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CONJUGATE GRADIENT COEFFICIENT FOR UNCONSTRAINED OPTIMIZATION BASED ON LOGISTIC EQUATION

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ABSTRACT

In this paper, a new conjugate gradient method for unconstrained optimization by using Logistics Equation .Conjugate gradient methods are widely used for large scale unconstrained optimization problems. Most of conjugate gradient methods don't always generate a descent search direction, so the descent condition is usually assumed in the analysis and implementation.

Keywords: *Unconstrained optimization, line search, conjugate gradient method, logistic equation.*

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1. INTRODACTION

Consider the following n variable unconstrained optimization problem:

$$\min f(x), \quad x \in R^n \tag{1.1}$$

where f: $R^n \to R$ is smooth and gradient g(x) is available. The nonlinear conjugate gradient (CG) Method for (1) is designed by the iterative form

$$x_{k+1} = x_k + \alpha_k d_k \text{, k=0, 1,….., n} \tag{1.2}$$

where x_k is the current iterate, $\alpha_k > 0$ is a step length, and d_k is the search direction defined by

$$d_k = \begin{cases} -g_k, if & k=0, \\ -g_k + \beta_{k-1} d_{k-1}, if & k \ge 1, \end{cases} \tag{1.3}$$

where g_k is the gradient of f(x) at the point x_k, and $\beta_k \in R$ is a scalar which determines the different conjugate gradient methods, There are some well-known formulas which are given as follows:

$$\beta_k^{PR} = \frac{g_k^T (g_k - g_{k-1})}{\|g_{k-1}\|^2}, \tag{1.4}$$

$$\beta_k^{FR} = \frac{g_k^T g_k}{\|g_{n-1}\|^2}, \tag{1.5}$$

$$\beta_k^{CD} = \frac{g_k^T g_k}{-d_{k-1}^T g_{k-1}}, \tag{1.6}$$

$$\beta_k^{HS} = \frac{g_k^T (g_k - g_{k-1})}{(g_k - g_{k-1})^T d_{k-1}}, \tag{1.7}$$

$$\beta_k^{DY} = \frac{g_k^T g_k}{(g_k - g_{k-1})^T d_{k-1}}, \tag{1.8}$$

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where g_{k-1} and g_k are gradients $\nabla f(x_{k-1})$ and $\nabla f(x_k)$ of f(x) at the point x_{k-1} and x_k, respectively, $\|.\|$ denotes the Euclidian norm of vectors. The CG method is a powerful line search method for solving optimization problems, and it remains very popular for engineers and mathematicians who are interested in solving large– scale problems. This method can avoid, like steepest descent method, the computation and storage of some matrices associated with the Hessain of objective function. Then there are many new formulas that have been studied by many authors.

The chaos optimization realized through the chaos variable. There are many methods for producing chaos variable. We select the Logistic Mapping method which is used extensively.

2. NEW CONJUGATE GRADIENT COEFFICIENT (β_{k+1}^{New})

To find a New Conjugate Gradient coefficient (2) we will use Conjugate Gradient coefficient of Polak – Ribiere-Polyak (PRP)

$$\beta_k = \frac{g_k^T(g_k - g_{k-1})}{||g_{k-1}||^2} \qquad (2.1)$$

And the Logistic Mapping method which is used extensively. Its equation is as follows:

$$\gamma_{k+1} = \mu\, \gamma_k (1-\gamma_k) \qquad (2.2)$$

Where μ is a control parameter ($\mu \epsilon$ (0,4)), since $y_k = g_k - g_{k-1}$, then we can write (2.1) as follows:

$$\beta_k = \frac{g_{k+1}^T y_k}{||g_k||^2} \qquad (2.3)$$

Now, from the equation (2.2) and the formula (2.3) we get

$$\beta_{k+1} = \mu\beta_k(1- \beta_k) \qquad (2.4)$$

Or

$$\beta_{K+1}^{New} = \mu \frac{g_{k+1}^T y_k}{||g_k||^2}\left(1- \left(\frac{g_{k+1}^T y_k}{||g_k||^2}\right)\right)$$

where $\gamma_k = \beta_k$

To achieve a balance we can add $\rho^2\, a_k$

Where $a_k = \frac{d_k^T g_{k+1}}{d_k^T y_k}$, and $0<\rho<1$

So, we have

$$\beta_{K+1}^{New} = \mu \frac{g_{k+1}^T y_k}{||g_k||^2}\left(1-\rho^2\, \frac{d_k^T g_{k+1}}{d_k^T y_k}\left(\frac{g_{k+1}^T y_k}{||g_k||^2}\right)\right) \qquad (2.5)$$

2.1 Algorithm of New Conjugate Gradient coefficient:

Step (1): set k=0, select the initial point x_k,

Step (2): $g_k = \nabla f(x_k)$, If $g_k = 0$, then stop,
else
set $d_k = -g_k$,

Step (3): compute α_k, to minimize $f(x_{k+1})$

Step (4): $x_{k+1} = x_k + \alpha_k d_k$,

Step (5): $g_{k+1} = \nabla f(x_{k+1})$, If $g_{k+1} = 0$, then stop,

Step (6): compute β_{k+1}^{New}

where $\beta_{k+1}^{New} = \mu \frac{g_{k+1}^T y_k}{||g_k||^2}$ (1- $\rho^2 \frac{d_k^T g_{k+1}}{d_k^T y_k} (\frac{g_{k+1}^T y_k}{||g_k||^2}$))

Step (7): d_{k+1}= - g_{k+1}+ β_{k+1}^{New} d_k,

Step (8): If k=n then go to step 2, else k=k+1 and go to step 3.

2.2 Theorem: Assume that the sequence $\{x_k\}$ is generated by the algorithm (1.2), then the modified of CG-method in (2.5) is satisfied the sufficient descent condition, i.e. $d_{k+1}^T g_{k+1} \le 0$ in to two cases: exact and inexact line search.

Proof: The proof is done by induction, the result clearly holds for k=0

$g_0^T d_0$ = - $||g_0|| \le 0$,

Now, we prove the current search direction is descent direction at the iteration (k+1), we have

$$d_{k+1}^T = -g_{k+1} + \beta_{k+1}^{New} d_k$$

Implies that

$$d_{k+1}^T = -g_{k+1} + \mu \frac{g_{k+1}^T y_k}{||g_k||^2} (1- \rho^2 \frac{d_k^T g_{k+1}}{d_k^T y_k} (\frac{g_{k+1}^T y_k}{||g_k||^2})) d_k^T$$

Multiply both sides by g_{k+1}, we get

$$d_{k+1}^T \; g_{k+1} = -||g_{k+1}||^2 + \mu \frac{g_{k+1}^T y_k}{||g_k||^2} (1- \rho^2 \frac{d_k^T g_{k+1}}{d_k^T y_k} (\frac{g_{k+1}^T y_k}{||g_k||^2})) d_k^T g_{k+1}$$

Implies that

$$d_{k+1}^T g_{k+1} = - ||g_{k+1}||^2 + \mu \frac{(g_{k+1}^T y_k)(d_k^T g_{k+1})}{||g_k||^2} - \mu\rho^2 \frac{(d_k^T g_{k+1})^2}{d_k^T y_k} (\frac{g_{k+1}^T y_k}{||g_k||^2})^2 \qquad (2.6)$$

We know that the first two terms from equation (2.6) are less than or equal to zero because the formula of (PRP) is satisfies the descent condition (i.e)

$$- ||g_{k+1}||^2 \; + \mu \frac{(\boldsymbol{g_{k+1}^T y_k})(\boldsymbol{d_k^T g_{k+1}})}{||\boldsymbol{g_k}||^2} \le 0, \mu \epsilon (0,4)$$

The prove is complete if the step length $\propto_k$ is chosen by an exact line search which requires $d_k^T g_{k+1} = 0$.

Now, if the step length $\propto_k$ is chosen by an inexact line search which requires $d_k^T g_{k+1} \neq 0$,

We know that

$(\boldsymbol{d_k^T g_{k+1}})^2$, $(\frac{\boldsymbol{g_{k+1}^T y_k}}{||\boldsymbol{g_k}||^2})^2$ and $\mu\rho^2$ are positive,

To complete the prove it is enough to show that

$d_k^T y_k > 0$,

Since f is convex, then $v_k^T y_k > 0$ for any points x_k and x_{k+1}. This can be proved by using the first order condition for convexity.

$$f(x_{k+1}) > f(x_k) + \nabla f(x_k)^T (x_{k+1} - x_k) \qquad (2.7)$$

$$f(x_{k+1}) > f(x_{k+1}) + f(x_{k+1})^T (x_k - x_{k+1}) + \nabla f(x_k)^T (x_{k+1} - x_k)$$

$$0 > (f(x_{k+1}) - f(x_k))^T (x_k - x_{k+1})$$

We know that

$y_k = \nabla f(x_{k+1})^T - \nabla f(x_k)^T$
and
$v_k = x_k - x_{k-1}$

So, we have
$y_k^T v_k > 0,$

Also, we know that
$v_k = x_k - x_{k-1} = \alpha_k d_k,\ \alpha_k > 0$

So, we have
$y_k^T d_k = d_k^T y_k > 0,$

We get
$-\mu\rho^2 \frac{(d_k^T g_{k+1})^2}{d_k^T y_k} (\frac{g_{k+1}^T y_k}{||g_k||^2})^2 \leq 0.$

Finally, we have
$d_{k+1}^T g_{k+1} = -||g_{k+1}||^2 + \mu \frac{||g_{k+1}||^2 d_k^T y_k}{||g_k||^2} - \mu\rho^2 \frac{(d_k^T g_{k+1})^2}{d_k^t y_k} (\frac{g_{k+1}^T y_k}{||g_k||^2})^2 \leq 0.$

Then the proof is complete.

2.3 NUMRICAL RESULTS

This section is devoted to test the implementation of the new methods. We compare the new method with standard P/R method ,the comparative tests involve well-known nonlinear problems (standard test function) with different dimension 4 ≤n≤3000, al l programs are written in FORTRAN95 language and for all cases the stopping condition is $\|g_{k+1}\|_\infty \leq 10^{-5}$ The results are given in table (2.1) is specifically quote the number of functions NOF and the number of iteration NOI experimental results in table (2.1) confirm that the new CG method is superior to standard CG method with respect to the NOI and NOF

Table (2.1)
Comparative Performance of the two algorithms (Polak – Ribiere- Polyak (PRP) and New Conjugate Gradient coefficient 2)

Test fun.	N	PRP algorithm		New algorithm	
		NOI	NOF	NOI	NOF
Powell	4	43	105	30	77
	100	50	136	38	109
	500	50	136	38	109
	1000	54	164	40	124
	3000	65	168	40	124
Wood	4	29	67	28	64
	100	30	69	29	67
	500	30	69	29	67
	1000	30	69	29	67
	3000	30	69	29	67
Extended PSC1 function	4	7	18	6	16
	100	7	18	6	16
	500	8	20	6	16
	1000	8	20	6	16
	3000	8	20	6	16
Cubic	4	15	45	13	37
	100	16	47	13	37
	500	16	47	13	37

	1000	16	47	14	39
	3000	16	47	14	39
Rosen	4	30	85	28	75
	100	30	85	30	80
	500	30	85	31	82
	1000	30	85	31	82
	3000	30	85	31	82
Mile	4	37	116	37	116
	100	44	148	43	146
	500	44	148	43	146
	1000	50	180	50	180
	3000	50	180	50	180
Beale	4	11	28	9	22
	100	12	30	12	29
	500	12	30	12	29
	1000	12	30	12	29
	3000	12	30	13	31
G central	4	27	153	27	148
	100	33	222	31	195
	500	40	312	37	271
	1000	40	312	37	291
	3000	40	312	42	338
Dixon	4	17	36	17	36
	100	6391	12786	744	1558
	500	5395	10793	738	1554
	1000	511	1157	775	1627
	3000	512	1144	765	1587
shalo	4	8	21	8	21
	100	8	21	8	21
	500	8	21	8	21
	1000	9	24	8	21
	3000	9	24	8	21

5. CONCLUSION

In this paper, we considered a conjugate gradient method with the formula (2.5). We have also shown that the search direction satisfied the descent condition
$d_{k+1}^T g_{k+1} \leq 0$.

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