

Fuzzy Bi- $\Gamma$ -ideals in  $\Gamma$ -semi near-field spaces (F-Bi-I- $\Gamma$ -SNF-S)

N V Nagendram<sup>1\*</sup>

Assistant Professor (Mathematics),

Department of Science & Humanities (Mathematics), Lakireddy Balireddy College of Engineering,  
Mylavaram 521 230 Krishna District - Andhra Pradesh India.

E-mail: [nvn220463@yahoo.co.in](mailto:nvn220463@yahoo.co.in)

T V Pradeep Kumar<sup>2</sup>

Assistant Professor, Department of Mathematics,

Acharya Nagarjuna University College of Engineering, Acharya Nagarjuna University, Nambur,  
Nagarjuna Nagar 523 510, Andhra Pradesh, India

Y Venkateswara Reddy<sup>3</sup>

Professor (Retd.) cum Subject Advisor, Department of Mathematics,

Acharya Nagarjuna University College of Engineering, Acharya Nagarjuna University, Nambur,  
Nagarjuna Nagar 523 510, Andhra Pradesh, India.

(Received on: 28-10-13; Revised & Accepted on: 28-11-13)

---

ABSTRACT

In this paper, we consider the fuzzification of bi- $\Gamma$ -ideals in  $\Gamma$ -semi near-field spaces, and investigate some of their related properties. Maximal fuzzy bi- $\Gamma$ -ideals of  $\Gamma$ -semi near-field spaces are introduced and their properties discussed. Finally, chain conditions relating to fuzzy bi- $\Gamma$ -ideals of  $\Gamma$ -semi near-field spaces are investigated

**Key words:**  $\Gamma$ -semi near-field space, fuzzy  $\Gamma$ -ideals, fuzzy bi- $\Gamma$ -ideals, fuzzy interior,  $\Gamma$ -ideals.

**AMS 2000 Subject classification code:** 16D50, 20M99, 20N25, 16P20, 16Y30, 03E72.

---

SECTION 1: INTRODUCTION

Fuzzy bi- $\Gamma$ -ideals in  $\Gamma$ -semi near-field spaces were first introduced in the literature by N V Nagendram in depth study of existing literature of Zadeh [25], the fuzzy set theories developed by zadeh and others found many applications in the domain of mathematics and elsewhere.

$\Gamma$ -semi near-field spaces defined by N V Nagendram and the ideal theory in  $\Gamma$ -semi near-field spaces studied by N V Nagendram, Dr. T V Pradeep Kumar and Dr. Y Venkateswara Reddy. Fuzzy ideals of rings were introduced by Liu and have been studied by several authors. The notion of fuzzy ideals and its properties were applied to various areas: semi-groups and BCK algebras and semi-rings. The classification of left (resp. right) ideals of  $\Gamma$ -near-rings, and investigated the related properties by Y B Jun. Also he introduced the notion of fuzzy characteristic left (resp. right) ideals and normal fuzzy left (resp. right) ideals of  $\Gamma$ -near-rings, and studied some of their related properties.

In continuation, I state fuzzy bi- $\Gamma$ -ideals in  $\Gamma$ -semi near-field spaces, and investigate its properties. As we know,  $\Gamma$ -semi groups are generalization of semi-groups, Chinram [4] studied some properties of bi-ideals in semi-groups and it has motivated us to study and to introduce the notion fuzzification of a bi- $\Gamma$ -ideals in a  $\Gamma$ -semi near-field space as a generalization of a  $\Gamma$ -semi near-field.

In this paper, we consider a fuzzification of the concept of a bi- $\Gamma$ -ideal in a  $\Gamma$ -semi near-field space and some properties of such bi- $\Gamma$ -ideals are investigated. The homeomorphic property of fuzzy bi- $\Gamma$ -ideal is established. The concept of a fuzzy interior  $\Gamma$ -bi-ideal and a fuzzy  $\Gamma$ -ideal are also introduced and some properties are discussed. Now the notion of maximal fuzzy bi- $\Gamma$ -ideals of  $\Gamma$ -semi near-field spaces discussed. Finally chain conditions relating to fuzzy bi- $\Gamma$ -ideals of  $\Gamma$ -semi near-field spaces are discussed.

---

Corresponding author: N V Nagendram<sup>1\*</sup>

## SECTION 2: PRELIMINARIES

We first recall some basic concepts for the sake of completeness.

**Definition 2.1:** A near-ring space  $N$  is a system with two binary operations “+” and “ $\cdot$ ” such that:  
 (1)  $(N, +)$  is a near- ring,( not necessarily abelian) (2)  $(N, \cdot)$  is a semi-near- ring and  
 (3)  $(x + y)z = xz + yz$ , for all  $x, y, z \in N$ .

We will use the word ”near-ring” to mean ”right distributive near-ring” and write  $xy$  instead of  $x \cdot y$ .

Note that  $0 \cdot x = 0$  and  $(-x)y = -xy$  but in general  $x \cdot 0 = 0$  for some  $x \in N$ .

**Definition 2.2:** Let  $(N, +, \cdot)$  be a near-ring. A subset  $I$  of  $N$  is said to be an ideal of  $N$  if:

- (1)  $(I, +)$  is a normal subgroup of  $(N, +)$ ,
- (2)  $IN \subseteq I$ ,
- (3)  $n_1(n_2 + i) - n_1 n_2 \in I$ , for all  $i \in I$  and  $n_1, n_2 \in N$ .

If  $I$  satisfies (1) and (2), then it is called a right ideal of  $N$ . If  $I$  satisfies (1) and (3), then it is called a left ideal of  $N$ .

**Definition 2.3:** A  $\Gamma$ -near-ring is a triple  $(N, +, \Gamma)$  where

- (i)  $(N, +)$  is a group,
- (ii)  $\Gamma$  is a non-empty set of binary operators on  $N$  such that  $\forall \beta \in \Gamma, (N, +, \beta)$  is a near-ring and
- (iii)  $x\beta(y\alpha z) = (x\beta y)\alpha z$  for all  $x, y, z \in N$  and  $\beta, \alpha \in \Gamma$ .

**Definition 2.4:** A subset  $A$  of a  $\Gamma$ -near-ring  $N$  is called a left ideal (or right ideal) of  $N$  if (i)  $(A, +)$  is a normal divisor of  $(N, +)$ , (ii)  $u\beta(x + v) - u\beta v \in A$  i.e.,  $u\beta x \in A$  for all  $x \in A, \beta \in \Gamma$  and  $u, v \in N$ .

**Definition 2.5.:** A near-field space  $N$  is a system with two binary operations “+” and “ $\cdot$ ” such that:

- (1)  $(N, +)$  is a near-field ( $N$  not necessarily abelian) (2)  $(N, \cdot)$  is a semi-near-field and
- (3)  $(x + y)z = xz + yz$ , for all  $x, y, z \in N$ .

We will use the word ”near-field” to mean ”right distributive near-field” and write  $xy$  instead of  $x \cdot y$ .

Note that  $0 \cdot x = 0$  and  $(-x)y = -xy$  but in general  $x \cdot 0 = 0$  for some  $x \in N$ .

**Definition 2.5:** Let  $S$  be a semi near-field. By a sub semi near-field of  $S$  we mean a non-empty sub near-field space  $A$  of  $S$  such that  $A^2 \subseteq A$ .

For the sake of completeness, we now recall some concepts of fuzzy theory.

**Definition 2.6:** A mapping  $\lambda: N \rightarrow [0, 1]$  is called fuzzy near-field space of  $N$  and complement of a near-field space  $\lambda$ , denoted by  $\lambda'$  is the fuzzy near-field space in  $N$  given by  $\lambda'(x) = 1 - \lambda(x)$  for all  $x \in N$ . the level near-field space of a fuzzy near-field space  $\lambda$  of  $N$  is defined as  $U(\lambda; t) = \{x \in N / \lambda(x) \geq t\}$ .

**Definition 2.7:** A fuzzy near-field space  $\lambda$  in  $N$  is called a fuzzy sub semi near-field space of  $N$  if  $\lambda(xy) \geq \min\{\lambda(x), \lambda(y)\} \forall x, y \in N$ .

**Definition 2.8:** A sub semi near-field space  $A$  of a semi near-field space  $N$  is called a bi-ideal of  $N$  if  $ANA \subseteq A$ .

**Definition 2.9:** A fuzzy sub semi near-field space  $\lambda$  of a semi near-field space  $N$  is called a fuzzy bi-ideal of  $N$  if  $\lambda(xyz) \geq \min\{\lambda(x), \lambda(z)\} \forall x, y$  and  $z \in N$ .

**Definition 2.10:** Let  $N = \{x, y, z, \dots\}$  and  $\Gamma = \{\alpha, \beta, \gamma, \dots\}$  be two non-empty semi near-field spaces. Then  $N$  is called a  $\Gamma$ -semi near-field space if it satisfies the axioms

- (i)  $x\gamma y \in N$ ,
- (ii)  $(x\alpha y)\gamma z = x\alpha(y\gamma z), \forall x, y, z \in N$  and  $\alpha, \gamma \in \Gamma$ .

**Definition 2.11:** Let  $N$  be a  $\Gamma$ -semi near-field space. A non-empty  $\Gamma$ -semi near-field sub space  $A$  of a  $\Gamma$ -semi near-field space  $N$  is said to be a  $\Gamma$ -sub semi near-field space if  $A\Gamma A \subseteq A$ .

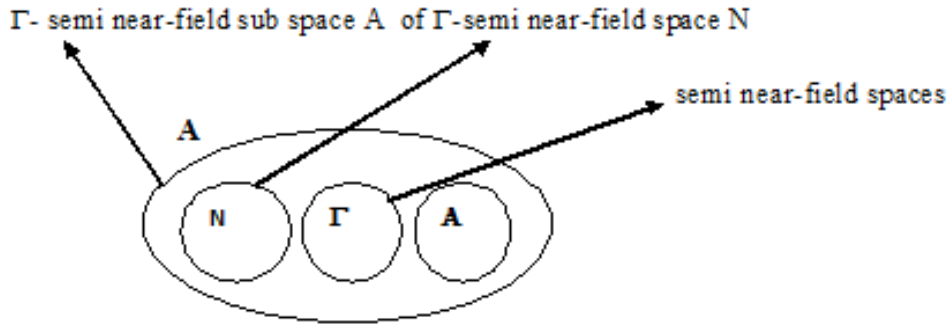


Fig.01

**Definition 2.12:** A left (right) ideal of a  $\Gamma$ -semi near-field space  $N$  is a non-empty  $\Gamma$ -semi near-field sub space  $A$  of a  $\Gamma$ -semi near-field space  $N$  such that  $N\Gamma A \subseteq A$  ( $A\Gamma M \subseteq A$ ).

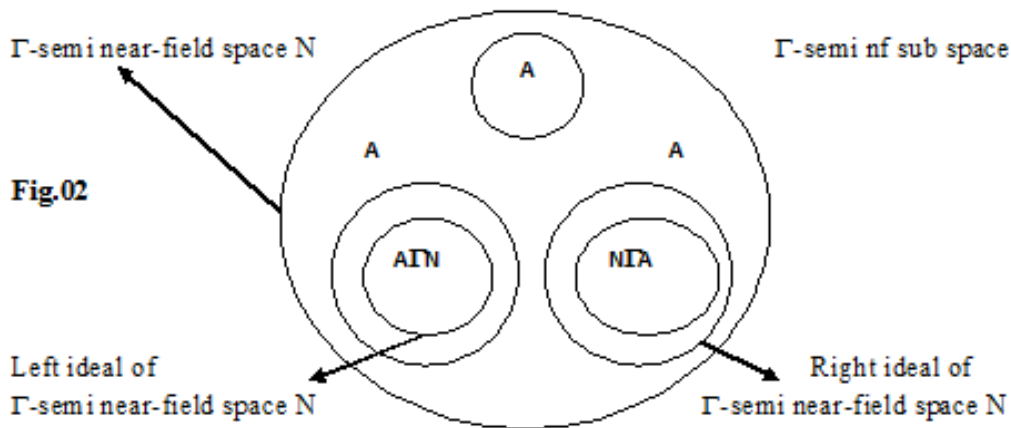


Fig.02

**Definition 2.13:** If  $A$  is both a left and right ideal of a  $\Gamma$ -semi near-field space  $N$ , then we say that  $A$  is a  $\Gamma$ -ideal of ( $\Gamma$ -semi near-field space)  $N$ .

**Definition 2.14:** A  $\Gamma$ -semi near-field space  $N$  is called left-zero (or right-zero) if  $x\gamma y = x$  (or  $x\gamma y = y$ ) for all  $x, y \in N$  and  $\gamma \in \Gamma$ .

**Definition 2.15:** An element  $e$  in a  $\Gamma$ -semi near-field space  $N$  is called an idempotent if  $e\gamma e = e$  for some  $\gamma \in \Gamma$ .

**Definition 2.16:** A  $\Gamma$ -sub semi near-field space  $A$  of  $\Gamma$ -semi near-field space  $N$  is called an interior  $\Gamma$ -ideal of  $N$  if  $M\Gamma A\Gamma M \subseteq A$ .

**Definition 2.17:** Let  $N$  be a  $\Gamma$ -semi near-field space. A sub  $\Gamma$ -semi near-field space  $A$  of  $N$  is called a bi- $\Gamma$ -ideal of  $N$  if  $A\Gamma M\Gamma A \subseteq A$ .

**Definition 2.18:** Let  $N$  be a  $\Gamma$ -semi near-field space and  $N_1$  a  $\Gamma_1$ -semi near-field space. A pair of mappings  $f_1 : N \rightarrow N_1$  and  $f_2 : \Gamma \rightarrow \Gamma_1$  is said to be a homomorphism from  $(N, \Gamma)$  to  $(N_1, \Gamma_1)$  if  $f_1(axb) = f_1(a)f_2(x)f_1(b)$  for all  $a, b \in N$  and  $x \in \Gamma$ .

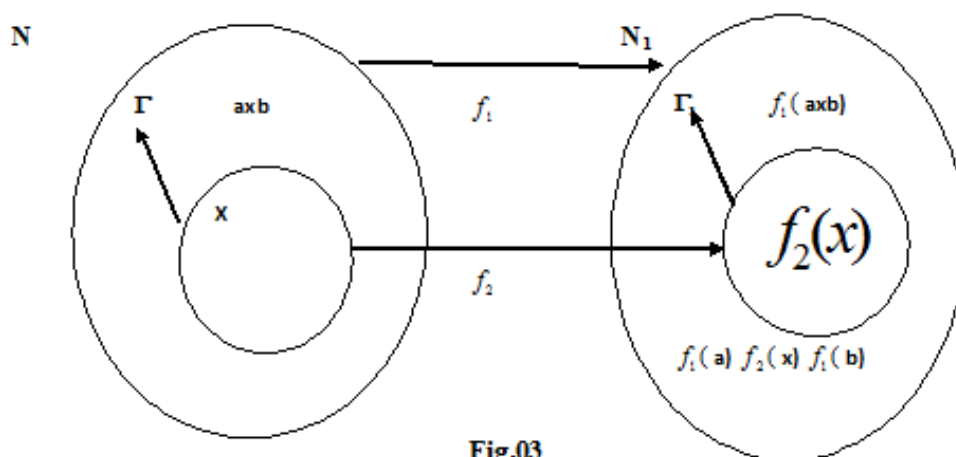


Fig.03

$(f_1, f_2)$  pair is a Homomorphism from  $(N, \Gamma)$  to  $(N_1, \Gamma_1)$

**Definition 2.19:** A fuzzy set  $\lambda$  of a  $\Gamma$ -semi near-field space is called a fuzzy  $\Gamma$ -semi near-field space of  $N$  if and only if  $\lambda(x \gamma y) \geq \min\{\lambda(x), \lambda(y)\} \forall x, y \in N$  and  $\gamma \in \Gamma$ .

**Definition 2.20:** A  $\Gamma$ -semi near-field space  $N$  is said to satisfy the left (or right) ascending chain condition a.c.c. of left (or right) ideals if every strictly increasing sequence  $U_1 \subset U_2 \subset U_3 \subset \dots$  of left (right) ideals of  $N$  is of finite length.

**Definition 2.21:** A  $\Gamma$ -semi near-field space  $N$  is said to satisfy the left (or right) Notherian if  $N$  satisfies the left (or right) ascending chain condition a.c.c. of left (or right) ideals.

**Note 2.22:** Let  $E_N$  denote the set of all idempotent elements in a  $\Gamma$ -semi near-field space  $N$  and  $\chi_A$  the characteristic function of a sub  $\Gamma$ -semi near-field space  $A$  of  $N$ . And Let  $N$  denote  $\Gamma$ -semi near-field space otherwise specified and  $\lambda$  of a near-field space  $N$  is a notion of fuzzy bi- $\Gamma$ -ideal of  $N$ .

**Definition 2.23:** A fuzzy set  $\lambda$  of  $N$  is called a fuzzy bi- $\Gamma$ -ideal of  $N$  if satisfies following axioms:

- (i)  $\lambda(x \gamma y) \geq \min\{\lambda(x), \lambda(y)\} \forall x, y \in N$  and  $\gamma \in \Gamma$ ,
- (ii)  $\lambda(x\beta y\alpha z) \geq \min\{\lambda(x), \lambda(z)\} \forall x, y, z \in N$  and  $\beta, \alpha \in \Gamma$ .

**Example 2.24:** Let  $N = \{0, p, q, r\}$  and  $\Gamma = \{\gamma, \beta, \alpha\}$  be the non-empty near-field space of binary operations defined and shown in composition tables as below:

$\beta$	0	p	q	r
0	0	0	0	0
p	p	p	p	p
q	0	0	0	q
r	0	0	0	r

$\alpha$	0	p	q	r
0	0	0	0	0
p	0	p	0	0
q	0	0	q	0
r	0	0	0	r

$\gamma$	0	p	q	r
0	0	0	0	0
p	0	p	0	p
q	0	q	0	r
r	0	0	0	q

Fig.04

Clearly  $N$  is a  $\Gamma$ -semi near-field space. Moreover the fuzzy near-field space map defined  $\lambda: N \rightarrow [0,1]$  such as  $\lambda(0) = 0.6, \lambda(p) = 0.7, \lambda(q) = 0.8, \lambda(r) = 0.9$  is a fuzzy bi- $\Gamma$ -ideal of a  $\Gamma$ -semi near-field space  $N$ .

**SECTION: 3 some results on fuzzy bi- $\Gamma$ -ideals in a  $\Gamma$ -semi near-field space**

In this section we obtain some results on fuzzy bi- $\Gamma$ -ideals in a  $\Gamma$ -semi near-field space  $N$ .

**Lemma 3.1:** If  $W$  is a bi- $\Gamma$ -ideal of  $N$  then  $\forall 0 < q < 1, \exists$  a fuzzy bi- $\Gamma$ -ideal  $\lambda$  of  $\Gamma$ -semi near-field space  $N \ni \lambda_q = W$ .

**Proof:** Let  $\lambda : N \rightarrow [0, 1]$  defined by  $\lambda(x) = \begin{cases} q, & \text{if } x \in W, \\ 0, & \text{if } x \notin W, \end{cases}$  where  $q$  is a fixed number in  $(0, 1)$ . Then, clearly  $\lambda_q = W$ .

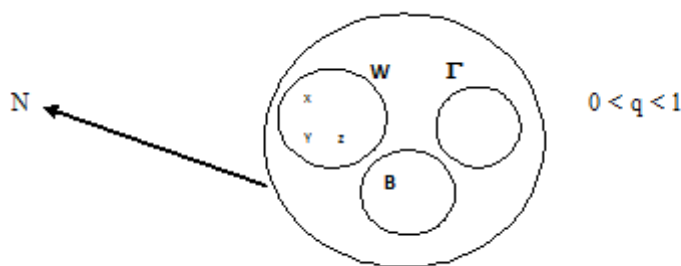


Fig.05

Now suppose that  $W$  is a bi- $\Gamma$ -ideal of  $N$ .  $\forall x, y \in W$  and  $\gamma \in \Gamma \ni x\gamma y \in W$ , we have  $\lambda(x\gamma y) \geq q = \min \{\lambda(x), \lambda(y)\} \forall x, y \in N$  and  $\gamma \in \Gamma$ . Also for all  $x, y, z \in W$  and  $\beta, \alpha \in \Gamma$  such that  $x\beta y\alpha z \in W$ , we have  $\lambda(x\beta y\alpha z) \geq q = \min \{\lambda(x), \lambda(z)\}$ . Thus  $\lambda$  is a fuzzy bi- $\Gamma$ -ideal of  $\Gamma$ -semi near-field space  $N$ . This completes the proof of Lemma.

**Lemma 3.2:** Let  $W$  be a non-empty near-field subspace of  $\Gamma$ -semi near-field space  $N$ . Then  $W$  is a bi- $\Gamma$ -ideal  $\lambda$  of  $\Gamma$ -semi near-field space  $N$  if and only if  $\chi_w$  is a fuzzy bi- $\Gamma$ -ideal  $\lambda$  of  $N$ .

**Proof:** ( $\Rightarrow$ if) Let  $x, y \in W$  and  $\gamma \in \Gamma$ . From hypothesis,  $x\gamma y \in W$ .

(a) If  $x, y \in W$  and  $\gamma \in \Gamma$ , then  $\chi_w(x) = 1$  and  $\chi_w(y) = 1$ .

In this case  $\chi_w(x\gamma y) = 1 \geq \min \{\chi_w(x), \chi_w(y)\}$ .

(b) If  $x \in W, y \notin W$  and  $\gamma \in \Gamma$ , then  $\chi_w(x) = 1$  and  $\chi_w(y) = 0$ .

So,  $\chi_w(x\gamma y) = 0 \geq \min \{\chi_w(x), \chi_w(y)\}$ .

(c) If  $x \notin W, y \in W$  and  $\gamma \in \Gamma$ , then  $\chi_w(x) = 0$  and  $\chi_w(y) = 1$ .

So,  $\chi_w(x\gamma y) = 0 \geq \min \{\chi_w(x), \chi_w(y)\}$ .

(d) If  $x \notin W, y \notin W$  and  $\gamma \in \Gamma$ , then  $\chi_w(x) = 0$  and  $\chi_w(y) = 0$ .

So,  $\chi_w(x\gamma y) \geq 0 = \min \{\chi_w(x), \chi_w(y)\}$ .

Thus by (i) of definition 2.23 holds good.

Let  $x, y, z \in W$  and  $\beta, \alpha \in \Gamma$ . From hypothesis,  $x\beta y\alpha z \in W$ .

(a) If  $x, z \in W$ , then  $\chi_w(x) = 1$  and  $\chi_w(z) = 1$ .

In this case  $\chi_w(x\beta y\alpha z) = 1 \geq \min \{\chi_w(x), \chi_w(z)\}$ .

(b) If  $x \in W, z \notin W$ , then  $\chi_w(x) = 1$  and  $\chi_w(z) = 0$ .

So,  $\chi_w(x\beta y\alpha z) = 0 \geq \min \{\chi_w(x), \chi_w(z)\}$ .

(c) If  $x \notin W, z \in W$ , then  $\chi_w(x) = 0$  and  $\chi_w(z) = 1$ .

So,  $\chi_w(x\beta y\alpha z) = 0 \geq \min \{\chi_w(x), \chi_w(z)\}$ .

(d) If  $x \notin W, z \notin W$ , then  $\chi_w(x) = 0$  and  $\chi_w(z) = 0$ .

So,  $\chi_w(x\beta y\alpha z) \geq 0 = \min \{\chi_w(x), \chi_w(z)\}$ .

Thus (ii) of definition 2.23 holds good.

( $\Leftarrow$ iff) Suppose, that  $\chi_W$  is a fuzzy bi- $\Gamma$ -ideal of N. Then by Lemma 3.1,  $\chi_W$  is two valued, hence W is a bi- $\Gamma$ -ideal of N. this completes the proof of Lemma.

The following theorem proves that an intersection of a family of fuzzy bi- $\Gamma$ -ideals of  $\Gamma$ -semi near-field space N also a fuzzy bi- $\Gamma$ -idea.

**Theorem 3.3:** If  $\{W_i\}_{i \in \Lambda}$  is a family of fuzzy bi- $\Gamma$ -ideals of  $\Gamma$ -semi near-field space N. Then  $\bigcap W_i$  is a fuzzy bi- $\Gamma$ -ideals of N, where  $\bigcap W_i = \{\Lambda \lambda_i\}$  and  $\Lambda \lambda_i(x) = \inf .\{\lambda_i(x) / i \in \Lambda, x \in W\}$ .

**Proof:** Let  $x, y \in N$ , where N is a  $\Gamma$ -semi near-field space.

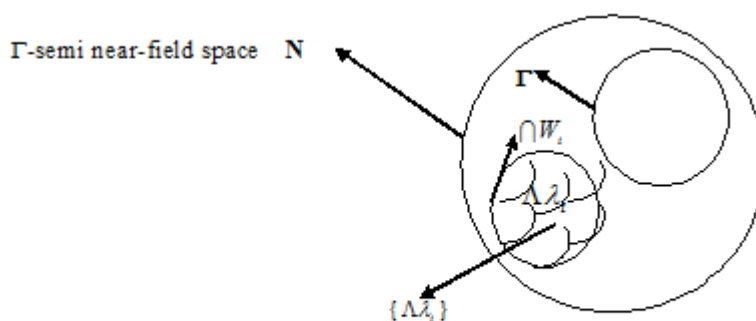


Fig.06

Let  $x, y \in N$ . Then we have,

$$\begin{aligned} (a) \quad \Lambda \lambda_i(x\gamma y) &= \inf .\{\min .\{\lambda_i(x), \lambda_i(y)\} \ni i \in \Lambda, x, y \in W\} \\ &= \min .\{\inf .\{(\lambda_i(x)), \inf(\lambda_i(y))\} \ni i \in \Lambda, x, y \in W\} \\ &= \min .\{\inf .(\lambda_i(x) \ni i \in \Lambda, x \in W), \inf(\lambda_i(y)) \ni i \in \Lambda, y \in W\} \\ &= \min .\{\Lambda \lambda_i(x), \Lambda \lambda_i(y)\}. \end{aligned}$$

Let  $x, y, z \in N$  and  $\beta, \alpha \in \Gamma$ . Then we have,

$$\begin{aligned} (b) \quad \Lambda \lambda_i(x\beta y\alpha z) &= \inf .\{\min .\{\lambda_i(x), \lambda_i(z)\} \ni i \in \Lambda, x, z \in W\} \\ &= \min .\{\inf .\{(\lambda_i(x)), \inf(\lambda_i(z))\} \ni i \in \Lambda, x, z \in W\} \\ &= \min .\{\inf .(\lambda_i(x) \ni i \in \Lambda, x \in W), \inf(\lambda_i(z)) \ni i \in \Lambda, z \in W\} \\ &= \min .\{\Lambda \lambda_i(x), \Lambda \lambda_i(z)\}. \end{aligned}$$

Therefore,  $\bigcap W_i$  is a fuzzy bi- $\Gamma$ -ideals of  $\Gamma$ -semi near-field space N. This completes the proof of theorem.

**Theorem 3.4:** A fuzzy near-field sub space W in a bi- $\Gamma$ -ideals of  $\Gamma$ -semi near-field space N is a fuzzy bi- $\Gamma$ -ideal of N if and only if the level fuzzy near-field space  $U(\lambda; q) = \{x \in N / \lambda(x) \geq q\}$  is a bi- $\Gamma$ -ideal of N when it is non-empty.

**Proof: (IF Part)** Let  $\lambda$  be a fuzzy bi- $\Gamma$ -ideal of N. Then  $\lambda(x\gamma y) \geq \min\{\lambda(x), \lambda(y)\}$ .

$$\begin{aligned} x, y \in U(\lambda; q), \gamma \in \Gamma &\Rightarrow \lambda(x) \geq q, \lambda(y) \geq q \\ &\Rightarrow \lambda(x\gamma y) \geq \min\{\lambda(x), \lambda(y)\} \geq q \\ &\Rightarrow \lambda(x\gamma y) \geq q \\ &\Rightarrow x\gamma y \in U(\lambda; q). \end{aligned}$$

Also,  $\lambda(x\beta y\alpha z) \geq \min\{\lambda(x), \lambda(z)\}$

$$\begin{aligned} x, y \in U(\lambda; q), \beta, \alpha \in \Gamma &\Rightarrow \lambda(x) \geq q, \lambda(y) \geq q, \lambda(z) \geq q \\ &\Rightarrow \lambda(x\beta y\alpha z) \geq \min\{\lambda(x), \lambda(z)\} \geq q \\ &\Rightarrow \lambda(x\beta y\alpha z) \geq q \\ &\Rightarrow x\beta y\alpha z \in U(\lambda; q) \end{aligned}$$

Thus  $U(\lambda; q)$  is a bi- $\Gamma$ -ideal of  $N$ .

( $\Leftarrow$  **IFF part**) If  $U(\lambda; q)$  is a bi- $\Gamma$ -ideal of  $N$ .

$$\begin{aligned} \text{let } q = \min\{\lambda(x), \lambda(y)\}. \text{ Then } x, y \in U(\lambda; q), \gamma \in \Gamma &\Rightarrow x\gamma y \in U(\lambda; q) \\ &\Rightarrow \lambda(x\gamma y) \geq q \\ &\Rightarrow \lambda(x\gamma y) \geq \min\{\lambda(x), \lambda(y)\} \end{aligned}$$

$$\begin{aligned} \text{Let us define, } q = \min\{\lambda(x), \lambda(z)\}. \text{ Then } x, y, z \in U(\lambda; q), \beta, \alpha \in \Gamma &\Rightarrow x\beta y\alpha z \in U(\lambda; q) \\ &\Rightarrow \lambda(x\beta y\alpha z) \geq q \\ &\Rightarrow \lambda(x\beta y\alpha z) \geq \min\{\lambda(x), \lambda(z)\}. \end{aligned}$$

Consequently,  $\lambda$  is a fuzzy bi- $\Gamma$ -ideal of  $N$ . This completes the proof of theorem.

**Note 3.5:** Let  $\lambda$  be a fuzzy bi- $\Gamma$ -ideal of  $N$ . If  $N$  is completely regular, then  $\lambda(a) = \lambda(a\beta a)$  for all  $a \in N$  and  $\beta \in \Gamma$ .

**Theorem 3.6:** Let the pair of mappings  $f : N \rightarrow N_1, h : \Gamma \rightarrow \Gamma_1$  be a homomorphism of  $\Gamma$ -semi near-field spaces. If  $\lambda$  is a fuzzy bi- $\Gamma$ -ideal of  $M_1$ , then the pre image  $f^{-1}(\lambda)$  of  $\lambda$  under  $f$  is a fuzzy bi- $\Gamma$ -ideal of  $N$ .

$$\begin{aligned} \text{Proof: (i) Let } x, y \in N \text{ and } \gamma \in \Gamma. \text{ Then we have, } f^{-1}(\lambda)(x\gamma y) &= \lambda(f(x\gamma y)) \\ &= \lambda(f(x)h(\gamma)f(y)) \\ &= \min\{\lambda(f(x)), \lambda(f(y))\} \\ &= \min\{f^{-1}(\lambda(x)), f^{-1}(\lambda(y))\} \end{aligned}$$

(ii) Let  $x, y, z \in N$  and  $\beta, \alpha \in \Gamma$ . Then,

$$\begin{aligned} f^{-1}(\lambda)(x\beta y\alpha z) &= \lambda(f(x\beta y\alpha z)) \\ &= \lambda(f(x)h(\beta)f(y)h(\alpha)f(z)) \\ &\geq \min\{\lambda(f(x)), \lambda(f(z))\} \\ &= \min\{f^{-1}(\lambda(x)), f^{-1}(\lambda(z))\} \end{aligned}$$

Therefore  $f^{-1}(\lambda)$  is a fuzzy bi- $\Gamma$ -ideal of  $N$ . This completes the proof of the theorem.

**Proposition 3.7:** If  $W$  is a left-zero fuzzy  $\Gamma$ -sub semi near-field space in a bi- $\Gamma$ -ideals of  $\Gamma$ -semi near-field space  $N$  and  $\lambda$  a fuzzy  $\Gamma$ -left ideal of  $N$ , then  $\lambda(x) = \lambda(z)$  for all  $x, y, z \in N$ .

**Proof:** Let  $x, y, z \in W$ . Since  $W$  is left-zero fuzzy  $\Gamma$ -sub semi near-field space of  $N$  and  $\lambda$  a fuzzy  $\Gamma$ -left ideal of  $N$ ,  $x\beta y\alpha z = x$  and  $z\alpha y\beta x = z$  for all  $\beta, \alpha \in \Gamma$ .

In this case, by hypothesis, we have that  $\lambda(x) = \lambda(x\beta y\alpha z) \geq \lambda(z), \lambda(z) = \lambda(z\alpha y\beta x) \geq \lambda(x)$ .

Thus we obtain  $\lambda(x) = \lambda(z) \forall x, y, z \in N$ . This completes the proof of theorem.

**Lemma 3.8:** If  $W$  is a left-zero fuzzy  $\Gamma$ - sub semi near-field space in a bi- $\Gamma$ -ideals of  $\Gamma$ -semi near-field space  $N$ , then  $\chi_W$  is a fuzzy  $\Gamma$ -left ideal of  $N$ .

**Proof:** Let  $x, y, z \in W$  and  $\beta, \alpha \in \Gamma$ . Since  $W$  is left-zero fuzzy  $\Gamma$ - sub semi near-field space of  $N$ ,  $x\beta y\alpha z \in N$ .

- (i) If  $z \in N$ , then  $\chi_W(z) = 1 \Rightarrow \chi_W(x\beta y\alpha z) = 1 = \chi_W(z)$
- (ii) If  $z \notin N$ , then  $\chi_W(z) = 0$ , hence  $\chi_W(x\beta y\alpha z) \geq 0 = \chi_W(z)$ .

Consequently,  $\chi_W$  is a fuzzy  $\Gamma$ -left ideal of  $\Gamma$ -semi near-field space  $N$ . This completes the proof of lemma.

**Theorem 3.9:** Let  $\lambda$  be a fuzzy  $\Gamma$ -left ideal of  $\Gamma$ -semi near-field space  $N$ . If  $E_W$  is a left – zero fuzzy  $\Gamma$ - sub semi near-field space in a bi- $\Gamma$ -ideals of  $\Gamma$ -semi near-field space  $N$ , then  $\lambda(e) = \lambda(e_2)$  for all  $e, e_2 \in E_W$ .

**Proof:** Let  $e, e_2 \in E_W$ . From the hypothesis,  $e\beta e_1\alpha e_2 = e$  and  $e_2\alpha e_1\beta e = e_2$  for all  $e_1 \in E_W$  and  $\beta, \alpha \in \Gamma$ . Thus, since  $\lambda$  be a fuzzy  $\Gamma$ -left ideal of  $\Gamma$ -semi near-field space  $N$ , we get that  $\lambda(e) = \lambda(e\beta e_1\alpha e_2) \geq \lambda(e_2)$ ,  $\lambda(e_2) = \lambda(e_2\alpha e_1\beta e) \geq \lambda(e)$ .

Hence we have  $\lambda(e) = \lambda(e_2)$  for all  $e, e_2 \in E_W$ . This completes the proof of theorem.

**Theorem 3.10:** If  $W$  is an interior  $\Gamma$ -left ideal of  $\Gamma$ -semi near-field space  $N$ , Then  $\chi_W$  is a fuzzy interior  $\Gamma$ -left ideal of  $\Gamma$ -semi near-field space  $N$ .

**Proof:** Since  $W$  is a none-empty fuzzy  $\Gamma$ - sub semi near-field space in a bi- $\Gamma$ -ideals of  $\Gamma$ -semi near-field space  $N$ . Let  $x, y, z \in W$  and  $\beta, \alpha \in \Gamma$ . From the hypothesis,  $x\beta y\alpha z \in W$

- (i) If  $y \in W$ , then  $\chi_W(y) = 1$  thus,  $\chi_W(x\beta y\alpha z) = 1 = \chi_W(y)$
- (ii) If  $y \notin W$ , then  $\chi_W(x\beta y\alpha z) \geq 0 = \chi_W(y)$ .

Hence  $\chi_W$  is a fuzzy interior  $\Gamma$ -left ideal of  $\Gamma$ -semi near-field space  $N$ . This completes the proof of the theorem.

**Note 3.11:** Every fuzzy  $\Gamma$ -left ideal of  $\Gamma$ -semi near-field space  $N$  is a fuzzy interior  $\Gamma$ -left ideal of  $\Gamma$ -semi near-field space  $N$ .

**Note 3.12:** If  $N$  is regular, then every fuzzy interior  $\Gamma$ -left ideal of  $\Gamma$ -semi near-field space  $N$  is a fuzzy  $\Gamma$ -left ideal of  $\Gamma$ -semi near-field space  $N$ .

#### SECTION 4: MAIN RESULTS ON CHAIN CONDITIONS OF A $\Gamma$ -SEMI NEAR-FIELD SPACE $N$

In this section I derived main results on chain conditions of a  $\Gamma$ -semi near-field space  $N$ . we know that the intersection of all bi- $\Gamma$ -ideals of  $\Gamma$ -semi near-field space  $N$ . Let  $\wedge$  be a totally ordered set of a  $\Gamma$ -semi near-field space  $N$  and let  $\{A_j / j \in \wedge\}$  be a collection of all bi- $\Gamma$ -ideals of  $\Gamma$ -semi near-field space  $N$  such that for all  $j < i$  if and only if  $A_i \subset A_j$ . Then  $\bigcap_{j < i} A_j$  is a bi- $\Gamma$ -ideal of  $\Gamma$ -semi near-field space  $N$ .

**Theorem 4.1:** Let  $\{A_j / j \in \wedge \subseteq [0, 1]\}$  be a collection of bi- $\Gamma$ -ideals of  $\Gamma$ -semi near-field space  $N$  such that (a)  $N = \bigcup_{j \in \wedge} A_j$  (b)  $j < i$  if and only if  $A_i \subset A_j \forall i, j \in \wedge$ . Then the fuzzy set  $\lambda \in M$  defined by  $\lambda(x) = \sup\{j \in \wedge / x \in A_j\} \forall x \in M$  is a fuzzy bi- $\Gamma$ -ideal of  $\Gamma$ -semi near-field space  $N$ .

**Proof:** For any  $i \in [0, 1]$ , we consider the following two cases:

$$i = \sup\{j \in \wedge / j < i\}, i \neq \sup\{j \in \wedge / j < i\}.$$

**Case (i):**  $\Rightarrow$  we know that for all  $x, y \in A_j$  and  $\gamma \in \Gamma$ ,  $x \gamma y \in U(\lambda; i)$

$$(\Leftrightarrow) x \gamma y \in A_j \text{ for all } j < i,$$

$$(\Leftrightarrow) x \gamma y \in \bigcap_{j < i} A_j. \text{ Also, } \forall x, y, z \in A_j \text{ and } \beta, \alpha \in \Gamma, x\beta y\alpha z \in U(\lambda; i)$$

$$(\Leftrightarrow) x\beta y\alpha z \in A_j \text{ for all } j < i$$



$$(\Leftrightarrow) x\beta y\alpha z \in \bigcap_{j < i} A_j .$$

Hence,  $U(\lambda ; i) = \bigcap_{j < i} A_j$  , which is a bi- $\Gamma$ -ideal of  $\Gamma$ -semi near-field space N.

**Case (ii):**  $\Rightarrow \exists e > 0$  such that  $(i - e, i) \cap \wedge = \phi$ .

We claim that  $U(\lambda ; i) = \bigcap_{j < i} A_j$  . If  $x \gamma y \in \bigcap_{j < i} A_j$  , for some,  $j < i$ .  $\gamma \in \Gamma$ .

$$\Rightarrow \lambda(x \gamma y) \geq j \geq i.$$

Hence  $x \gamma y \in U(\lambda ; i)$ , showing that if  $x \gamma y \in A_j$ , for  $j \leq i - e$  and so  $x \gamma y \notin U(\lambda ; i)$ .

Also, if  $x\beta y\alpha z \in \bigcap_{j < i} A_j$  then  $x\beta y\alpha z \in A_j$  for some  $j < i$ ,  $\beta, \alpha \in \Gamma$ .

$$\Rightarrow \lambda(x\beta y\alpha z) \geq j \geq i. \text{ Hence, } x\beta y\alpha z \in U(\lambda ; i), \text{ and}$$

$$\therefore x\beta y\alpha z \notin U(\lambda ; i).$$

$$\therefore U(\lambda ; i) \subseteq \bigcap_{j < i} A_j \text{ and so } U(\lambda ; i) = \bigcap_{j < i} A_j .$$

Hence  $\lambda$  is a fuzzy bi- $\Gamma$ -ideal of  $\Gamma$ -semi near-field space N.

This completes the proof of theorem.

**Note 4.2:** Let  $\{A_n / n \in \wedge\}$  be a family of bi- $\Gamma$ -ideals of  $\Gamma$ -semi near-field space N which is nested, that is  $N = A_1 \supset A_2 \supset A_3 \supset \dots \supset A_n \supset A_{n+1} \supset \dots$ . Let  $\lambda$  be the fuzzy in N defined by

$$\lambda(x) = \left\{ \begin{array}{l} \frac{n}{n+1} \text{ if } x \in A_n \setminus A_{n+1}, \forall n = 1, 2, 3, \dots \\ 1 \text{ if } x \in \bigcap_{n=1}^{\infty} A_n \end{array} \right\} \text{ for all } x \in N.$$

Then  $\lambda$  is a fuzzy bi- $\Gamma$ -ideal of  $\Gamma$ -semi near-field space N.

**Note 4.3:** A fuzzy bi- $\Gamma$ -ideal  $\lambda$  of  $\Gamma$ -semi near-field space N is finite – valued if and only if it is generated by a finite – values fuzzy bi- $\Gamma$ -ideal  $\lambda$  in N.

**Theorem 4.4:** If N is a  $\Gamma$ -Notherian, then every fuzzy bi- $\Gamma$ -ideal  $\lambda$   $\Gamma$ -semi near-field space N is finite - valued.

**Proof:** Let,  $\lambda: N \rightarrow [0, 1]$  be a fuzzy bi- $\Gamma$ -ideal of  $\Gamma$ -semi near-field space N which is not finite – valued. Then,  $\exists$  an infinite sequence of distinct numbers  $\lambda(x_1) = t_1 > t_2 > t_3 > t_4 > \dots > t_n > \dots$  where  $t_i = \lambda(x_i)$  for some  $x_i \in N$ . This sequence induces an infinite sequence of distinct bi- $\Gamma$ -ideal of  $\Gamma$ -semi near-field space N. so that we have the following conclusion  $U(\lambda; i) \subset U(\lambda; t_2) \subset \dots \subset U(\lambda; t_n) \subset \dots$  which is a contradiction. This completes the proof of the theorem.

**Note 4.5:** By combining note 4.2 and Note 4.3 we have the following corollary.

**Corollary 4.6:** If N is a  $\Gamma$ -Notherian, then every fuzzy bi- $\Gamma$ -ideal of  $\Gamma$ -semi near-field space N is generated by a fuzzy set in N.

## REFERENCES

- [1] Antony J.M and shrewood, Fuzzy groups redefined, J.Maths. Anal.Appl.69, 124-130,1979.
- [2] Barnes, W.E on the  $\Gamma$ -rings of nobusawa, pacific J Math, 18,411-422, 1966.

- [3] Chinram R, Jirojkul C on bi- $\Gamma$ -ideals in  $\Gamma$ -semigroups, songklanakarin.
- [4] Dib, K A on fuzzy spaces and fuzzy group theory, inform.sci 80,253-282, 1994.
- [5] G Pilz Near rings, North Holland.
- [6] N V Nagendram, Kum. E.S. Sudeeshna, "Generalization Of  $((\in, \in VqK))$  -Fuzzy Sub-Near-Fields and Ideals Of Near-Fields (GF-NF-IO-NF) " ,published in International Journal of Mathematical Archive IJMA- ISSN. 2229 - 5046, Vol.4, No. 328-343, June 29th day 2013.
- [7] N V Nagendram, Kum. E.S. Sudeeshna, "Generalization Of  $((\in, \in VqK))$  -Fuzzy Sub-Near-Fields and Ideals Of Near-Fields (GF-NF-IO-NF) " ,published in International Journal of Mathematical Archive IJMA- ISSN. 2229 - 5046, Vol.4, No. 328-343, June 29th day 2013.
- [8] C K Wong, Fuzzy topology, product and quotient theorems, J. Math. Anal. Appli. 45, (1974), 512-521.
- [9] L A Zadeh, fuzzy sets, information and control 8 91965), 338-353.
- [10] N V Nagendram, Kum. E.S. Sudeeshna , "Generalization Of  $((\in, \in VqK))$  -Fuzzy Sub-Near-Fields and Ideals Of Near-Fields (GF-NF-IO-NF)" , published in International Journal of Mathematical Archive IJMA- ISSN. 2229 - 5046, Vol.4, No. 328-343, June 29th day 2013.
- [11] N V Nagendram, "Non-Commutative Near Fields Of Algebra And Its Extensions (NC-NF-IE)", accepted and to be published in International Journal of Advances in Algebra, Korean@Research India Publications, IJAA- ISSN 0973-6964 Volume 6, Number 1 (2013), pp. 73-89.
- [12] N V Nagendram, "A Note On Near – Field group Of Quotient Near-Fields (NF-GQ-NF)", accepted and to be published in International Journal of Advances in Algebra, Korean@Research India Publications, IJAA -June,2013, ISSN: 0973-6964; Vol:6; Issue:2; Pp.91-99.
- [13] N V Nagendram, Dr. T V Pradeep Kumar "A Note on Levitzki Radical of Near Fileds(LR - NF) " International Journal of Mathematical Archive(IJMA) Published ISSN NO.2229-5046, Vol No. 4 No. 5, pp 288-295, 2013.
- [14] N V Nagendram, Dr. T V Pradeep Kumar "Some Problems And Applications Of Ordinary Differential Equations To Hilbert Spaces In Engineering Mathematics (SP-ODE-HEM)" International Journal of Mathematical Archive(IJMA) Published ISSN NO.2229-5046, Vol No. 4 No. 4, pp. 118-125, 2013.
- [15] N.V. Nagendram, 'Amalgamated Duplications of Some Special Near-Fields(AD-SP-N-F)', International Journal of Mathematical Archive(IJMA) -4(3), 2013, ISSN 2229-5046, Pp. 1 - 7.
- [16] N.V. Nagendram, 'Infinite Sub-near-Fields of Infinite-near-fields and near-left almost-near-fields(IS-NF-INF-NL-A-NF)', International Journal of Mathematical Archive(IJMA) Published ISSN. No.2229 - 5046, Vol. no.4, no.2, pp. 90-99 (2013).
- [17] N.V. Nagendram, Dr.T.V.Pradeep Kumar & Dr.Y.Venkateswara Reddy, 'Semi Simple near-fields generating from Algebraic K-theory(SS-NF-G-F-AK-T) ', International Journal of Mathematical Archive(IJMA) -3(12), 2012, ISSN 2229-5046, Pp. 1-7.
- [18] N.V. Nagendram, Dr.T.V.Pradeep Kumar & Dr.Y.Venkateswara Reddy, 'A note on generating near fields effectively: Theorems from Algebraic-Theory-(G-NF-E-TFA-KT)', International Journal of Mathematical Archive - 3(10), 2012, ISSN 2229-5046, Pp. 3612-3619.
- [19] N V Nagendram 1 and B Ramesh 2 on "Polynomials over Euclidean Domain in Noetherian Regular Delta Near Ring Some Problems related to Near Fields of Mappings(PED-NR-Delta-NR & SPR-NF)" published in IJMA, ISSN: 2229-5046, Vol.3(8), pp no.2998 - 3002 August, 2012.
- [20] N V Nagendram research paper on "Near Left Almost Near-Fields (N-LA-NF)" communicated to for 2nd international conference by International Journal of Mathematical Sciences and Applications, IJMSA@mindreader publications, New Delhi on 23-04-2012 also for publication.

[21] N V Nagendram, T Radha Rani, Dr T V Pradeep Kumar and Dr Y V Reddy "A Generalized Near Fields and (m, n) Bi-Ideals over Noetherian regular Delta-near rings (GNF-(m, n) BI-NR-delta-NR)" , communicated to International Journal of Theoretical Mathematics and Applications (TMA),Greece, Athens, dated 08-04-2012.

[22] N VNagendram<sup>1</sup>, N Chandra Sekhara Rao<sup>2</sup> "Optical Near field Mapping of Plasmonic Nano Prisms over Noetherian Regular Delta Near Fiedls (ONFMPN-NR-Delta-NR)" accepted for 2nd international Conference by International Journal of Mathematical Sciences and Applications, IJMSA @ mind reader publications, New Delhi going to conduct on 15 – 16 th December 2012 also for publication.

[23] N V Nagendram,B Ramesh, Ch Padma , T Radha Rani and S V M Sarma research article "A Note on Finite Pseudo Artinian Regular Delta Near Fields(FP AR-Delta-NF)" Published by International Journal of Advances in Algebra, IJAA, Jordan ISSN 0973-6964 Volume 5, Number 3 (2012), pp. 131-142.

[24] Satyanarayana Bh., A note on  $\Gamma$ -near-rings, Indian j of mathematics, bf41 (3), 427-433, 1999.

[25] Zadeh, L A Fuzzy sets, information and control, 8, 338-353, 1965.

**Source of support: Nil, Conflict of interest: None Declared**