαgI-closed set but not conversely.

Proof. Let A be a wgαI-closed set in (X,τ, I) and Let U be an open set containing A. Since A is wαgI-closed. So αIcl(Int (A)) ⊆U. Hence A is wαgI-closed in (X, τ, I).

Example: 2.38 Let X = {a, b, c}, τ = {ϕ, {a}, X}} and I = {ϕ, {c}. Then A = {a, b} is wαg-I-closed but not in wgαI-closed.

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