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Almost slightly β-continuity, Slightly β-open and Slightly β-closed mappings

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ABSTRACT

In this paper we discuss new type of continuous functions called Almost slightly pre−continuous, slightly β-open and slightly β-closed functions; its properties and interrelation with other such functions are studied.

Keywords: *slightly continuous functions; slightly semi-continuous functions; slightly pre-continuous; slightly β−continuous functions and slightly ν−continuous functions.*

AMS-classification Numbers: *54C10; 54C08; 54C05.*

1. INTRODUCTION

In 1995 T. M. Nour introduced slightly semi-continuous functions. After him T. Noiri and G. I. Chae further studied slightly semi-continuous functions in 2000. T. Noiri individually studied about slightly β−continuous functions in 2001. C. W. Baker introduced slightly precontinuous functions in 2002. Arse Nagli Uresin and others studied slightly δ−continuous functions in 2007. Recently S. Balasubramanian and P.A.S.Vyjayanthi studied slightly ν−continuous functions in 2011. Mappings plays an important role in the study of modern mathematics, especially in Topology and Functional analysis. Closed mappings are one such mappings which are studied for different types of closed sets by various mathematicians for the past many years. N.Biswas, discussed about semiopen mappings in the year 1970, A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb studied preopen mappings in the year 1982 and S.N.El-Deeb, and I.A.Hasanien defind and studied about preclosed mappings in the year 1983. Further Asit kumar sen and P. Bhattacharya discussed about pre-closed mappings in the year 1993. A.S.Mashhour, I.A.Hasanein and S.N.El-Deeb introduced α-open and α-closed mappings in the year in 1983, F.Cammaroto and T.Noiri discussed about semipre-open and semipre-clsoed mappings in the year 1989 and G.B.Navalagi further verified few results about semipreclosed mappings. M.E.Abd El-Monsef, S.N.El-Deeb and R.A.Mahmoud introduced β-open mappings in the year 1983 and Saeid Jafari and T.Noiri, studied about β-closed mappings in the year 2000. In the year 2010, S. Balasubramanian and P.A.S.Vyjayanthi introduced *v*-open mappings and in the year 2011, further defined almost *v*-open mappings and also they introduced *v*-closed and Almost *v*-closed mappings. C.W.Baker studied slightly-open and slightly-closed mappings in the year 2011. Inspired with these developments we introduce in this paper Almost slightly β-continuous, slightly β-open and slightly β-closed functions and study its basic properties and interrelation with other type of such functions. Throughout the paper (X, τ) and (Y, σ) (or simply X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned.

2. PRELIMINARIES

Definition 2.1: $A\subset X$ is called g-closed [rg-closed] if cl $A\subseteq U$ whenever $A\subseteq U$ and U is open in X.

Definition 2.2: A function f:X→ Y is said to be

(i) continuous [resp: nearly-continuous; rα-continuous; α-continuous; semi-continuous; β−continuous; pre-continuous] if inverse image of each open set is open[resp: regular-open; rα-open; α-open; semi-open; β -open; preopen].

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(ii) almost continuous[resp: almost nearly-continuous; almost rα-continuous; almost α-continuous; almost semi-continuous; almost β-continuous; almost pre -continuous] if for each x in X and each open set (V, f(x)), ∃ an open[resp: regular-open; rα-open; α-open; semi-open; β-open; preopen] set (U, x) such that $f(U) \subset (cl(V))^o$.
(iii) slightly continuous[resp: slightly semi-continuous; slightly pre-continuous; slightly β−continuous; slightly α-continuous; slightly r-continuous; slightly v-continuous] at x in X if for each clopen subset V in Y containing f(x), ∃ U∈τ(X)[∃ U∈SO(X); ∃ U∈PO(X); ∃ U∈βO(X); ∃ U∈αO(X); ∃ U∈RO(X); ∃ U∈vO(X)] containing x such that $f(U) \subseteq V$.
(iv) almost slightly continuous[resp: almost slightly semi-continuous; almost slightly pre-continuous; almost slightly α-continuous; almost slightly r-continuous; almost slightly v-continuous] at x in X if for each r-clopen subset V in Y containing f(x), ∃ U∈τ(X)[∃ U∈SO(X); ∃ U∈PO(X); ∃ U∈αO(X); ∃ U∈RO(X); ∃ U∈vO(X)] containing x such that $f(U) \subseteq V$.
(v) open[resp: nearly-open; rα-open; α-open; semi-open; β-open; pre-open] if the image of each open set is open[resp: regular-open; rα-open; α-open; semi-open; β -open; pre-open].
(vi) almost-open [resp: almost-nearly-open; almost-rα-open; almost-α-open; almost-semi-open; almost-β-open; almost-pre-open] if the image of each r-open set is open[resp: regular-open; rα-open; α-open; semi-open; β-open; pre-open].
(vii) slightly-open [resp: slightly-r-open; slightly-semi-open; slightly-pre-open] if the image of each clopen set is open[resp: regular-open; semi-open; pre-open].
(viii) almost slightly-open[resp: almost slightly-r-open; almost slightly-semi-open; almost slightly-pre-open] if the image of each r-clopen set is open[resp: regular-open; semi-open; pre-open].

Lemma 2.1:
(i) Let A and B be subsets of a space X, if A∈τ(X) and B∈RO(X), then A∩B∈τ(B).
(ii)Let A⊂ B⊂ X, if A∈τ(B) and B∈ RO(X), then A∈τ(X).

Note 1: RCO(Y, f(x)) means regular-clopen set in Y containing f(x) and τ(X, x) means open set in X containing x.

3. ALMOST SLIGHTLY *β*-CONTINUOUS FUNCTIONS

Definition 3.1: A function f:X→Y is said to be almost slightly *β*-continuous at x in X if for each V∈RCO(Y, f(x)), ∃ U∈βO(X, x) such that $f(U) \subseteq V$ and almost slightly *β*-continuous if it is almost slightly *β*-continuous at each x in X.

Note 2: Here after we call almost slightly *β*-continuous function as al.sl.β.c function shortly.

Example 3.1: X = Y = {a, b, c}; τ = {ϕ, {a}, {b}, {a, b}, X} and σ = {ϕ, {a}, {b, c}, Y}. Let f be defined as f(a) = b; f(b) = c and f(c) = a, then f is sl.β.c., and al.sl.β.c.

Example 3.2: X = Y = {a, b, c}; τ = {ϕ, {a}, {b, c}, X} and σ = {ϕ, {a}, {b}, {a, b}, Y}. Let f be defined as f(a) = b; f(b) = c and f(c) = a, then f is not sl.β.c., and al.sl.β.c.

Example 3.3: X = Y = {a, b, c}; τ = {ϕ, {a, c}, X} and σ = {ϕ, {a}, {b, c}, Y}. Let f be defined as f(a) = b; f(b) = c and f(c) = a, then f is sl.β.c., sl.p.c., al.sl.β.c., al.sl.p.c., but not sl.c., sl.s.c., al.sl.c., and al.sl.s.c.,

Example 3.4: In Example 3.1, f is sl.β.c., sl.p.c., sl.s.c., sl.c., al.sl.β.c., al.sl.p.c., al.sl.s.c., and al.sl.c.

Example 3.5: In Example 3.2, f is sl.p.c., sl.c., al.sl.p.c., and al.sl.c., but not sl.β.c., and sl.s.c., but not al.sl.β.c., and al.sl.s.c.,

Example 3.6: In Example 3.3, f is sl.p.c., sl.β.c., al.sl.p.c., and al.sl.β.c., but not sl.c., sl.s.c., al.sl.c., and al.sl.s.c.,

Theorem 3.1: The following are equivalent:
(i) f is al.sl.β.c.
(ii) $f^{-1}(V)$ is *β*-open for every r-clopen set V in Y.
(iii) $f^{-1}(V)$ is *β*-closed for every r-clopen set V in Y.
(iv) $f(\beta cl(A)) \subseteq \beta cl(f(A))$.

Corollary 3.1: The following are equivalent.
(i) f is al.sl.β.c.
(ii) For each x in X and each V∈ RCO(Y, f(x)) ∃ U∈ βO(X, x) such that $f(U) \subseteq V$.

Theorem 3.2: Let $\sum = \{U_i : i \in I\}$ be any cover of X by regular open sets in X. A function *f* is al.sl.β.c. iff $f_{/Ui}$**:** is al.sl.β.c., for each $i \in I$.

Proof: Let $i \in I$ be an arbitrarily fixed index and $U_i \in RO(X)$. Let $x \in U_i$ and $V \in RCO(Y, f_{Ui}(x))$

Since *f* is al.sl.β.c, $\exists$ $U \in \beta O(X, x)$ such that $f(U) \subset V$. Since $U_i \in RO(X)$, by Lemma 2.1 $x \in U \cap U_i \in \beta O(U_i)$ and $(f_{/Ui})U \cap U_i = f(U \cap U_i) \subset f(U) \subset V$. Hence $f_{/Ui}$ is al.sl.β.c.

Conversely Let x in X and $V \in RCO(Y, f(x))$, $\exists$ $i \in I$ such that $x \in U_i$. Since $f_{/Ui}$ is al.sl.β.c, $\exists$ $U \in \beta O(U_i, x)$ such that $f_{/Ui}(U) \subset V$. By Lemma 2.1, $U \in \beta O(X)$ and $f(U) \subset V$. Hence *f* is al.sl.β.c.

Theorem 3.3: If *f* is almost continuous and *g* is continuous[al.sl.β.c.,], then *g*• *f* is al.sl.β.c.

Theorem 3.4: If *f* is almost continuous, open and *g* be any function, then *g*•*f* is al.sl.β.c iff *g* is al.sl.β.c.

Proof: If part**:** Theorem 3.3

Only if part**:** Let $A \in RCO(Z)$. Then $(g \bullet f)^{-1}(A) \in \tau(X)$. Since *f* is open, $f(g \bullet f)^{-1}(A) = g^{-1}(A)$ is open in Y. Thus *g* is al.sl.β.c.

Corollary 3.2: If *f* is r-irresolute, open and bijective, *g* is a function. Then *g* is al.sl.β.c. iff *g*• *f* is al.sl.β.c.

Theorem 3.5: If $g: X \to X \times Y$, defined by $g(x) = (x, f(x))$ for all x in X be the graph function of $f: X \to Y$. Then *g* is al.sl.β.c iff *f* is al.sl.β.c.

Proof: Let $V \in RCO(Y)$, then $X \times V \in RCO(X \times Y)$. Since *g* is al.sl.β.c., $f^{-1}(V) = f^{-1}(X \times V) \in \beta O(X)$.
Thus *f* is al.sl.β.c.

Conversely, let x in X and $F \in RCO(X \times Y, g(x))$. Then $F \cap (\{x\} \times Y) \in RCO(\{x\} \times Y, g(x))$. Also $\{x\} \times Y$ is homeomorphic to Y. Hence $\{y \in Y : (x, y) \in F\} \in RCO(Y)$. Since *f* is al.sl.β.c.$\cup\{f^{-1}(y) : (x, y) \in F\}$ is open in X. Further $x \in \cup\{f^{-1}(y) : (x, y) \in F\} \subseteq g^{-1}(F)$. Hence $g^{-1}(F)$ is open. Thus *g* is al.sl.β.c.

Theorem 3.6:
(i) $f: \Pi X_\lambda \to \Pi Y_\lambda$ is al.sl.β.c, iff $f_\lambda: X_\lambda \to Y_\lambda$ is al.sl.β.c for each $\lambda \in \Gamma$.
(ii) If $f: X \to \Pi Y_\lambda$ is al.sl.β.c, then $P_\lambda \bullet f : X \to Y_\lambda$ is al.sl.β.c for each $\lambda \in \Gamma$, where $P_\lambda : \Pi Y_\lambda$ onto Y_λ.

Remark 1: Composition, Algebraic sum, product and the pointwise limit of al.sl.β.c functions is not in general al.sl.β.c. However we can prove the following**:**

Theorem 3.7: The uniform limit of a sequence of al.sl.β.c functions is al.sl.β.c.

Note 3: Pasting Lemma is not true for al.sl.β.c functions. However we have the following weaker versions.

Theorem 3.8: Let X and Y be topological spaces such that $X = A \cup B$ and let $f_{/A}$ and $g_{/B}$ are al.sl.r.c maps such that $f(x) = g(x)$ for all $x \in A \cap B$. If A, $B \in RO(X)$ and RO(X) is closed under finite unions, then the combination $\alpha: X \to Y$ is al.sl.β.c continuous.

Theorem 3.9: Pasting Lemma Let X and Y be spaces such that $X = A \cup B$ and let $f_{/A}$ and $g_{/B}$ are al.sl.β.c maps such that $f(x) = g(x)$ for all $x \in A \cap B$. A, $B \in RO(X)$ and βO(X) is closed under finite unions, then the combination $\alpha: X \to Y$ is al.sl.β.c.

Proof: Let $F \in RCO(Y)$, then $\alpha^{-1}(F) = f^{-1}(F) \cup g^{-1}(F)$, where $f^{-1}(F) \in \beta O(A)$ and $g^{-1}(F) \in \beta O(B) \Rightarrow f^{-1}(F)$; $g^{-1}(F) \in \beta O(X) \Rightarrow f^{-1}(F) \cup g^{-1}(F) = \alpha^{-1}(F) \in \beta O(X)$. Hence $\alpha: X \to Y$ is al.sl.β.c.

Definition 3.2: A function *f* is said to be almost somewhat *β*-continuous if for $U \in RO(\sigma)$ and $f^{-1}(U) \neq \varphi$, there exists a non-empty *β*-open set V in X such that $V \subset f^{-1}(U)$.

It is clear that every continuous function is almost somewhat continuous and almost somewhat continuous function is almost somewhat *β*-continuous. But the converse is not true.

Example 3.7: Let X = {a, b, c}, τ = σ = {φ, {b}, {a, b}, {b, c}, X}. The function *f* defined by *f*(a) = b, *f*(b) = c and *f*(c) = a is almost somewhat β-continuous but not somewhat β-continuous.

Note 4: Every almost somewhat β-continuous function is almost slightly β-continuous.

Theorem 3.10: If *f* is almost somewhat β-continuous and *g* is continuous, then *g•f* is almost somewhat β-continuous.

Corollary 3.3: If *f* is almost somewhat β-continuous and *g* is r-continuous[r-irresolute], then *g•f* is almost somewhat β-continuous.

Theorem 3.11: For a surjective function *f*, the following statements are equivalent:
(i) *f* is almost somewhat β-continuous.
(ii) If C is a r-closed subset of Ysuch that $f^{-1}(C)\neq X$, then there is a proper β-closed subset D of X such that $f^{-1}(C) \subset D$.

(iii)If M is a dense subset of X, then *f*(M) is a dense subset of Y.

Proof:
(i) ⇒(ii): For C∈RC(Y) with $f^{-1}(C) \neq X$, Y-C∈RO(Y) such that $f^{-1}(Y\text{-}C) = X - f^{-1}(C) \neq \varphi$ By (i), there exists a β-open set V such that V ≠ φ and $V\subset f^{-1}(Y\text{-}C) = X - f^{-1}(C)$. Thus $X\text{-}V\supset f^{-1}(C)$ and X - V = D is a proper β-closed set in X.

(ii) ⇒(i): Let U∈RO(σ) and $f^{-1}(U) \neq \varphi$ Then Y-U is r-closed and $f^{-1}(Y\text{-}U) = X\text{-}f^{-1}(U) \neq X$. By (ii), there exists a proper β-closed set D such that $D\supset f^{-1}(Y\text{-}U)$. This implies that $X\text{-}D \subset f^{-1}(U)$ and X-D is β-open and X-D ≠ φ.

(ii) ⇒(iii): Let M be dense set in X. If *f*(M) is not dense in Y. Then there exists a proper r-closed set C in Y such that *f*(M) ⊂ C⊂ Y. Clearly $f^{-1}(C) \neq X$. By (ii), there exists a proper β-closed set D such that $M\subset f^{-1}(C) \subset D\subset X$. This is a contradiction to the fact that M is dense in X.

(iii) ⇒(ii): Suppose (ii) is not true, there exists a r-closed set C in Y such that $f^{-1}(C) \neq X$ but there is no proper β-closed set D in X such that $f^{-1}(C) \subset D$. This means that $f^{-1}(C)$ is dense in X.

But by (iii), $f(f^{-1}(C)) = C$ must be dense in Y, which is a contradiction to the choice of C.

Theorem 3.12: Let *f* be a function and X = A∪B, where A, B∈RO(X). If $f_{/A}$ and $f_{/B}$ are almost somewhat β-continuous, then *f* is almost somewhat β-continuous.
Proof: Let U∈RO(σ) such that $f^{-1}(U) \neq \varphi$. Then $(f_{/A})^{-1}(U) \neq \varphi$ or $(f_{/B})^{-1}(U)\neq\varphi$ or both $(f_{/A})^{-1}(U) \neq \varphi$ and $(f_{/B})^{-1}(U) \neq \varphi$. Suppose $(f_{/A})^{-1}(U) \neq \varphi$, Since $f_{/A}$ is almost somewhat β-continuous, there exists a β-open set V in A such that V≠ φ and $V\subset (f_{/A})^{-1}(U) \subset f^{-1}(U)$. Since V∈ βO(A) and A∈RO(X), V∈ βO(X). Thus *f* is almost somewhat β-continuous.

The proof of other cases are similar.

Definition 3.3: If X is a set and τ and σ are topologies on X, then τ is said to be β-equivalent to σ provided if U∈ βO(τ) and U ≠ φ, there is an β-open set V in *X* such that V ≠ φ and V ⊂ U and if U∈ βO(σ) and U ≠ φ, there is an β-open set V in (X, τ) such that V≠ φ and U ⊃ V.

Definition 3.4: A⊂ X is said to be dense in X if there is no proper closed set C in X such that M⊂ C⊂ X.

Now, consider the identity function *f* and assume that τ and σ are equivalent. Then *f* and f^{-1} are almost somewhat continuous. Conversely, if the identity function *f* is almost somewhat continuous in both directions, then τ and σ are equivalent.

Theorem 3.13: Let *f*:(X, τ)→ (Y, σ) be a almost somewhat β-continuous surjection and τ* be a topology for X, which is β-equivalent to τ. Then *f*:(X, τ*)→ (Y, σ) is almost somewhat β-continuous.

Proof: Let V∈RO(σ)∋$f^{-1}(V) \neq \varphi$. Since *f* is almost somewhat β-continuous, ∃ a nonempty U∈ βO(X, τ) ∋$U\subset f^{-1}(V)$. For τ* is β-equivalent to τ, ∃ U*∈ βO(X; τ*) ∋ U*⊂ U. But $U\subset f^{-1}(V)$. Then $U^*\subset f^{-1}(V)$; hence *f*:(X, τ*)→ (Y, σ) is almost somewhat β-continuous.

Theorem 3.14: Let *f*:(X, τ)→ (Y, σ) be a almost somewhat β-continuous surjection and σ* be a topology for Y, which is β-equivalent to σ. Then *f* :(X, τ)→ (Y, σ*) is almost somewhat β-continuous.

Proof: Let $V^* \in RO(\sigma^*) \ni f^{-1}(V^*) \neq \varphi$. Since σ^* is β-equivalent to σ, $\exists\, V \neq \varphi \in \beta O(Y, \sigma) \ni V \subset V^*$.

Now $\varphi \neq f^{-1}(V) \subset f^{-1}(V^*)$. Since f is almost somewhat β-continuous, $\exists\, U \neq \varphi \in \beta O(X, \tau) \ni U \subset f^{-1}(V)$.

Then $U \subset f^{-1}(V^*)$; hence $f:(X, \tau) \to (Y, \sigma^*)$ is almost somewhat β-continuous.

4. SLIGHTLY β-OPEN MAPPINGS, ALMOST SLIGHTLY β-OPEN MAPPINGS AND ALMOST SOMEWHAT β-OPEN FUNCTION

Definition 4.1: A function $f: X \to Y$ is said to be
(i) slightly β-open if image of every clopen set in X is β-open in Y
(ii) almost slightly β-open if image of every regular-clopen set in X is β-open in Y

Example 4.1: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$; $\sigma = \{\varphi, \{a, c\}, Y\}$. Let $f:X \to Y$ be defined $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then f is slightly open, slightly pre-open, slightly semi-open, slightly β-open, almost slightly open, almost slightly semi-open, almost slightly pre-open, and almost slightly β-open.

Example 4.2: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{b, c\}, X\}$; $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f:X \to Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then f is not slightly open, slightly pre-open, slightly semi-open, slightly β-open, almost slightly open, almost slightly semi-open, almost slightly pre-open, and almost slightly β-open.

Note 5:
(i) If $R\alpha O(Y) = \beta O(Y)$, then f is [almost-] slightly rα-open iff f is [almost-] slightly β-open.
(ii) If $\beta O(Y) = RO(Y)$, then f is [almost-] slightly-r-open iff f is [almost-] slightly β-open.
(iii) If $\beta O(Y) = \alpha O(Y)$, then f is [almost-] slightly α-open iff f is [almost-] slightly β-open.

Theorem 4.1:
(i) If f is [almost-] slightly open and g is β-open[r-open] then $g\bullet f$ is slightly β-open
(ii) If f is [almost-] slightly β-open and g is M- β-open [M-r-open] then $g\bullet f$ is slightly β-open

Proof: Let A be clopen[regular clopen] set in X $\Rightarrow f(A)$ is open in Y $\Rightarrow g(f(A)) = g\bullet f(A)$ is β-open in Z. Hence $g\bullet f$ is [almost-] slightly β-open.

Theorem 4.2: If f and g are r-open then $g\bullet f$ is [almost-] slightly β-open

Proof: Let A be clopen[r-clopen] set in X $\Rightarrow f(A)$ is r-open and so open in Y $\Rightarrow g(f(A))$ is r-open in Z$\Rightarrow g(f(A)) = g\bullet f(A)$ is open in Z. Hence $g\bullet f$ is [almost-] slightly β-open.

Theorem 4.3: If f is almost slightly-r-open and g is [almost-] β-open then $g\bullet f$ is [almost-] slightly β-open

Corollary 4.1:
(i) If f is almost slightly-open and g is open[r-open] then $g\bullet f$ is [almost-] slightly β-open.
(ii) If f is almost slightly-r-open and g is [almost-] β-open then $g\bullet f$ is [almost-] slightly β-open.
(iii) If f and g are almost slightly-r-open then $g\bullet f$ is [almost-] slightly β-open.

Theorem 4.4: If f is [almost-] slightly β-open, then $f(A^o) \subset \beta(f(A))^o$

Proof: Let $A \subset X$ and f is slightly β-open gives $f(A^o)$ is β-open in Y and $f(A^o) \subset f(A)$ which in turn gives

$f(A^o)^o \subset \beta(f(A))^o$ (1)

Since $f(A^o)$ is β-open in Y, $\beta(f(A^o))^o = f(A^o)$ (2)

From (1) and (2) we have $f(A^o) \subset \beta(f(A))^o$ for every subset A of X.

Remark 2: converse is not true in general.

Theorem 4.5: If f is slightly β-open and $A \subset X$ is r-open, then $f(A)$ is τ_p-open in Y.

Proof: Let $A \subset X$ and f is slightly β-open implies $f(A^o) \subset \beta(f(A))^o$ which in turn implies $\beta(f(A))^o \subset f(A)$, since $f(A) = f(A^o)$. But $f(A) \subset \beta(f(A))^o$. Combining we get $f(A) = \beta(f(A))^o$. Hence $f(A)$ is τ_p-open in Y.

Corollary 4.2:
(i) If f is [almost-] slightly r-open, then $f(A^o) \subset \beta(f(A))^o$
(ii) If f is [almost-] slightly r-open, then $f(A)$ is τ_p-open in Y if A is r-open set in X.
(iii) If f is almost slightly β-open and $A \subset X$ is r-open, then $f(A)$ is τ_s-open in Y.

Theorem 4.6: If $\beta(A)^o = r(A^o)$ for every $A \subset Y$, then the following are equivalent:
(i) f is [almost-]slightly β-open map
(ii) $f(A^o) \subset \beta(f(A))^o$

Proof:
(i) ⇒ (ii): follows from theorem 4.4

(ii) ⇒ (i): Let A be any r-open set in X, then $f(A) = \beta(f(A))^o \supset f(A^o)$ by hypothesis. We have $f(A) \subset s\ (f(A))^o$. Combining we get $f(A) = \beta(A)^o = r(A^o)$[by given condition] which implies $f(A)$ is r-open and hence open. Thus f is slightly β-open.

Theorem 4.7: f is [almost-]slightly β-open iff for each subset S of Y and each r-clopen set U containing $f^{-1}(S)$, there is a β-open set V of Y such that $S \subset V$ and $f^{-1}(V) \subset U$.

Remark 3: composition of two [almost-] slightly β-open maps is not [almost-] slightly β-open in general.

Theorem 4.8: Let X, Y, Z be topological spaces and every open set is r-clopen in Y, then the composition of two [almost-] slightly β-open maps is [almost-] slightly β-open.

Proof: Let A be r-clopen in X ⇒ $f(A)$ is open and so r-clopen in Y[by assumption]
⇒ $g(f(A)) = g \bullet f(A)$ is open in Z. Hence $g \bullet f$ is almost slightly β-open.

Theorem 4.9: If f is [almost-] slightly g-open; g is open[r-open] and Y is $T_{½}$**[**r-$T_{½}$**]**, then $g \bullet f$ is [almost-] slightly β-open.

Proof :(i) Let A be regular clopen in X ⇒ A be clopen in X ⇒ $f(A)$ is g-open and open in Y[since Y is $T_{½}$]
⇒ $g(f(A)) = g \bullet f(A)$ is open in Z. Hence $g \bullet f$ is [almost-]slightly β-open.

Corollary 4.3:
(i) If f is [almost-] slightly g-open; g is open[r-open] and Y is $T_{½}$**[** r-$T_{½}$**]** then $g \bullet f$ is [almost-]slightly β-open.
(ii) If f is [almost-] slightly g-open; g is [almost-] β-open [[almost-]r-open] and Y is $T_{½}$**[** r-$T_{½}$**]** then $g \bullet f$ is [almost-]slightly β-open.

Theorem 4.10: If f is [almost-] slightly rg-open; g is open[r-open] and Y is r-$T_{½}$, then $g \bullet f$ is [almost-] slightly β-open.

Proof: Let A be r-clopen in X ⇒ A be clopen in X ⇒ $f(A)$ is rg-open and r-open in Y[since Y is r-$T_{½}$]
⇒ $g(f(A)) = g \bullet f(A)$ is open in Z. Hence $g \bullet f$ is almost slightly β-open.

Theorem 4.11: If f is [almost-] slightly rg-open; g is [almost-] β-open [[almost-]r-open] and Y is r-$T_{½}$, then $g \bullet f$ is [almost-]slightly β-open.

Proof: Let A be r-clopen in X ⇒ A be clopen in X ⇒ $f(A)$ is rg-open in Y ⇒ $f(A)$ is r-open in Y[since Y is r-$T_{½}$]
⇒ $g(f(A)) = g \bullet f(A)$ is open in Z. Hence $g \bullet f$ is almost slightly β-open.

Corollary 4.4:
(i) If f is [almost-] slightly rg-open; g is open[r-open] and Y is r-$T_{½}$, then $g \bullet f$ is [almost-] slightly β-open.
(ii) If f is [almost-] slightly rg-open; g is [almost-] β-open [[almost-]r-open] and Y is r-$T_{½}$, then $g \bullet f$ is [almost-] slightly β-open.

Theorem 4.12: If f; g be two mappings such that $g \bullet f$ is [almost-] slightly β-open[[almost-] slightly r-open]. Then the following are true
(i) If f is continuous[r-continuous] and surjective, then g is [almost-] slightly β-open
(ii) If f is g-continuous, surjective and X is $T_{½}$, then g is [almost-] slightly β-open
(iii) If f is rg-continuous, surjective and X is r-$T_{½}$, then g is [almost-] slightly β-open

Proof: Let A be regular clopen in Y ⇒ A be clopen in Y ⇒ $f^{-1}(A)$ is open in X ⇒ $g \bullet f(f^{-1}(A)) = g(A)$ is open in Z. Hence g is almost slightly β-open.

Similarly we can prove the remaining parts and so omitted.

Corollary 4.5: If f; g be two mappings such that $g \bullet f$ is [almost-] slightly β-open [[almost-] slightly r-open]. Then the following are true
(i) If f is continuous[r-continuous] and surjective, then g is [almost-] slightly β-open.
(ii) If f is g-continuous, surjective and X is $T_{½}$, then g is [almost-] slightly β-open.
(iii) If f is rg-continuous, surjective and X is r-$T_{½}$, then g is [almost-] slightly β-open.

Theorem 4.13: If X is regular, f is r-open, nearly-continuous, open surjection and $\bar{A} = A$ for every open[r-open] set in Y, then Y is regular.

Theorem 4.14: If f is [almost-]slightly β-open and A is r-clopen[clopen] set of X, then f_A is [almost-]slightly β-open.

Proof: Let F be r-open set in A. Then $F = A \cap E$ is r-open in X for some r-open set E of X which implies $f(A)$ is open in Y. But $f(F) = f_A(F)$. Therefore f_A is [almost-] slightly β-open.

Theorem 4.15: If f is [almost-] slightly β-open, X is $T_{½}$ and A is g-open set of X, then f_A is [almost-] slightly β-open.

Corollary 4.6: If f is [almost-] slightly open, X is $T_{½}$ and A is g-open set of X, then f_A is [almost-] slightly β-open.

Theorem 4.16: If f_i: $X_i \to Y_i$ be [almost-] slightly β-open for i = 1, 2. Let f: $X_1 \times X_2 \to Y_1 \times Y_2$ be defined as $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then f: $X_1 \times X_2 \to Y_1 \times Y_2$ is [almost-] slightly β-open.

Proof: Let $U_1 \times U_2 \subset X_1 \times X_2$ where U_i is r-clopen in X_i for i = 1, 2. Then $f(U_1 \times U_2) = f_1(U_1) \times f_2(U_2)$ a open set in $Y_1 \times Y_2$. Thus $f(U_1 \times U_2)$ is open and hence f is [almost-]slightly β-open.

Corollary 4.7: If f_i: $X_i \to Y_i$ be [almost-] slightly open for i = 1, 2. Let f: $X_1 \times X_2 \to Y_1 \times Y_2$ be defined as $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then f: $X_1 \times X_2 \to Y_1 \times Y_2$ is [almost-] slightly β-open.

Theorem 4.17: Let h: $X \to X_1 \times X_2$ be [almost-]slightly β-open. Let f_i: $X \to X_i$ be defined as $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then f_i: $X \to X_i$ is [almost-] slightly β-open for i = 1, 2.

Proof: Let U_1 be r-clopen in X_1, then $U_1 x X_2$ is r-clopen in $X_1 x X_2$, and $h(U_1 x X_2)$ is open in X. But $f_1(U_1) = h(U_1 x X_2)$, therefore f_1 is [almost-]slightly β-open. Similarly we can show that f_2 is [almost-] slightly β-open and thus f_i: $X \to X_i$ is [almost-] slightly β-open for i = 1, 2.

Corollary 4.8: Let h: $X \to X_1 \times X_2$ be [almost-] slightly open. Let f_i: $X \to X_i$ be defined as $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then f_i: $X \to X_i$ is [almost-] slightly β-open for i = 1, 2.

Definition 4.2: A function f is said to be almost somewhat β-open provided that if $U \in RO(\tau)$ and $U \neq \varphi$, then there exists a non-empty β-open set V in Y such that $V \subset f(U)$.

Example 4.3: Let X = {a, b, c}, τ = {φ, {a}, X} and σ = {φ, {a}, {b, c}, X}. The function f defined by $f(a) = a$, $f(b) = c$ and $f(c) = b$ is almost somewhat open and almost somewhat β-open.

Example 4.4: Let X = {a, b, c}, τ = {φ, {a}, {b, c}, X} and σ = {φ, {a}, {b}, {a, b}, X}. The function f defined by $f(a) = c$, $f(b) = a$ and $f(c) = b$ is not almost somewhat β-open.

Theorem 4.18: Let f be an r-open function and g almost somewhat β-open. Then $g \bullet f$ is almost somewhat β-open.

Theorem 4.19: For a bijective function f, the following are equivalent:
(i) f is almost somewhat β-open.
(ii) If C is an r-closed subset of X, such that $f(C) \neq Y$, then there is a β-closed subset D of Y such that $D \neq Y$ and $D \supset f(C)$.

Proof: (i) ⇒(ii): Let C be any r-closed subset of X such that $f(C) \neq Y$. Then X-C is r-open in X and X-C ≠ φ. Since f is almost somewhat β-open, there exists a β-open set $V \neq \varphi$ in Y such that $V \subset f(X\text{-}C)$. Put D = Y-V. Clearly D is β-closed in Y and we claim D≠ Y. If D = Y, then V = φ, which is a contradiction. Since V⊂ *f*(X-C), D = Y-V ⊃ (Y- *f*(X-C)) = *f*(C).

(ii) ⇒(i): For U≠ φ an r-open in X, C = X-U is r-closed in X and *f*(X-U) = *f*(C) = Y- *f*(U) implies *f*(C) ≠ Y. Therefore, by (ii), there is a β-closed set D of Y such that D≠ Y and *f*(C) ⊂ D. Clearly V = Y-D is a β-open set and V≠ φ . Also, V = Y-D ⊂ Y- *f*(C) = Y- *f*(X-U) = *f*(U).

Theorem 4.20: The following statements are equivalent:
(i) *f* is almost somewhat β-open.
(ii) If A is a dense subset of Y, then f^{-1}(A) is a dense subset of X.

Proof:
(i) ⇒(ii): If A is dense set in Y. If f^{-1}(A) is not dense in X, then there exists a r-closed set B in X such that f^{-1}(A) ⊂ B⊂ X. Since *f* is almost somewhat β-open and X-B is open, there exists a nonempty β-open set C in Y such that C⊂ *f*(X-B). Therefore, C⊂ *f*(X-B) ⊂ *f*(f^{-1}(Y-A)) ⊂ Y-A. That is, A⊂ Y-C ⊂ Y. Now, Y-C is a β-closed set and A⊂ Y-C ⊂ Y. This implies that A is not a dense set in Y, which is a contradiction. Therefore, f^{-1}(A) is a dense set in X.

(ii) ⇒(i): If A≠ φ is an r-open set in X. We want to show that $(f(A))^{o}$≠ φ. Suppose $(f(A))^{o}$ = φ. Then, *cl*(*f*(A)) = Y. By (ii), f^{-1}(Y - *f*(A)) is dense in X. But f^{-1}(Y- *f*(A)) ⊂ X-A. Now, X-A is r-closed. Therefore, f^{-1}(Y- *f*(A)) ⊂ X-A gives X = *cl*(f^{-1}(Y- *f*(A))) ⊂ X-A. This implies that A = φ, which is contrary to A≠ φ. Therefore, $(f(A))^{o}$≠ φ. Hence *f* is almost somewhat β-open.

Theorem 4.21: Let *f* be almost somewhat β-open and A be any r-open subset of X. Then $f_{/A}$ is almost somewhat β-open.

Proof: Let U∈RO($\tau_{/A}$) such that U ≠ φ. Since U∈RO($\tau_{/A}$); A∈RO(X); U∈RO(X) and *f* is almost somewhat β-open, ∃ V∈ βO(Y), such that V⊂ *f*(U). Thus $f_{/A}$ is almost somewhat β-open.

Theorem 4.22: Let *f* be a function and X = A∪ B, where A,B∈τ(X). If the restriction functions $f_{/A}$ and $f_{/B}$ are almost somewhat β-open, then *f* is almost somewhat β-open.

Proof: Let U be any r-open subset of X such that U≠ φ. Since X = A∪ B, either A∩ U ≠ φ or B∩U ≠ φ or both A∩U ≠ φ and B∩U ≠ φ. Since U is open in X, U is open in both A and B.

Case (i): If A∩U ≠ φ, where U∩A∈RO($\tau_{/A}$). Since $f_{/A}$ is almost somewhat β-open, ∃V∈ βO(Y) such that V⊂*f*(U∩A) ⊂ *f*(U), which implies that *f* is almost somewhat β-open.

Case (ii): If B∩U ≠ φ, where U∩B∈RO($\tau_{/B}$). Since $f_{/B}$ is almost somewhat β-open, ∃V∈ βO(Y) such that V⊂ *f*(U∩B) ⊂ *f*(U), which implies that *f* is almost somewhat β-open.

Case (iii): If both A∩U ≠ φ and B∩U ≠ φ. Then by cases (i) and (ii) *f* is almost somewhat β-open.

Remark 4: Two topologies τ and σ for X are said to be β-equivalent if and only if the identity function *f*: (X, τ)→ (Y, σ) is almost somewhat β-open in both directions.

Theorem 4.23: Let *f*: (X, τ)→ (Y, σ) be a almost somewhat almost β-open function. Let τ* and σ* be topologies for X and Y, respectively such that τ* is β-equivalent to τ and σ* is β-equivalent to σ. Then *f*: (X; τ*)→ (Y; σ*) is almost somewhat β-open.

5. SLIGHTLY β-CLOSED MAPPINGS AND ALMOST SLIGHTLY β-CLOSED MAPPINGS

Definition 5.1: A function *f: X*→ Y is said to be
(i) slightly β-closed if image of every clopen set in X is β-closed in Y
(ii) almost slightly β-closed if image of every regular-clopen set in X is β-closed in Y

Example 5.1: Let *X* = *Y* = {a, b, c}; τ = {φ, {a}, {a, b}, *X*}; σ = {φ, {a, c}, *Y*}. Let *f:X*→*Y* be defined *f*(a) = c, *f*(b) = b and *f*(c) = a. Then *f* is slightly closed, slightly pre-closed, slightly semi-closed, slightly β-closed, almost slightly closed, almost slightly semi-closed, almost slightly pre-closed, and almost slightly β-closed.

Example 5.2: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{b, c\}, X\}$; $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f: X\to Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then f is not slightly closed, slightly pre-closed, slightly semi-closed, slightly β-closed, almost slightly closed, almost slightly semi-closed, almost slightly pre-closed, and almost slightly β-closed.

Note 6:
(i) If RαC(Y) = βC(Y), then *f* is [almost-] slightly rα-closed iff *f* is [almost-] slightly β-closed.
(ii) If βC(Y) = RC(Y), then *f* is [almost-] slightly-r-closed iff *f* is [almost-] slightly β-closed.
(iii) If βC(Y) = αC(Y), then *f* is [almost-] slightly α-closed iff *f* is [almost-] slightly β-closed.

Theorem 5.1: (i) If *f* is [almost-] slightly closed and *g* is β-closed[*r*-closed] then *g•f* is slightly β-closed
(ii) If *f* is [almost-] slightly β-closed and *g* is M- β-closed [M-*r*-closed] then *g•f* is slightly β-closed

Proof: Let A be clopen[regular clopen] set in X ⇒ *f*(A) is closed in Y ⇒ *g*(*f*(A)) = *g•f*(A) is β-closed in Z. Hence *g• f* is [almost-] slightly β-closed.

Theorem 5.2: If *f* and *g* are *r*-closed then *g•f* is [almost-] slightly β-closed

Proof: Let A be clopen[r-clopen] set in X ⇒ *f*(A) is *r*-closed and so closed in Y ⇒ *g*(*f*(A)) is *r*-closed in Z ⇒ *g*(*f*(A)) = *g•f*(A) is closed in Z. Hence *g• f* is [almost-] slightly β-closed.

Theorem 5.3: If *f* is almost slightly-*r*-closed and *g* is [almost-] β-closed then *g•f* is [almost-]slightly β-closed

Corollary 5.1:
(i) If *f* is almost slightly-closed and *g* is closed[r-closed] then *g•f* is [almost-] slightly β-closed.
(ii) If *f* and *g* are almost slightly-*r*-closed then *g•f* is [almost-] slightly β-closed.
(iii) If *f* is almost slightly-*r*-closed and *g* is [almost-] β-closed then *g•f* is [almost-] slightly β-closed.

Theorem 5.4: If *f* is [almost-]slightly β-closed, then $\beta cl(\{f(A)\})\subset f(cl\{A\})$

Proof: Let $A\subset X$ and *f* is slightly β-closed gives $f(cl\{A\})$ is β-closed in Y and $f(A) \subset f(cl\{A\})$ which in turn gives

$\beta cl(\{f(A)\}) \subset \beta cl\{(f(cl\{A\}))\}$ (1)

Since $f(cl\{A\})$ is β-closed in Y, $\beta cl\{(f(cl\{A\}))\} = f(cl\{A\})$ (2)

From (1) and (2) we have $(\beta cl\{f(A)\}) \subset (f(cl\{A\}))$ for every subset A of X.

Remark 5: converse is not true in general.

Theorem 5.5: If *f* is slightly β-closed and $A\subset X$ is *r*-closed, then *f*(A) is τ_p-closed in Y.

Proof: Let $A\subset X$ and *f* is slightly β-closed implies $(\beta cl\{f(A)\}) \subset f(cl\{A\})$ which in turn implies $(\beta cl\{f(A)\}) \subset f(A)$, since $f(A) = f(cl\{A\})$. But $f(A) \subset (\beta cl\{f(A)\})$. Combining we get $f(A) = (\beta cl\{f(A)\})$. Hence *f*(A) is τ_p-closed in Y.

Corollary 5.2:
(i) If *f* is [almost-] slightly *r*-closed, then $\beta cl(\{f(A)\})\subset f(cl\{A\})$
(ii) If *f* is [almost-] slightly *r*-closed, then *f*(A) is closed in Y if A is *r*-closed set in X.
(iii) If *f* is almost slightly β-closed and $A\subset X$ is *r*-closed, then *f*(A) is τ_s-closed in Y.

Theorem 5.6: If $(\beta cl\{A\}) = r(cl\{A\})$ for every $A\subset Y$, then the following are equivalent:
(i) *f* is [almost-]slightly β-closed map
(ii) $\beta cl(f(A)) \subset f(cl(A))$

Proof:
(i) ⇒ (ii) follows from theorem 5.4

(ii) ⇒ (i) Let A be any *r*-closed set in X, then $f(A) = f(cl\{A\}) \supset (\beta cl\{f(A)\})$ by hypothesis. We have $f(A) \subset (\beta cl\{f(A)\})$. Combining we get $f(A) = (\beta cl\{f(A)\}) = r(cl\{f(A)\})$[by given condition] which implies *f*(A) is *r*-closed and hence closed. Thus *f* is slightly β-closed.

Theorem 5.7: f is [almost-]slightly β-closed iff for each subset S of Y and each r-clopen set U containing $f^{-1}(S)$, there is a β-closed set V of Y such that $S \subset V$ and $f^{-1}(V) \subset U$.

Remark 6: composition of two [almost-] slightly β-closed maps is not [almost-] slightly β-closed in general

Theorem 5.8: Let X, Y, Z be topological spaces and every closed set is r-clopen in Y, then the composition of two [almost-] slightly β-closed maps is [almost-] slightly β-closed.

Proof: Let A be r-clopen in X $\Rightarrow$ $f(A)$ is closed and so r-clopen in Y[by assumption] $\Rightarrow$ $g(f(A)) = g\bullet f(A)$ is closed in Z. Hence $g\bullet f$ is almost slightly β-closed.

Theorem 5.9: If f is [almost-]slightly g-closed; g is closed[r-closed] and Y is $T_{½}$[r-$T_{½}$], then $g\bullet f$ is [almost-]slightly β-closed.

Proof:(i) Let A be r-clopen in X $\Rightarrow$ A be clopen in X $\Rightarrow$ $f(A)$ is g-closed in Y $\Rightarrow$ $f(A)$ is closed in Y[since Y is $T_{½}$]
$\Rightarrow$ $g(f(A)) = g\bullet f(A)$ is closed in Z. Hence $g\bullet f$ is [almost-] slightly β-closed.

Corollary 5.3: (i) If f is [almost-]slightly g-closed; g is closed[r-closed] and Y is $T_{½}$[r-$T_{½}$] then $g\bullet f$ is [almost-]slightly β-closed.
(ii) If f is [almost-] slightly g-closed; g is [almost-] β-closed [[almost-] r-closed] and Y is $T_{½}$[r-$T_{½}$] then $g\bullet f$ is [almost-]slightly β-closed.

Theorem 5.10: If f is [almost-]slightly rg-closed; g is closed[r-closed] and Y is r-$T_{½}$, then $g\bullet f$ is [almost-]slightly β-closed.

Proof: Let A be r-clopen in X $\Rightarrow$ A be clopen in X $\Rightarrow$ $f(A)$ is rg-closed and so r-closed in Y[since Y is r-$T_{½}$]
$\Rightarrow$ $g(f(A)) = g\bullet f(A)$ is closed in Z. Hence $g\bullet f$ is almost slightly β-closed.

Theorem 5.11: If f is [almost-]slightly rg-closed; g is [almost-] β-closed[[almost-]r-closed] and Y is r-$T_{½}$, then $g\bullet f$ is [almost-]slightly β-closed.

Proof: Let A be r-clopen in X $\Rightarrow$ A be clopen in X $\Rightarrow$ $f(A)$ is rg-closed and so r-closed in Y[since Y is r-$T_{½}$]
$\Rightarrow$ $g(f(A)) = g\bullet f(A)$ is closed in Z. Hence $g\bullet f$ is almost slightly β-closed.

Corollary 5.4:
(i) If f is [almost-] slightly rg-closed; g is closed[r-closed] and Y is r-$T_{½}$, then $g\bullet f$ is [almost-] slightly β-closed.
(ii) If f is [almost-] slightly rg-closed; g is [almost-] β-closed[[almost-]r-closed] and Y is r-$T_{½}$, then $g\bullet f$ is [almost-] slightly β-closed.

Theorem 5.12: If f; g be two mappings such that $g\bullet f$ is [almost-] slightly β-closed [[almost-] slightly r-closed]. Then the following are true
(i) If f is continuous[r-continuous] and surjective, then g is [almost-] slightly β-closed
(ii) If f is g-continuous, surjective and X is $T_{½}$, then g is [almost-] slightly β-closed
(iii)If f is rg-continuous, surjective and X is r-$T_{½}$, then g is [almost-] slightly β-closed

Proof: Let A be regular clopen in Y $\Rightarrow$ A be clopen in Y $\Rightarrow$ $f^{-1}(A)$ is closed in X $\Rightarrow$ $g\bullet f(f^{-1}(A)) = g(A)$ is closed in Z. Hence g is almost slightly β-closed.

Similarly we can prove the remaining parts and so omitted.

Corollary 5.5: If f; g be two mappings such that $g\bullet f$ is [almost-] slightly β-closed[[almost-]slightly r-closed]. Then the following are true
(i) If f is continuous[r-continuous] and surjective, then g is [almost-] slightly β-closed.
(ii) If f is g-continuous, surjective and X is $T_{½}$, then g is [almost-] slightly β-closed.
(iii) If f is rg-continuous, surjective and X is r-$T_{½}$, then g is [almost-] slightly β-closed.

Theorem 5.13: If X is regular, f is r-closed, nearly-continuous, closed surjection and $\bar{A} = A$ for every closed[r-closed] set in Y, then Y is regular.

Theorem 5.14: If f is [almost-] slightly β-closed and A is r-clopen[clopen] set of X, then f_A is [almost-]slightly β-closed.

Proof: For F, *r*-closed in A, Then F = A∩ E is *r*-closed in X for some *r*-closed set E of X which implies *f*(A) is closed in Y. But $f(F) = f_A(F)$. Therefore f_A is [almost-] slightly β-closed.

Theorem 5.15: If *f* is [almost-] slightly β-closed, X is $T_{½}$ and A is g-closed set of X, then f_A is [almost-] slightly β-closed.

Corollary 5.6: If *f* is [almost-] slightly-closed, X is $T_{½}$ and A is g-closed set of X, then f_A is [almost-] slightly β-closed.

Theorem 5.16: If f_i: $X_i \to Y_i$ be [almost-] slightly β-closed for i = 1, 2. Let *f:* $X_1\times X_2 \to Y_1\times Y_2$ be defined as $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then *f:* $X_1\times X_2 \to Y_1\times Y_2$ is [almost-] slightly β-closed.

Proof: Let $U_1\times U_2\subset X_1\times X_2$ where $U_i\in RCO(X_i)$ for i = 1, 2. Then $f(U_1\times U_2) = f_1(U_1) \times f_2(U_2)$ a closed set in $Y_1\times Y_2$. Thus $f(U_1\times U_2)$ is closed and hence *f* is [almost-]slightly β-closed.

Corollary 5.7: If f_i:$X_i\to Y_i$ be [almost-]slightly β-closed for i = 1, 2. Let *f*:$X_1\times X_2\to Y_1\times Y_2$ be defined as $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then *f*:$X_1\times X_2\to Y_1\times Y_2$ is [almost-]slightly β-closed.

Theorem 5.17: Let *h*:$X\to X_1\times X_2$ be [almost-]slightly β-closed. Let f_i: $X\to X_i$ be defined as $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then f_i: $X\to X_i$ is [almost-] slightly β-closed for i = 1, 2.

Proof: Let U_1 be r-clopen in X_1, then U_1xX_2 is r-clopen in X_1xX_2, and $h(U_1xX_2)$ is closed in X. But $f_1(U_1) = h(U_1xX_2)$, therefore f_1 is [almost-]slightly β-closed. Similarly we can show that f_2 is [almost-] slightly β-closed and thus f_i: $X\to X_i$ is [almost-] slightly β-closed for i = 1, 2.

Corollary 5.8: Let *h*: $X \to X_1\times X_2$ be [almost-] slightly β-closed. Let f_i: $X \to X_i$ be defined as $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then f_i: $X \to X_i$ is [almost-] slightly β-closed for i = 1, 2.

6. COVERING AND SEPARATION PROPERTIES OF al.sl.β.c. and al.swt.β.c. FUNCTIONS

Theorem 6.1: If *f* is al.sl.β.c.[resp**:** al.sl.r.c] surjection and X is β-compact, then Y is compact.

Proof: Let $\{G_i : i\in I\}$ be any r-clopen cover for Y. Then each G_i is r-clopen in Y and *f* is al.sl.β.c., $f^{-1}(G_i)$ is β-open in X. Thus $\{f^{-1}(G_i)\}$ forms a β-open cover for X with a finite subcover, since X is β-compact. Since *f* is surjection, $Y = f(X) = \cup^n_{i=1} G_i$. Therefore Y is compact.

Theorem 6.2: If *f* is al.sl.β.c., surjection and X is β-compact[β-Lindeloff] then Y is mildly compact[mildly lindeloff].

Proof: Let $\{U_i : i\in I\}$ be r-clopen cover for Y. For each x in X, ∃ $\alpha_x\in I$ such that $f(x) \in U_{\alpha x}$ and ∃ $V_x\in \beta O(X, x)$ ∍ $f(V_x)\subset U_{\alpha x}$. Since $\{V_i : i\in I\}$ is a β-open cover of X, ∃ a finite subset I_0 of I such that $X\subset \{V_x : x\in I_0\}$. Thus $Y\subset\cup\{f(V_x) : x\in I_0\}\subset \cup\{U_{\alpha x} : x\in I_0\}$. Hence Y is mildly compact.

Corollary 6.1:
(i) If *f* is al.sl.r.c. surjection and X is β-compact, then Y is compact.
(ii) If *f* is al.sl.β.c.[resp**:** al.sl.r.c] surjection and X is locally β-compact{resp**:** β-Lindeloff; locally β-Lindeloff}, then Y is locally compact{resp**:** Lindeloff; locally lindeloff; locally mildly compact; locally mildly lindeloff}.
(iii) If *f* is al.sl.β.c., [resp**:** al.sl.r.c] surjection and X is β-compact[β-lindeloff] then Y is mildly compact[mildly lindeloff].

Theorem 6.3: If *f* is al.sl.β.c., surjection and X is s-closed then Y is mildly compact[mildly lindeloff].

Proof: Let $\{V_i : V_i\in RCO(Y); i\in I\}$ be a cover of Y, then $\{f^{-1}(V_i) : i\in I\}$ is β-open cover of X and so there is finite subset I_0 of I, such that $\{f^{-1}(V_i) : i\in I_0\}$ covers X. Therefore $\{V_i : i\in I_0\}$ covers Y since *f* is surjection. Hence Y is mildly compact.

Theorem 6.4: If *f* is al.sl.β.c.,[al.sl.r.c.] surjection and X is β-connected, then Y is connected.

Proof: If Y is disconnected, then Y = A∪ B where A and B are disjoint r-clopen sets in Y. Since *f* is al.sl.β.c. surjection, $X = f^{-1}(Y) = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ $f^{-1}(B)$ are disjoint β-open sets in X, which is a contradiction for X is β-connected. Hence Y is connected.

Corollary 6.2:
(i) If f is al.sl.c[resp**:** al.sl.r.c.] surjection and X is s-closed then Y is mildly compact[mildly lindeloff].
(ii)The inverse image of a disconnected space under a al.sl.β.c.,[resp**:** al.sl.r.c.] surjection is β-disconnected.

Theorem 6.5: If f is al.sl.β.c.[resp**:** al.sl.r.c.], injection and Y is UrT_i, then X is βT_i i = 0, 1, 2.

Proof: Let $x_1 \neq x_2 \in$ X. Then $f(x_1) \neq f(x_2) \in Y$ since f is injective. For Y is UrT_2 $\exists$ $V_j \in RCO(Y)$ such that $f(x_j) \in V_j$ and $\cap V_j = \phi$ for j = 1,2. By Theorem 3.1, $x_j \in f^{-1}(V_j) \in \beta O(X)$ for j = 1,2 and $\cap f^{-1}(V_j) = \phi$ for j = 1,2. Thus X is βT_2.

Theorem 6.6: If f is al.sl.β.c.[al.sl.r.c.] injection; r-closed and Y is UrT_i, then X is βT_i i = 3, 4.

Proof:
(i) Let x in X and F be disjoint r-closed subset of X not containing x, then f(x) and f(F) be disjoint r-closed subset of Y not containing f(x), since f is r-closed and injection. Since Y is ultraregular, f(x) and f(F) are separated by disjoint r-clopen sets U and V respectively.
Hence $x \in f^{-1}(U)$; $F \subseteq f^{-1}(V)$, $f^{-1}(U)$; $f^{-1}(V) \in \beta O(X)$ and $f^{-1}(U) \cap f^{-1}(V) = \phi$. Thus X is βT_3.
(ii) Let F_j and $f(F_j)$ are disjoint r-closed sets in X and Y respectively for j = 1,2, since f is r-closed and injection. For Y is ultranormal, $f(F_j)$ are separated by disjoint r-clopen sets V_j respectively for j = 1,2. Hence $F_j \subseteq f^{-1}(V_j)$ and $f^{-1}(V_j) \in \beta O(X)$ and $\cap f^{-1}(V_j) = \phi$ for j = 1,2. Thus X is βT_4.

Theorem 6.7: If f is al.sl.β.c.[resp**:** al.sl.r.c.], injection and
(i) Y is UrC_i[resp**:** UrD_i] then X is βC_i[resp**:** βD_i] i = 0, 1, 2.
(ii)Y is UrR_i, then X is βR_i i = 0, 1.

Theorem 6.8: If f is al.sl.β.c.[al.sl.r.c] and Y is UrT_2, then the graph G(f) is β-closed in X×Y.

Proof: Let $(x_1, x_2) \notin G(f)$ implies $y \neq f(x)$ implies $\exists$ disjoint V; W∈RCO(Y) such that $f(x) \in V$ and $y \in W$. Since f is al.sl.β.c., $\exists$ $U \in \beta O(X)$ such that $x \in U$ and $f(U) \subset W$ and $(x, y) \in U \times V \subset X \times Y - G(f)$. Hence G($f$) is β-closed in X×Y.

Theorem 6.9: If f is al.sl.β.c.[al.sl.r.c] and Y is UrT_2, then $A = \{(x_1, x_2) | f(x_1) = f(x_2)\}$ is β-closed in X×X.

Proof: If $(x_1, x_2) \in X \times X - A$, then $f(x_1) \neq f(x_2)$ implies $\exists$ disjoint $V_j \in RCO(Y)$ such that $f(x_j) \in V_j$, and since f is al.sl.β.c., $f^{-1}(V_j) \in \beta O(X, x_j)$ for j = 1,2. Thus $(x_1, x_2) \in f^{-1}(V_1) \times f^{-1}(V_2) \in \beta O(X \times X)$ and $f^{-1}(V_1) \times f^{-1}(V_2) \subset X \times X - A$. Hence A is β-closed.

Theorem 6.10: If f is al.sl.r.c.[resp**:** al.sl.β.c.]; g is al.sl.c[resp**:** al.sl.r.c]; and Y is UrT_2, then E = {x in X **:** $f(x) = g(x)$} is β-closed in X.

We have the following consequences of theorems 6.1 to 6.10:

Theorem 6.11: If f is al.swt.β.c.[al.swt.r.c] surjection and X is β-compact, then Y is compact.

Theorem 6.12: If f is al.swt.β.c., surjection and X is β-compact[β-Lindeloff] then Y is mildly compact[mildly lindeloff].

Corollary 6.3 :
(i) If f is al.swt.r.c. surjection and X is β-compact, then Y is compact.
(ii) If f is al.swt.β.c.[resp**:** al.swt.r.c] surjection and X is β-compact[β-Lindeloff] then Y is mildly compact[mildly lindeloff].
(iii) If f is al.swt.β.c.[resp**:** al.swt.r.c] surjection and X is locally β-compact{resp**:** β-Lindeloff; locally β-Lindeloff}, then Y is locally compact{resp**:** Lindeloff; locally lindeloff; locally mildly compact; locally mildly lindeloff}.

Theorem 6.13: If f is al.swt.β.c., surjection and X is s-closed then Y is mildly compact[mildly lindeloff].

Theorem 6.14: If f is al.swt.β.c.,[al.swt.r.c.] surjection and X is β-connected, then Y is connected.

Corollary 6.4:
(i) If f is al.swt.c[resp**:** al.swt.r.c.] surjection and X is s-closed then Y is mildly compact[mildly lindeloff].
(ii)The inverse image of a disconnected space under an al.swt.β.c.,[resp**:** al.swt.r.c.;] surjection is β-disconnected.

Theorem 6.15:
(i) If *f* is al.swt.β.c.[al.swt.r.c.], injection and Y is UrT_i, then X is βT_i i = 0, 1, 2.
(ii) If *f* is al.swt.β.c.[al.swt.r.c.] injection; r-closed and Y is UrT_i, then X is βT_i i = 3, 4.

Theorem 6.16: If *f* is al.swt.β.c.[resp**:** al.swt.r.c.;], injection and
(i) Y is UrC_i[resp**:** UrD_i] then X is βC_i[resp**:** βD_i] i = 0, 1, 2.
(ii)Y is UrR_i, then X is βR_i i = 0, 1.

Theorem 6.17: If *f* is al.swt.β.c.[resp**:** al.swt.r.c] and Y is UrT_2, then
(i) the graph G(*f*) is β-closed in X×Y.
(ii) A = $\{(x_1, x_2) | f(x_1) = f(x_2)\}$ is β-closed in X×X.

Theorem 6.18: If *f* is al.swt.r.c.[resp**:** al.swt.β.c.]; *g***:** X→ Y is al.swt.c[resp**:** al.swt.r.c]; and Y is UrT_2, then E = {x in X **:** *f*(x) = *g*(x)} is β-closed in X.

CONCLUSION

In this paper we introduced the concept of almost slightly β-continuous functions, almost somewhat β-continuous functions, somewhat β-open mappings, slightly β-open mappings, almost slightly β-open mappings, slightly β-closed mappings, almost slightly β-closed mappings, studied their basic properties and the interrelationship between other such maps.

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