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TSK - FUZZY CONTROLLED MODELING VIA CLASSICAL MATHEMATICAL EEG SIGNALS MODELING

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ABSTRACT

In this paper, we have proposed a prototype optimistic TSK Fuzzy Logic Control (FLC)model to be used for developing classical mathematical model of EEG Signals based on the Hodgkin-Huxley model so as a TSK - FLC model will generate. We have designed this model using the same inputs (I/Ps) and its values (sensor readings) as that of used in classical mathematical model of EEG signals and achieved a desired output (O/P) result. Comparing the proposed approach with the Hodgkin-Huxley classical mathematical model of EEG signals, it is observed that the developed TSK fuzzy model exhibits better results with higher accuracy and smaller size of architecture. Further it is to be noted that the efforts required to work out the fuzzy model are more feasible than that of the classical mathematical model of EEG signals.

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Key words: *Mathematical model of EEG signals, I/Ps-O/P linguistic variables, Mamdani fuzzy inference rules, TSK fuzzy inference rules, weighted average formula.*

1. INTRODUCTION

The most popular direct fuzzy reasoning technique is that of the Mamdani method. In order to improve upon this method, we attempt to propose its natural extension by means of the so called Takagi-Sugeno-Kang (TSK) architecture. The main motivation for developing this model is to reduce the number of rules required by Mamdani model (the large number of rules create error to fire the rules and reduce the accuracy of the result), by inserting linear equation of the I/P - variables in the consequence (then part)of the Mamdani fuzzy inference rules. Since each rule in TSK method has a numeric output, the overall output is obtained via "weighted average", this avoids the time-consuming process of defuzzification required in a Mamdani model.

TSK - FLC can be used for controlling a process (plant) for which it is inconvenient to use traditional classical control design and secondly FLC is ease of describing human knowledge expressed in imprecise linguistic terms. As the traditional classical mathematical EEG signal model is designed using the I/Ps –intensity (I), duration (τ) and the O/P – membrane current (I_{memb}) which are imprecise or approximate linguistic terms that produce major uncertainty to build up the model. Hence utility of TSK - FLC on the classical EEG signals is more convenient.

2. CLASSICAL MATHEMATICAL MODEL OF EEG SIGNALS

This EEG signal model is based on the Hodgkin - Huxley **Nobel prize** winning model for the squidax on published in 1952[10].

2.1. Mechanism: A nerve axon may be stimulated and the activated sodium (Na+) and potassium (k+) channels produced in the vicinity of the cell membrane may lead to the electrical excitation of the nerve axon. Prominently, the electrical excitation arises: (a) from the effect of membrane potential on the movement of ions, and (b) from interaction of the potential with the opening and closing of voltage activated membrane channels. The membrane potential increases when the membrane polarized with anet negative charges lining in the inner surface and equal but opposite net positive charge on the outer surface. This potential (E) may be related to the amount of electrical charge (Q), using the relation,

$$E = \frac{Q}{c_m}, \ldots\ldots \qquad (1)$$

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where E, electrical potential (or membrane potential or electrical force) is measured in the unit of volts; Q, electrical charge is measured in terms of coulombs/cm^2; c_m, is the measure of capacity of membrane in units of farad/cm^2.

In practice, in order to model the action potential (APs) the amount of charge $Q+$ on the inner surfaces (and Q^-on the outer surface) of the cell membrane has to be mathematically related to the stimulatingcurrent (I_{steam}) flowing into the cell through the stimulating electrodes. The Hodgkin-Huxley model is shown in Figure 1.

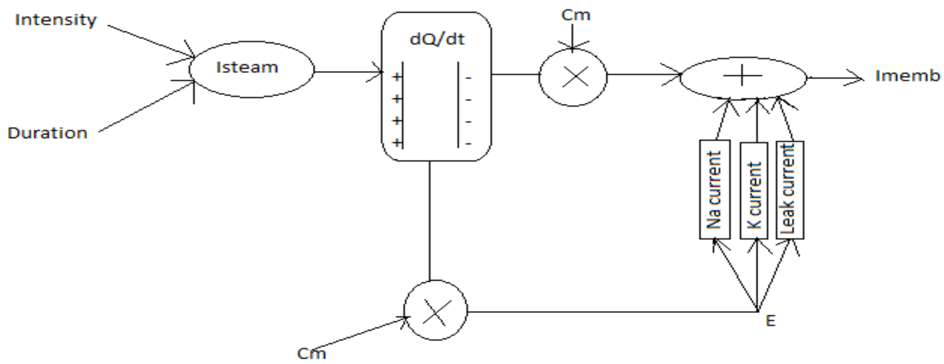


Figure 1*: Hodgkin-Huxley excitation model.*

In this Figure 1, membrane current (I_{memb}) is the result of positive charges flowing out of cell. This current consists of three currents namely, sodium (*Na*), potassium (*K*) and leak currents (the leak current is due to fact that the inner and outer Na and K ions not exactly equal). Hodgkin and Huxley estimated the activation and inactivation functions for the *Na* and *K* currents and derived a mathematical model to describe an action potential *AP* similar to that of a giant squid. The model is neuron model that usages voltage gated channels. This model describes the change in membrane potential (*E*) with respect to time. The overall membrane current is the sum of capacity current and ionic current as follows,

$$I_{memb} = c_m \frac{dE}{dt} + I_i \tag{2}$$

where I_i, is the ionic current as indicated in Figure 3. It consists of the sum of three individual components as follows,

$$I_i = I_{Na} + I_k + I_{leak} \tag{3}$$

where I_{Na}can be related to the maximal conductance $\bar{g}_{Na}$; activation variable a_{Na}; inactivation variable h_{Na} and driving force ($E - E_{N_a}$) through

$$I_{Na} = \bar{g}_{Na} h_{Na} (E - E_{N_a}) \, a^{3}_{Na} \tag{4}$$

Similarly I_k and I_{leak} can be described.

The change in the variables a_{Na} ,a_k and h_{Na} vary from 0 to 1 (time in ms) according to the following equations:

$$\frac{d}{dt}(a_{Na}) = \lambda_t \, [\alpha_{Na}(E)(1 - a_{Na}) - \beta_{Na}(E) a_{Na}] \tag{5}$$

where $\alpha(E)$ and $\beta(E)$are forward and backward rate functions respectively and $\lambda_t$is a temperature dependent factor. Similarly, $\frac{d}{dt}(h_{Na})$ and $\frac{d}{dt}(a_k)$ can be described.

As stated in the simulator for neural network and action potential (SNNPA) $literature$[10]. The parameters $\alpha(E)$ and $\beta(E)$ have been converted from the original Hodgkin-Huxley version to a version agreeing with physiological practice taking depolarization of the membrane as positive. Resting potential has been shifted to $-60mV$ (from original $0mV$). A simulated action potential is illusrated in Figure 1.

For this model, the parameters are set to be,c_m =1.1μF/cm^2,$\bar{g}_{Na} = 100ms/cm^2$, $\bar{g}_k = 35\,ms/cm^2$ $\bar{g}_l = 0.35\,ms/cm^2$,d$E_{Na}$=60mV.Using the values of c_m,$\bar{g}_k$, $g_l$etc, in the above related equations (1)-(9), one gets

$$\boldsymbol{I_{memb} = 80\mu A/cm^2}, \text{ (see Figure 2).} \tag{6}$$

2.2. Brief algorithm of EEG signal modeling: The information transmitted by nerve in the central nerves system (CNS) is called an action potential (AP). APs are caused by an exchange of ions across the neuron membrane and are a temporary change in the membrane potential that transmitted along the axon. As soon as the stimulus strength goes above the threshold, an action potential appears and travels down the nerve. The membrane potential depolarizes (becomes more positive) producing spike. After the peak of the spike (having sodium (+) channels close and the potassium (+) open), the membrane potential repolarizes (becomes more negative). The potential becomes more negative than the resting potential is called hyper polarization and return to the normal called resting potential as shown in **Figure 2.** It is important to note that the action potential of the most nerves system last up to 5 to 10ms.

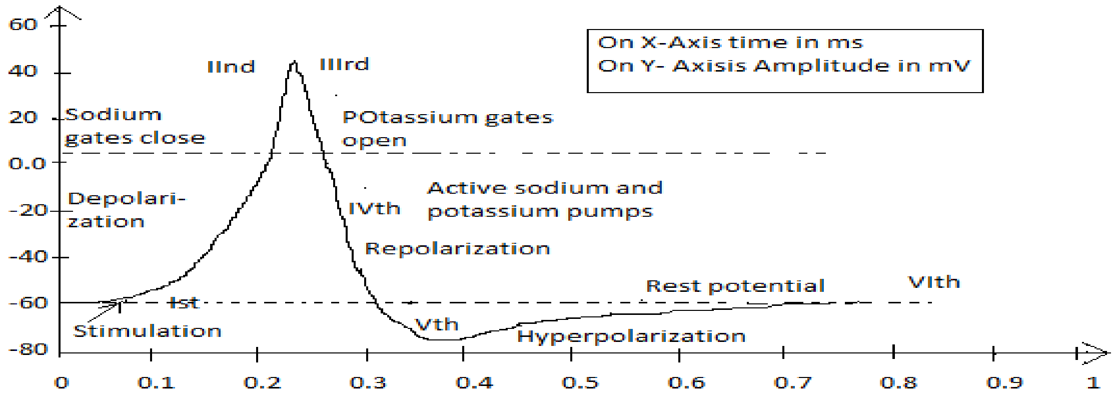


Figure 2: A single AP in response to a transient stimulation based on Hodgkin –Huxley model. The initiated time is t=0.4ms and the injected current i.e., $I_{memb} = 80\mu A/cm^2$ for duration of 0.1ms.

This model is complex due to imprecise linguistic I/P-variables and coupling of different parameters. The technique of TSK-fuzzy controllers on EEG signal modeling is more convenient under these conditions.

3. TSK -Fuzzy Logic Control (FLC) On EEG Signal Modeling

As the system of the classical EEG signal model consist of two fuzzy I/ Ps intensity (I) and duration (τ)as the stimulator for dendrites of the nerve cell and one fuzzy O/P namely membrane current ($\boldsymbol{I_{memb}}$)to be computed. We elaborate a general scheme for controlling a desired value by the technique of TSK FLC over the classical EEG signal model is shown in Figure 3.

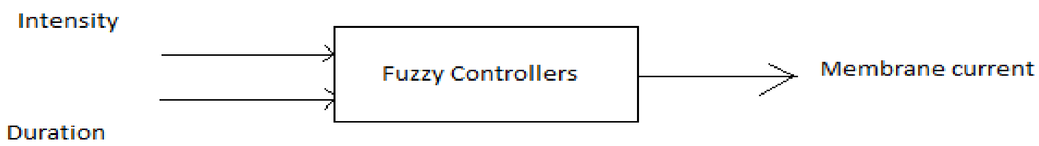


Figure 3: A general scheme of TSK- FLC for controlling desired value.

The general inference process based on the TSK - FLC Proceeds in three steps:

a) Construction of fuzzy sets and fuzzifications.
b) Formation of fuzzy inference rules from Mamdani to TSK.
c) Compositions of fuzzy inference rules.

Step (a) Construction of fuzzy sets and fuzzifications: After identifying the relevant I/Ps and O/p variables of the classical controller, our first step in designing the FLC should be to characterize the range of values for the I/Ps and O/P variables. Since the duration of the action potential of a nerve system in the classical controllers is in the range of 5 to 10 ms, so that we have chosen the range of values for the both I/P –variables thats are ‘intensity’ and ‘duration’ in the time interval of 0 to 10 ms in FLC. And since final injected current in EEG signal model is, $\boldsymbol{I_{memb} = 80\mu A/cm^2}$, accordingly we have chosen range of values for O/P - variable ‘membrane current’ as 0 to 100 µA $/cm^2$ in FLC.

Further we have to select meaningful linguistic states for each of the three variable and express them by appropriate fuzzy sets. Accordingly we choose as: Negative Large(NL); Negative Medium (NM);Negative Slow(NS); Almost zero(AZ); Positive Slow(PS); Positive Medium(PM)and Positive Large(PL).We elaborate these seven linguistic verbal adjectives to their corresponding numerical descriptions as: “about and below 0.13 ”; “about 0.26 ”; “about 0.39 ”; “about 0.52 ”;“about 0.65 ”; “about 0.78”;“about and above 0.91 ”respectively. Representing these seven linguistic states of I/P and O/P linguistic variables by triangular shape fuzzy numbers as in Figure 4 and Figure 5 respectively.

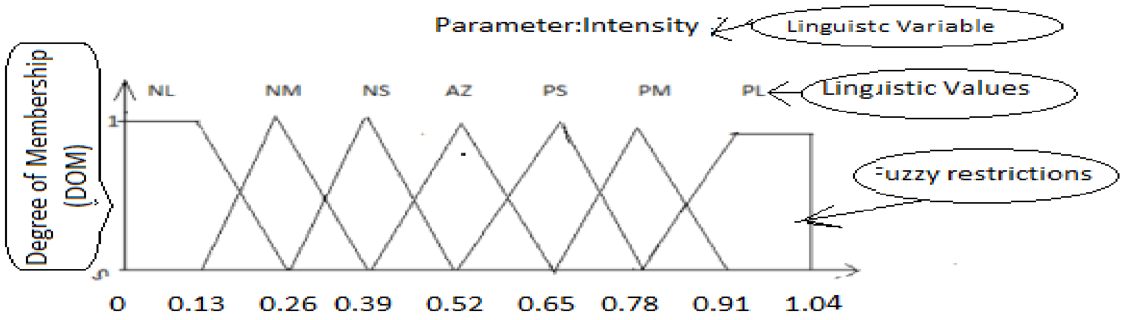


Figure 4: Fuzzy sets and decomposition for I/P variable intensity/ duration over the range [0, 1]- is the time in ms.

Next, the O/P-linguistic variable membrane current is shown in Figure 5.

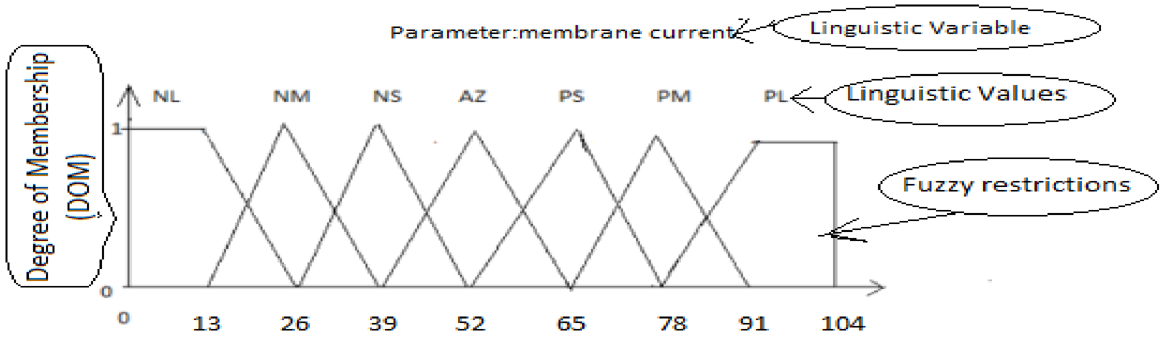
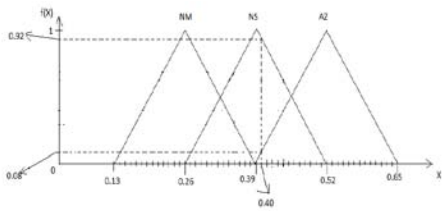


Figure 5: Fuzzy sets and decomposition for O/P variable 'membrane current' (I_{memb}) over the range [0,100] is the injected current in $\mu A/cm^2$.

Fuzzification of I/P-variables: The main purpose of the fuzzification is to interpret measurement of I/P-variables (each expressed by the fuzzy approximation of the respective real number) and toexpress the associated measurement uncertainties. Let us consider an illustration. A fuzzification process (function) applied to the I/P variable intensity (*I*), is represented by f_I. Then the fuzzification function has the form $f_I: [0,1] \to R$, where R denote the set of all fuzzy numbers. Then $f_I(x_0 = 0.40)$is a fuzzy number chosen by $f_I$as a fuzzy approximation of the measurement (sensor reading) intensity (*I*)at $x_0 = 0.40$.The computation of fuzzy membership values from Figure 4, for which $f_I(x_0 = 0.40 \neq 0)$is carried out as below and is as shown in Figure 6.

NS (0.40sec)= $\frac{0.40-0.52}{0.39-0.52} = \frac{0.12}{0.13} = 0.92$; $\quad AZ(0.40sec) = \frac{0.40-0.39}{0.52-0.39} = \frac{0.01}{0.13} = 0.08$.

Remaining all fuzzy membership values (from Figure 4) are zero such as, *NL*(0.40) = *NM*(0.40) =*PS*(0.40) = *PM*(0.40) = *PL*(0.40) = 0.



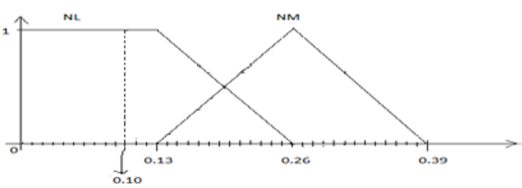


Figure 6 **Figure 7**

Figures (6 and 7): Fuzzification of I/P variable intensity for $x_0 = 0.40$ *and duration*for $y_0 = 0.10$ *respectively*.

A fuzzification process (function) applied to the I/P variable duration (τ), is represented by f_τ. Then the fuzzification function has the form f_τ: [0*;* 1] $\to$ *R*, where *R* denote the set of all fuzzy numbers. Then $f_\tau(y_0 = 0.10)$ is a fuzzy number chosen by f_τ as a fuzzy approximation of the measurement (sensorreading) duration (τ) at $y_0 = 0.10$. The computation of fuzzy membership values from Figure 5 for which $f_\tau(y_0 = 0.10) \neq 0$, is shown in Figure 7. The membership values for fuzzy sets *NL* are computedas, *NL* (0*:*10) = 1*:* Remaining all memberships values from Figure 4 are zero. Such as *NS*(0.10) =*AZ*(0.10) = *PL*(0.10) = *PM*(0.10) = *PS*(0.10) = *NM*(0.10) = 0. This shows that only one rule fires, namely *NL* (0.10) = 1.

Step (b) Formation of TSK - fuzzy inference rules: The knowledge pertaining to the given control problem is formulated in terms of a set of fuzzy inference rules. We elicit fuzzy inference rules, for the I/P -variables intensity (*I*), duration (τ) and O/P -variable membrane current (I_{memb}) using thecanonical form:

If I = A and τ = B then $I_{memb} = $ C,

where *A*, *B* and *C* are fuzzy numbers chosen from the set of fuzzy numbers , that represent the linguisticstates *NL; NM; NS; AZ; PM; PS* and *PL*. Since each I/P - variable has, seven linguistic states, the total number of possible non-conflicting fuzzy inference rules are $7^2 = 49$.

In practice, instead of these 49 rules, a small subset of all possible fuzzy inference rules is often sufficient to obtain acceptable performance of the fuzzy controllers. An appropriate subset of fuzzy rules derived intuitively by common sense reasoning is as follows:
Rule (1): If *I* is *AZ* and τis *NL* then I_{memb} is *PL*;
Rule (2): If *I* is *NS* and τis *NL* then I_{memb} is *PM*;
Rule (3): If *I* is *NM* and τis *NL* then I_{memb} is *NS*;
Rule (4): If *I* is *NM* and τis *AZ* then I_{memb} is *AZ*;
Rule (5): If *I* is *NS* and τis *PS* then I_{memb} *PL*;
Rule (6): If *I* is *PS* and τis *NS* then I_{memb} is *PS*;
Rule (7): If *I* is *PL* and τis *AZ* then I_{memb} is *PL*;
Rule (8): If *I* is *AZ* and τis *NS* then I_{memb} is *PS*;
Rule (9): If *I* is *AZ* and τis *NM* then I_{memb} is *PM*.

The fuzzy sets used in this set of rules are shown in Figure 4 and Figure 5. If we replace these fuzzy sets with practical fuzzy numbers such as, NS = about 0.13; NM = about 0.26 etc. We can rewrite the above rules as follows:

Rule (1): If I is AZ = about 0.52 and τ is NL= about 0.13 thenI_{memb} is PL= about 91;
Rule (2): If I is NS= about0.39andτ is NL= about 0.13 then I_{memb} is PM= about 78;
Rule (3): If I is NM = about 0.26 and τ is NL= about 0.13 thenI_{memb} is NS= about 39;
Rule (4): If I is NM = about 0.26 and τ is AZ= about 0.52 thenI_{memb} is AZ= about52;
Rule (5): If I is NS = about 0.39 and τ is PS= about 0.65 thenI_{memb} is PL= about 91;
Rule (6): If I is PS= about 0.65 and τ is NS= about 0.39 then I_{memb} is PS= about 65;
Rule (7): If I is PL= about 0.91 and τ is AZ= about 0.52 then I_{memb} is PL= about 91;
Rule (8): If I is AZ= about 0.52 and τ is NS= about 0.39 then I_{memb} is PS= about 65;
Rule (9): If I is AZ= about 0.52 and τ is NM= about 0.26 then I_{memb} is PM=about78.

The I/Ps-O/P relationships of the simplified TSK model from the above nine rules are shown in Figure 8.

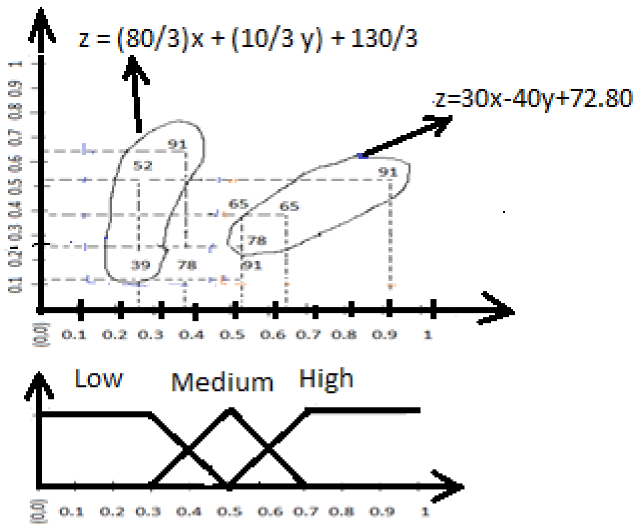


Figure 8: I/Ps -O/P relationship of simplified TSK fuzzy model.

From the above Figure 8, we obtain the equation of a plane passing through three points (or is called three points form equation of the plane). We know formula for the equation of plane is,

$$a\,(x - x_1) + b(y - y_1) + c(z - z_1) = 0.$$

We choose the points to represent the concepts Low, Medium and High. For the region Low we choose three points, A(0.26, 0.52, 52) , B(0.26, 0.13, 39) and C(0.39, 0.65,91) for which equation of plane is,

$$z_1 = \frac{80}{3}x + \frac{10}{3}y + \frac{130}{3} \tag{7}$$

And for the region High we choose three points A (.52, 0.26, 78), B (0.65, 0.39, 65) and C (0.91, 0.52, 91) for which equation of plane is,

$$z_2 = 30\,x - 40\,y + 72.80\ldots\ldots\ldots \tag{8}$$

Thus original number of equations was nine but it has been reduced to two. We express the I/Ps - O/P relations by these two linear equations as shown in Figure 8.

The TSK Method: The TSK method is used when the consequence part is given as a linear function of I/P - variables, and expressed as,

R_i : "If $x\ is\ A_i\ and\ y\ is\ B_i\ then\ \ z\ is\ f(x,y)$", i = 1, 2, 3, $\ldots n$,

where, $z = \ f(x,y)$ is a crisp linear function of the I/ P variables x and y expressed as,$z = ax + by + c$, where a, b and c are real numerical constants and$A_i\ and\ B_i$ are fuzzy sets in the antecedent part. We note that this method works when I/Ps are given as a singleton values and called fuzzy singleton. Thus in view of derivation of equations (8), (9) and rule $R_i$we note that the inference performed by the TSK – model is an interpolation of the relevant linear models. The degree of relevance of linear model is determined by the degree of I/P data belonging to the fuzzy subspaces associated with the linear model. These degrees of relevance become the weight in the interpolation process. The total O/P of the fuzzy model is given by the equation below,

$z_0 = \frac{\sum_{i=1}^{L} \alpha_i\, f_i(x_1, x_2, \ldots, x_r)}{\sum_{i=1}^{L} \alpha_i}$, where L is a finite positive integer.

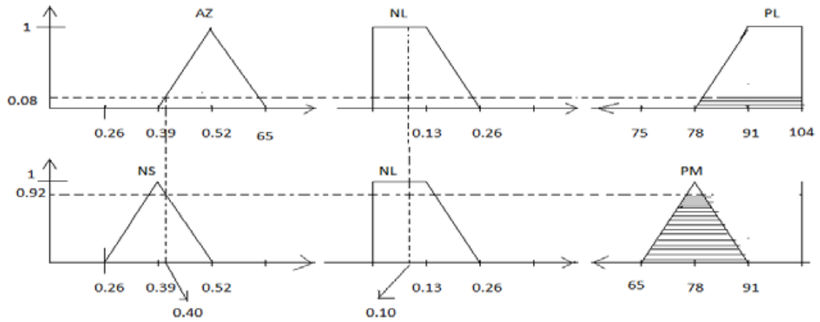
$= \frac{\sum_{i=1}^{L} \alpha_i (b_{i1}x_1 + b_{i2}x_2 + \cdots + b_{ir}x_r)}{\sum_{i=1}^{L} \alpha_i}$,

where$\alpha_i$is the matching degree of rule R_i, which is calculated analogous to the matching degree of the Mamdani model. The I/Ps of TSK - model are crisp numbers. Therefore degree of I/Ps is typically computed by "min" operator as follows:

$\alpha_i = \min(\mu_{A_{i1}}(a_{i1}), \mu_{A_{i2}}(a_{i2}), \ldots, \mu_{A_{ir}}(a_{ir}))$.

Step(c) Compositions of fuzzy inference: The inferred values of the control action from the 8th and9th rules are z_1= $f_1(x_0\,, y_0)$ and z_2= $f_2(x_0\,, y_0)$; respectively, wherein $x_0\,, y_0$are I/Ps sensor readings. The matching degrees $\alpha_1\ of\ R_1$ and$\alpha_2\ of\ R_2$ are determined similar to the Mamdani matching degree using 'min' operator as shown in Figure 9.

$\alpha_1\ = min(\mu_{A_1}(0.40), \mu_{B_1}(0.13)) = 0.08$.



$\alpha_2\ = min(\mu_{A_2}(0.40),\ \mu_{B_2}(0.10))$ = 0.92.

Figure 9: Graphical representation of TSK method.

In the sense of TSK -fuzzy inference rules, the aggregated result is given by weighted average formula,

$$z_0 = \frac{\alpha_1 f_1(x_0\,, y_0) + \alpha_2 f_2(x_0\,, y_0)}{\alpha_1 + \alpha_2}$$

$$= \frac{\alpha_1 z_1 + \alpha_2 z_2}{\alpha_1 + \alpha_2}.$$

We calculate the inferred value of the control action from the first rule is $f_1(x_0\ , y_0)$ where $x_0\ , y_0$ I/Ps sensor readings as are,

$$z_1 = f_1(x_0, y_0) = \frac{80}{3}x + \frac{10}{3}y + \frac{130}{3}\ .$$

$$f_1(x_0\ , y_0) = f_1(0.40, 0.10) = \frac{80}{3} * 0.40 + \frac{10}{3} * 0.10 + \frac{130}{3}.$$
$$= 54.32.$$

And from the second rule $f_2(x_0\ , y_0)$ as,

$$z_2 = f_2(x_0, y_0) = 30\ x - 40\ y + 72.80$$

$$f_2(x_0\ , y_0) = 30 * 0.40 - 40 * 0.10 + 72.80$$
$$= 80.80.$$

Now using these values of $\alpha_{1\ ,}\alpha_2$ z_1 and z_2 in the weighted average formula. TSK-fuzzy logic control gives desired O/P result membrane current(I_{memb}),

$$I_{memb} = z_0 = \frac{0.08 * 54.32 + 0.92 * 80.80}{0.08 + 0.92}$$
$$= 78.68 \qquad (9)$$

4. CONCLUDING REMARKS

The relations (6) and (9) show that O/P results provided by Hodgkin- Huxley classical mathematical model and our designed TSK - fuzzy controlled model are equivalent, provided that the values (sensor readings) of I/Ps linguistic variables for both models must be same. The obtained TSK - fuzzy controlled system are shown to be within the class of designs and capable of approximating the true O/P relation to the required degree of accuracy.

Thus we conclude that the traditional classical EEG Signal mathematical model may appear simpler and perhaps more economical but we should not easily make this assumption due to its complex PID (proportional, integral and derivative) model and time consuming factor. In fact TSK-fuzzy logic control are often easily prototyped and implemented, very simpler to describe and verify, can be maintained and embedded with higher degree of accuracy in less time.

5. FUTURE SCOPE

The method can be extended for more general form.

REFERENCES

[1] George J. Klir and Bo Yuan; *Fuzzy Sets and Fuzzy Logic (Theory and applications)*, PHI, New Delhi, (2006).

[2] John Yen and Reza Langari; *Fuzzy Logic (Intelligence, Control and Information)*, Pearson Education, (1999).

[3] Kamble P.N., Mherotra S. C. and Dhakne M. B., Superity of fuzzy controllers over mathermatical modelling of EEG signals, Proceedings of the international conference on Mathematical Sciences in Honour of A. M. Mathi, 175-190, (2011).

[4] Kamble P.N., Mherotra S.C and Dhakne M.B., Generalization of Superiority of Fuzzy Controllers over Mathematical Modeling of EEG Signals, International Journal of Mathematics and computation, vol. 18; Issue No. 1, (2012).

[5] Kazuo Tanaka; *An Introduction to Fuzzy Logic for Practical Applications*, Springer, (1997).

[6] MetinAkay, Editor, Nonlinear Biomedical Signal processing Volume - I (Fuzzy Logic, Neural Networks and New Algorithms), IEEE press products, (2000).

[7] NazmiEtik, NovruzAllahverdi, Ibrahim UnalSert, Ismail Saritas, Fuzzy expert system design for operating room air-condition control systems, Expert Systems with Applications 36, 9753-9758, (2009).

[8] Ravindra R. Mudholkar and Shashikant R. Sawant, Fuzzy Logic Transformer Build Estimation, IEEE Transactions on industrial electronics, Vol.49, No. 1, February, (2002).

[9] SaeidSanei and J. A. Chambers; *EEG signal processing*, John Wiley and sons Ltd, (2007).

[10] *Simulator for Neural Networks and Action potentials (SNNAP) Tutorial*, The University of Texas-Houston Medical School, 2003,http//snap.uth.tmc.edu.

[11] M. Sugeno; *Fuzzy Control*, Nikkankogyo, Tokyo, (1988).

[12] Terrance J. Senijnowski; *Towardas, Brain computer Interfacing*, The MIT Press Cambridge, London, England, (2007).

[13] Verbruggen H. B., Bruijn P.M., Fuzzy control and conventional control: What is (and can be) the real contribution of Fuzzy Systems?, ELSEVIER Fuzzy Sets and Systems, 90, 151-160, (1997).

[14] L. X.Wang. Fuzzy systems are universal approximators. IEEE International Conference on Fuzzy Systems, (1992).

[15] L.A. Zadeh. Fuzzy logic = computing with words. IEEE Transactions on Fuzzy Systems, 4(2):103-111, (1996).

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