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STUDY OF A MATHEMATICAL MODEL OF THE ARTERIAL

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ABSTRACT

This paper deal with study of blood flow which was derived from the two-dimensional Navier-stokes equations. To study the mathematical model for blood flow with artery. The blood to flow smoothly through the blood vessels. Blood flow rate through the arteries which also may affect the blood pressure. The governing equation are solved numerically by using finite difference technique, arterial blood flow.

Key Words*: Newtonian fluid, finite difference technique, arterial blood flow.*

AMS Subject Classification *(1991):760z05, 92c35.*

1. INTRODUCTION

This work is important for human health. There are several researchers, who examined the blood flow in arteries. The blood flow is significantly already and fluid dynamical factors play an important role. This work will focus on the diastolic hypertension. blood is a complex fluid consisting of partulate solids suspended in a nonenewtonian fluid. which it self is a complex mixture of proteins and platelets of particulate solids are red cells, white blood cells and platelets. But Belardinelli and cavalcanti(1991) discussed a new non-linear two dimensional model of blood motion in tapered and elastic vessels jung *et.al* 2004 gave idea on axi-symmetric flows of non –Newtonian fluids in symmetric sensed artery. in this work , the hemodynamic behavior of the blood flow is influenced by the presence of the arterial stenosis, again belardinelli and cavacanti(1992) investigated about the theoretical analysis of pressure pulse propagation in arterial vessel of pressure pulse produced by a single flow pulse applied at the proximal vessel extremity, although takuji and guimaraes (1998) observed that the effect of non-newtonian property of blood on flow through a stenosed tube. Blood behaves as a non-newtonioan fluid but in this model , blood is assumed to be a Newtonian fluid. Even though this will make the problem much simpler. It still is valid since blood in large vessel acting almost like a Newtonian fluid.

2. MATHEMATICAL FORMULATION

Let us assume a cylindrical co-ordinate system (r, z, t) having the z-axis along the axis of arterial segment, while r and t are taken along the radial and circumferential direction. The arterial vessel is assumed to be a rectilinear, deformable, thick shell of isotropic, incompressible material with a circular section and with longitudinal movements, while the blood is considered as an incompressible Newtonian fluid and flow is axially symmetric.

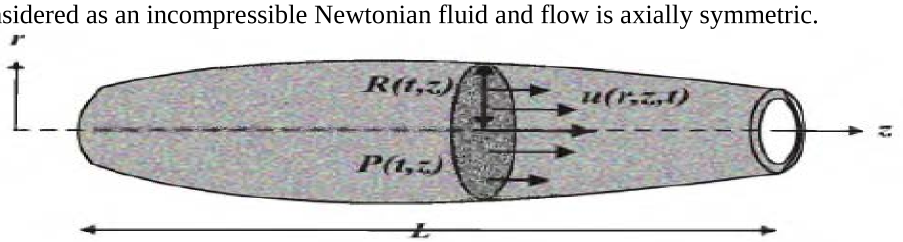

Figure 1: an arterial segment of a visco –elastic artery with length L

The model approach is to use the two-dimensional Navier –stock equation and continuity equation for a Newtonian and incompressible fluid is given by

Equation of continuity

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} \qquad (1)$$

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Equation of radial momentum

$$\frac{\partial w}{\partial r} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\left(\frac{\partial^2 y}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right) + F(t) \quad (2)$$

Equation of radial momentum

$$\frac{\partial u}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - \frac{u}{r^2}\right) \quad (3)$$

Equation of continuity

$$\frac{1}{r} + \frac{\partial (rw)}{\partial x} + \frac{\partial u}{\partial z} = 0 \quad (4)$$

where f(t) is body acceleration, ρ is density ,p pressure, r is radial axis, t is time, μ is viscosity, u is radial velocity, w is axial velocity and z is axial axis. The pressure gradient $\frac{\partial p}{\partial z}$ produces by the pumping action of the heart is given by

$$\frac{1}{\rho}\frac{\partial p}{\partial z} = A_0 + A_1 \cos\varpi t \quad (5)$$

Where A_0 is constant amplitude of the pressure gradient. A_l is amplitude of the pulsatile component given rise to systolic and disstolic pressure, $\omega = 2\pi f_{p,}$ where $f_{p,}$ the pulse frequency. there is is no radial flow along the axis of the artery initial and assumed to be zero. The normal component of velocity abnd shear stress vanish along the axis of symmetry.

These may be stated mathematically as

$$U(r, z, t) = 0, \frac{\partial w(r,z,t)}{\partial r} = 0 \text{ on r} \quad (6)$$

The radial velocity may be assumed to be equal the changes of radius of the tapered arterial segement in the constricted artery. The velocity boundary conditions on the arterial wall are taken as

$$U(r,z,t) = \frac{\partial R}{\partial t}, \ \omega(r,z,t) = 0 \ on\ r \quad (7)$$
$$= R(z, t)$$

When the system at rest it means that no flow takes places

$$U(r,z,t) = 0 = \omega(r,z,t) = 0 \text{ on t} \quad (8)$$
$$= 0$$

3. NUMERICAL PROCEDURE

For convenience, we define a new variable which is the radial coordinate $x = \frac{r}{R(z,t)}$ (9)

Where R(z,t) denote the inner radius of the vessels and x be arterial wall viscosity.

In order to make the uniform grid construction easier. Using the above transformation the Eq. (1). and Eq.-(2).Eq.-(3) can be rewritten as follow

$$\frac{1}{R}\frac{\partial u}{\partial x} + \frac{u}{xR} + \frac{\partial w}{\partial z} = -\frac{x}{R}\frac{\partial R}{\partial z}\frac{\partial w}{\partial x} \quad (10)$$

$$\frac{\partial w}{\partial r} = \frac{1}{R}\left[x\left(w\frac{\partial R}{\partial z} + \frac{\partial R}{\partial t} - u\right]\frac{\partial w}{\partial x} = -w\frac{\partial w}{\partial z} + \frac{\mu}{\rho R^2}\left(\frac{\partial^2 w}{\partial x^2} + \frac{1}{x}\frac{\partial w}{\partial x}\right) - \frac{1}{\rho}\frac{\partial p}{\partial z}\right) + F(t) \quad (11)$$

$$\frac{\partial u}{\partial t} = \frac{1}{R}\left[x\left(w\frac{\partial R}{\partial z} + \frac{\partial R}{\partial t}\right) - u\right]\frac{\partial u}{\partial x} = -w\frac{\partial u}{\partial z} + \frac{\mu}{\rho R^2}\left(\frac{\partial^2 u}{\partial x^2} + \frac{1}{x}\frac{\partial w}{\partial x}\right) - \frac{u}{x^2}) \quad (12)$$

In order to obtain the radial velocity component, multiplying Eq.(10) by xR and integrating respect to from the limit 0 to x, we obtain

$$\int_0^x x\frac{\partial u}{\partial x}dx + \int_0^x u\,dx + \int_0^x xR\frac{\partial w}{\partial z}\,dx - \int_0^x x^2\frac{\partial R}{\partial z}\frac{\partial w}{\partial x}dx = 0$$

Simplifying the above equation gives

$$u(x,z,t) = -\frac{R}{x}\int_0^x x\frac{\partial w}{\partial z}\,\mathrm{dx} + \frac{\partial R}{\partial z}\left[xw - \frac{2}{x}\int_0^x xw\,dx\right]\mathrm{dx} = 0 \tag{13}$$

From the conditions at the wall (Eq.7), w=0 when x=1

$$-\int_0^1 x\frac{\partial w}{\partial z}\,\mathrm{dx} = -\frac{1}{R}\frac{\partial R}{\partial t} + \frac{2}{R}\frac{\partial R}{\partial z}\int_0^1 xw\,dx$$

With f(x) = 6(2x-1) satisfying $\int_0^1 xf(x)dx = 1.$ comparing the left hand side and right hand side of the abpove equation, we can write

$$\frac{\partial w}{\partial z} = \frac{-6}{R}\,(2x-1)\frac{\partial R}{\partial t} - \frac{2}{R}\frac{\partial R}{\partial t}w$$

Substitute eq. (14) in to eq. (13) to obtain the component of radial velocity

$$u(x,z,t) = x\left[\frac{\partial R}{\partial z}w + \frac{\partial R}{\partial t}\,[4x-3]\right] \tag{15}$$

Finite difference scheme

The finite difference for solving eq. (11) and eq. (15) is based upon the central difference approximations.

$$\frac{\partial w}{\partial z} = \frac{w_{i+1,}^k - w_{i-1,j}^k}{2\Delta z}\ ,\frac{\partial w}{\partial x} = \frac{w_{i,j+1,}^k - w_{i,j-1}^k}{2\Delta x}\ ,\frac{\partial^2 w}{\partial x^2} = \frac{w_{i,j+1,}^k + w_{i,j-1}^k - 2w_{i,j}^k}{\Delta x^2}$$

$$\frac{\partial u}{\partial z} = \frac{u_{i+1,}^k - u_{i-1,j}^k}{2\Delta z}\ ,\frac{\partial u}{\partial x} = \frac{u_{i,j+1,}^k - u_{i,j-1}^k}{2\Delta x}\ ,\frac{\partial u}{\partial t} = \frac{w_{i,j}^{k+1} - w_{i,j}^k}{\Delta t}$$

Here, w(x, z, t) and u(x, z, t) are discreatized to w($x_j\, z_i\, t_k$) and u($x_j\, z_i\, t_k$) and are by

$$w_{i,j}^{k+1} = w_{i,j}^k + \Delta t\Big[-\frac{1}{\rho}\Big(\frac{\partial p}{\partial z}\Big)^{k+1} + \Big\{\frac{x_j}{R_i^k}\Big(\frac{\partial p}{\partial z}\Big)^k - \frac{u_{i,j}^k}{R_i^k}\Big(\frac{\partial R}{\partial z}\Big)_i^k\ w_{i,j}^k\Big\}\frac{w_{i,j+1,}^k - w_{i,j-1}^k}{2\Delta x} + \frac{\mu}{\rho(R_i^k)^2}\Big(\frac{(w)_{i,j+1}^k - 2(w)_{i,j}^k + (w)_{i,j-1}^k}{(\Delta x)^2} + \frac{1}{x_j}\frac{(w)_{i,j+1}^k - (w)_{i,j-1}^k}{2\Delta x}\Big) - w_{i,j}^k\Big(\frac{w_{i+1,}^k - w_{i-1,j}^k}{2\Delta z}\Big)\Big] + F(t)$$

$$u_{i,j}^{k+1} = x_j\Big[\frac{\partial R}{\partial z}\Big)_i^k\ w_{i,j}^k + \frac{\partial R}{\partial z}\Big)_i^k\,\big[4x_j - 3\big)\Big]$$

Flow rate, q consider as the quantity or volume of fluid moving in the artery with fixed time the equation of flow rate given by

$$Q_i^k = 2\pi(R_i^k)^2\int_0^1 x_j\, w_{i,j}^k\, dx_j$$

4. RESULT AND DISCUSSION

The pressure in artery may increase because of large amount of blood flow through the arteries in a smaller cross sectional area. Thus the blood pressure increases and contributes to high blood pressure.

When the cross sectional area is below 0.8 cm^2 then the blood flow rate decreases faster than the normal rate. But figure shows that the effect of blood flow rate when the cross sectional area is the range between 0.1 cm^2 to 0.8 cm^2. then the blood flg.

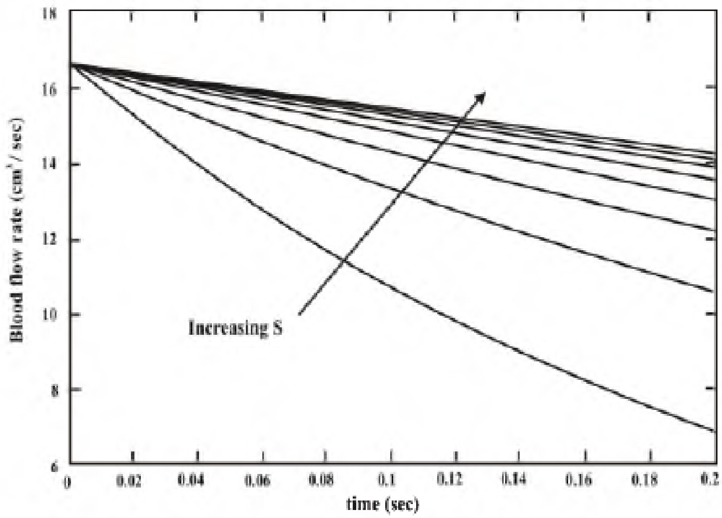

Figure 2: blood flow rate when the cross –sectional area is in the range between 0.1.cm^2 to 0.8 cm^2.

CONCLUSION

In this study, we have study the effect of blood flow in artery. even though, the model does not include viscoelastic. We observed that a small change in the value for the cross sectional may affect the amount of blood flow rate through the arteries, which also may affect the blood pressure.

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