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NUMERICAL SOLUTION FOR HYBRID FUZZY SYSTEMS BY SINGLE TERM HAAR WAVELET SERIES TECHNIQUE

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ABSTRACT

This paper presents numerical solutions for hybrid fuzzy differential equations by an application of the single-term Haar wavelet series (STHWS) technique, fourth order Runge-Kutta Method and Runge-Kutta Fehlberg method to solve the hybrid fuzzy differential equations [6 - 7]. The discrete solutions obtained through STHWS technique are compared with that of the Improved Euler method. The applicability of the STHWS technique is more suitable to solve the hybrid fuzzy differential equations.

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1. INTRODUCTION

Hybrid systems are devoted to modelling, design, and validation of interactive systems of computer programs and continuous systems. That is, control systems that are capable of controlling complex systems which have discrete event dynamics as well as continuous time dynamics can be modelled by hybrid system. The differential systems containing fuzzy valued functions and interaction with a discrete time controller are named hybrid fuzzy differential systems. For analytical results on stability properties and comparison theorems we refer reader to [10, 11, 14].

In this article we developed numerical methods for addressing hybrid fuzzy differential equations by an application of the STHW technique which was studied by S. Sekar and team of his researchers [15 - 21]. In section 2 we list some basic definitions for fuzzy valued functions. In Section 3 reviews hybrid fuzzy differential systems. In Section 4 contains the properties of Haar wavelets and STHWS technique for approaching hybrid fuzzy differential equations and a convergence theorem. In Section 5 contains a numerical example to illustrate the theorem. We refer [1, 2, 9, 12 - 13] for the numerical treatment of fuzzy differential equations.

2. PRELIMINARIES

Denote by E^1 the set of all functions $u : R \rightarrow [0, 1]$ such that (i) u is normal, that is, there exist an $x_0 \in R$ such that $u(x_0)=1$, (ii) u is a fuzzy convex, that is, for $x, y \in R$ and $0 \leq \lambda \leq 1$, $u(\lambda x + (1 - \lambda)y) \geq \min\{u(x), u(y)\}$, (iii) u is upper semicontinuous, and (iv) $[u]^0 \equiv$ the closure of $\{x \in R: u(x) > 0\}$ is compact. For $0 < \alpha \leq 1$, we define $[u]^\alpha = \{x \in R: u(x) \geq \alpha\}$. An example of a $u \in E^1$ is given by

$$u(x) = \begin{cases} 4x-3, \; if \;\; x \in (0.75, 1], \\ -2x+3, \; if \;\; x \in (1, 1.5), \\ 0, \; if \;\; x \notin (0.75, 1.5). \end{cases} \tag{1}$$

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The α-level sets of u in (1) are given by

$$[u]^{\alpha} = [0.75 + 0.25\alpha,\ 1.5 - 0.5\alpha]. \tag{2}$$

For later purpose, we define $\hat{0} \in E^1$ as $\hat{0}(x) = 1$ if $x = 0$ and $\hat{0}(x) = 0$ if $x \neq 0$.

Next we review the Seikkala derivative [22] of $x: I \to E^1$ where $I \subset R$ is an interval. If $[x(t)^{\alpha}] = [\underline{x}^{\alpha}(t), \overline{x}^{\alpha}(t)]$ for all $t \in I$ and $\alpha \in [0, 1]$, then $[x'(t)^{\alpha}] = [(\underline{x}^{\alpha})'(t), (\overline{x}^{\alpha})'(t)]$ if $x'(t) \in E^1$. Next consider the initial value problem (IVP)

$$u(x) = \begin{cases} x'(t) = f(t, x(t)), \\ x(0) = x_0 \end{cases} \tag{3}$$

where $f: [0,\infty) \times R \to R$ is continuous. We would like to interpret (3) using the Seikkala derivative and $x_0 \in E^1$.

Let $[x_0]^{\alpha} = [\underline{x_0}^{\alpha}, \overline{x}_0^{\alpha}]$ and $[x(t)]^{\alpha} = [\underline{x}^{\alpha}(t), \overline{x}^{\alpha}(t)]$. By the Zadeh extension principle we get $f: [0, \infty) \times E^1 \to E^1$ where $\left[f(t,x)\right]^{\alpha} = \left[\min\left\{f(t,u) : u \in \left[\underline{x}^{\alpha}(t), \overline{x}^{\alpha}(t)\right]\right\}, \max\left\{f(t,u) : u \in \left[\underline{x}^{\alpha}(t), \overline{x}^{\alpha}(t)\right]\right\}\right]$.

Then $x: [0, \infty) \to E^1$ is a solution of (3) using the Seikkala derivative and $x_0 \in E^1$ if

$$\begin{aligned} \left(\underline{x}^{\alpha}\right)'(t) &= \min\left\{f(t,u) : u \in \left[\underline{x}^{\alpha}(t), \overline{x}^{\alpha}(t)\right]\right\}, \underline{x}^{\alpha}(0) = \underline{x}_0^{\alpha}, \\ \left(\overline{x}^{\alpha}\right)'(t) &= \max\left\{f(t,u) : u \in \left[\underline{x}^{\alpha}(t), \overline{x}^{\alpha}(t)\right]\right\}, \overline{x}^{\alpha}(0) = \overline{x}_0^{\alpha}, \end{aligned},$$

for all $t \in [0,\infty)$ and $\alpha \in [0, 1]$. Lastly consider an $f: [0,\infty) \times R \times R \to R$ which is continuous and the IVP

$$\begin{cases} x'(t) = f(t, x(t), k), \\ x(0) = x_0 \end{cases} \tag{4}$$

As in [3], to interpret (4) using the Seikkala derivative and x_0, $k \in E^1$, by the Zadeh extension principle we use $f: [0,\infty) \times E^1 \times E^1 \to E^1$ where

$$\begin{aligned} \left[f(t,x,k)\right]^{\alpha} = [&\min\left\{f(t,u,u_k) : u \in \left[\underline{x}^{\alpha}(t), \overline{x}^{\alpha}(t)\right], u_k \in \left[\underline{k}^{\alpha}, \overline{k}^{\alpha}\right]\right\}, \\ &\max\left\{f(t,u,u_k) : u \in \left[\underline{x}^{\alpha}(t), \overline{x}^{\alpha}(t)\right], u_k \in \left[\underline{k}^{\alpha}, \overline{k}^{\alpha}\right]\right\}] \end{aligned}$$

where $k^{\alpha} = \left[\underline{k}^{\alpha}, \overline{k}^{\alpha}\right]$. Then $x: [0, \infty) \to E^1$ is a solution of (4) using the Seikkala derivative and x_0, $k \in E^1$ if

$$\begin{aligned} \left(\underline{x}^{\alpha}\right)'(t) &= \min\left\{f(t,u,u_k) : u \in \left[\underline{x}^{\alpha}(t), \overline{x}^{\alpha}(t)\right], u_k \in \left[\underline{k}^{\alpha}, \overline{k}^{\alpha}\right]\right\}, \underline{x}^{\alpha}(0) = \underline{x}_0^{\alpha}, \\ \left(\overline{x}^{\alpha}\right)'(t) &= \max\left\{f(t,u,u_k) : u \in \left[\underline{x}^{\alpha}(t), \overline{x}^{\alpha}(t)\right], u_k \in \left[\underline{k}^{\alpha}, \overline{k}^{\alpha}\right]\right\}, \underline{x}^{\alpha}(0) = \underline{x}_0^{\alpha}, \end{aligned}$$

for all $t \in [0,\infty)$ and $\alpha \in [0, 1]$.

3. THE HYBRID FUZZY DIFFERENTIAL SYSTEM

Consider the hybrid fuzzy differential system

$$\left.\begin{aligned} &x'(t) = f(t, x(t), \lambda_k x(t_k)), \ t \in [t_k, t_{k+1}], \\ &x(t_k) = x_{t_k}, \end{aligned}\right\} \tag{5}$$

where x' denotes Seikkala differentiation, $0 \leq t_0 < t_1 < \cdot\ \cdot\ \cdot < t_k < \cdot\ \cdot\ \cdot$, $t_k \to \infty$,

$f \in C[R^+ \times E^1 \times E^1, E^1]$, $\lambda_k \in C[E^1, E^1]$. To be specific the system look like

$$x'(t)=\begin{cases} x_0'(t)=f(t,x_0(t),\lambda_0(x_0)), x_0(t_0)=x_0,\ t_0\le t\le t_1,\\ x_1'(t)=f(t,x_1(t),\lambda_1(x_1)), x_1(t_1)=x_1,\ t_1\le t\le t_2,\\ \dots\\ x_k'(t)=f(t,x_k(t),\lambda_k(x_k)), x_k(t_k)=x_k,\ t_k\le t\le t_{k+1},\\ \dots\end{cases}$$

Assuming that the existence and uniqueness of solution of (5) hold for each $[t_k, t_{k+1}]$, by the solution of (3) we mean the following function:

$$x(t)=x(t,t_0,x_0)\begin{cases} x_0(t),\ t_0\le t\le t_1,\\ x_1(t),\ t_1\le t\le t_2,\\ \dots\\ x_k(t),\ t_k\le t\le t_{k+1},\\ \dots\end{cases}$$

We note that the solution of (5) are piecewise differentiable in each interval for $t \in [t_k, t_{k+1}]$ for a fixed $x_k \in E^1$ and k=0, 1, 2, . . .

Using a representation of fuzzy numbers studied by Goestschel and Woxman [4] and Wu and Ma [23], we may represent $x\in E^1$ by a pair of functions $(\underline{x}(r),\overline{x}(r))$, $0 \le r \le 1$, such that (i) $\underline{x}(r)$, is bounded, left continuous, and non decreasing, (ii) $\overline{x}(r)$, is bounded, left continuous, and non increasing, and (iii) $\underline{x}(r)\le\overline{x}(r)$, $0 \le r \le 1$. For example, $u \in E^1$ given in (1) is represented by $(\underline{u}(r),\overline{u}(r))$ = (0.75 + 0.25r, 1.5 − 0.5r), $0 \le r \le 1$, which is similar to $[u]^\alpha$ given by (2).

Therefore we may replace (5) by an equivalent system

$$\begin{cases}\underline{x}'(t)=\underline{f}(t,x,\lambda_k(x_k))\equiv F_k(t,\underline{x},\overline{x}), \underline{x}(t_k)=\underline{x}_k,\\ \underline{x}'(t)=\overline{f}(t,x,\lambda_k(x_k))\equiv G_k(t,\underline{x},\overline{x}), \underline{x}(t_k)=\underline{x}_k,\end{cases}\tag{6}$$

which possesses a unique solution $(\underline{x},\overline{x})$ which is a fuzzy function. That is for each t, the pair $[\underline{x}(t;r),\overline{x}(t;r)]$ is a fuzzy number, where $\underline{x}(t;r),\overline{x}(t;r)$ are respectively the solutions of the parametric form given by

$$\begin{cases}\underline{x}'(t)=F_k(t,\underline{x}(t;r),\overline{x}(t;r)), \underline{x}(t_k;r)=\underline{x}_k(r),\\ \underline{x}'(t)=G_k(t,\underline{x}(t;r),\overline{x}(t;r)), \underline{x}(t_k;r)=\underline{x}_k(r),\end{cases}\tag{7}$$

for $r \in [0, 1]$.

4. PROPERTIES OF HAAR WAVELET AND STHW TECHNIQUE

4.1 HAAR WAVELET SERIES

The orthogonal set of Haar wavelets $h_i(t)$ is a group of square waves with magnitude of ± 1 in some intervals and zeros elsewhere. In general, $h_n(t)=h_1(2^j t-k)$, Where $n=2^j+k$, $j\ge 0,\ 0\le k<2^j,\ n,\ j,\ k\in Z.$ Any function y(t), which is square integrable in the interval [0, 1) can be expanded in a Haar series with an infinite number of terms

$$y(t)=\sum_{i=0}^{\infty}c_i h_i(t),\ \text{Where } i=2^j+k \tag{8}$$

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where the Haar coefficients $j \ge 0,\ 0 \le k < 2^j, t \in [0,1)$, $c_i = 2^j \int_0^1 y(t) h_i(t) dt$ are determined such that the following integral square error ε is minimized $\varepsilon = \int_0^1 \left[y(t) - \sum_{i=0}^{m-1} c_i h_i(t) \right]^2 dt,$ Where $m = 2^j,\ j \in \{0\} \cup N$

Furthermore $\int_0^1 h_i(t) h_1(t) dt = 2^{-j} \delta_{il} = \begin{cases} 2^{-j},\ i = l = 2^j + k,\ j \ge 0, 0 \le k < 2^j \\ 0 \quad ,\ i \ne l \end{cases}$, usually, the series expansion Eq. (8) contains an infinite number of terms for a smooth *y (t)*. If *y (t)* is a piecewise constant or may be approximated as a piecewise constant, then the sum in Eq. (8) will be terminated after *m* terms, that is

$$y(t) \approx \sum_{i=0}^{m=1} c_i h_i(t) = C^T_{(m)} h_{(m)}(t),\ t \in [0,1)$$

$$c_{(m)}(t) = [c_0 c_1 ... c_{m-1}]^T,$$

$$h_m(t) = [h_0(t), h_0(t), ..., h_{m-1}(t)]^T$$

where "T" indicates transposition, the subscript *m* in the parentheses denotes their dimensions, $C^T_{(m)} h_{(m)}(t)$ denotes the truncated sum. Since the differentiation of Haar wavelets results in generalized functions, which in any case should be avoided, the integration of Haar wavelets are preferred.

Integration of Haar Wavelets should be expandable in Haar series $\int_0^t h_m(\tau) d\tau = \sum_{i=0}^{\infty} C_i h_i(t)$ If we truncate to $m = 2^n$ terms and use the above vector notation, then integration is performed by matrix vector multiplication and expandable formula into Haar series with Haar coefficient matrix defined by Hsiao [5].

$$\int_0^1 h_{(m)}(\tau) d\tau \approx E_{(m \times m)} h_{(m)}(t),\ t \in [0,1)$$

where the m-square matrix E is called the operational matrix of integration which satisfies the following recursive equations

$$E_{(m \times m)} = \begin{bmatrix} E_{\left[\frac{m}{2} \times \frac{m}{2}\right]} & -\left(\frac{1}{2m}\right) H_{\left[\frac{m}{2} \times \frac{m}{2}\right]} \\ \left(\frac{1}{2m}\right) H^{-1}_{\left[\frac{m}{2} \times \frac{m}{2}\right]} & 0_{\left[\frac{m}{2} \times \frac{m}{2}\right]} \end{bmatrix} \tag{9}$$

$$E_{(2 \times 2)} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{4} \\ \frac{1}{4} & 0 \end{bmatrix} \quad \text{and} \quad E_{(1 \times 1)} = \frac{1}{2}$$

The $H_{m \times m} = [h_n(x_0), h_n(x_1).h_n(x_2)........, h_m(x_{m-1})], \frac{i}{m} \le x_i \le \frac{i+1}{m}$

$$H^{-1}_{(m \times m)} = \left(\frac{1}{m}\right) H^T_{(m \times m)} dia(r),$$

$$r = \left[1,1,2,2,4,4,4,4,........\underbrace{\frac{m}{2},\frac{m}{2},\frac{m}{2}}_{n}.........\frac{m}{2} \right]^T, m > 2$$

Proof of equation (9) is found in [5]. Since $H_{(m\,x\,m)}$ and $H^{-1}_{(m \times m)}$ contain many zeros. Let us define $h_{(m)}(t) h^T_{(m)}(t) \approx M_{(m \times m)}(t)$, and $M_{(1 \times 1)}(t) = h_0(t)$ satisfying $M_{(m \times m)}(t) c_{(m)} = C_{(m \times m)} h_{(m)}(t)$ and $C_{(1 \times 1)} = c_0$.

4.2 SINGLE TERM HAAR WAVELET SERIES TECHNIQUE

With the STHWS approach, in the first interval, the given function is expanded as STHWS in the normalized interval $\tau \in [0,1)$, which corresponds to $\tau \in \left[0, \frac{1}{m}\right)$ by defining $\tau = mt$, m being any integer. In STHWS, the matrix becomes $E = \frac{1}{2}$. Let $\dot{x}(\tau)$ and $x(\tau)$ be expanded by STHWS in the first interval as $\dot{x}(\tau) = \nu^{(1)} h_o(\tau)$, $x(\tau) = x^{(1)} h_0(\tau)$ and in the nth interval as, $\dot{x}(\tau) = \nu^{(n)} h_o(\tau)$, $x(\tau) = x^{(n)} h_0(\tau)$ Integrating (9) with $E = \frac{1}{2}$, we get $x^{(1)} = \frac{1}{2}\nu^{(1)} + x(0)$. Where $x(0)$ is the initial condition. According to [7], we have

$$\nu^{(1)} = \int \dot{x}(\tau) d\tau = x(1) - x(0)$$

In general, for any interval n, n=1, 2.......

We obtain, $$x^{(n)} = \frac{1}{2}\nu^{(n)} + x(n-1) \qquad (10)$$

$$x(n) = \nu^{(n)} + x(n-1) \qquad (11)$$

Equation (10) and (11) give the discrete time values of $x^{(n)}$ and $x(n)$ $x(n)$ for the nth interval. These values from the basis for the estimating block pulse values and discrete values in the subsequent normalized time intervals.

5. NUMERICAL EXAMPLE

Consider the following hybrid fuzzy IVP, [6 – 7]

$$\left.\begin{array}{l} x'(t) = x(t) + m(t)\lambda_k x(t_k),\ t \in [t_k, t_{k+1}],\ t_k = k,\ k = 0,1,2,3,... \\ x(t;r) = \left[(0.75 + 0.25r)e^t,\ (1.125 - 0.125r)e^t\right],\ 0 \le r \le 1, \end{array}\right\} \qquad (12)$$

where $$m(t) = \begin{cases} 2(t(\text{mod}1)) & if\ \ t(\text{mod}1) \le 0.5 \\ 2(1 - t(\text{mod}1)) & if\ \ t(\text{mod}1) > 0.5 \end{cases}$$

$$\lambda_k(\mu) = \begin{cases} \hat{0},\ if\ \ k = 0 \\ \mu,\ if\ k \in \{1,2,...\} \end{cases}$$

The hybrid fuzzy IVP (12) is equivalent to the following systems of fuzzy IVPs:

$$\left.\begin{array}{l} x_0'(t) = x_0(t),\ t \in [0,1], \\ x(0;r) = \left[(0.75 + 0.25r)e^t,\ (1.125 - 0.125r)e^t\right],\ 0 \le r \le 1, \\ x_i'(t) = x_i(t) + m(t) x_{i-1}(t),\ t \in [t_i, t_{i+1}],\ x_i(t) = x_{i-1}(t_i),\ i = 1,2,... \end{array}\right\}$$

In (12) $x(t) + m(t)\lambda_k x(t_k)$ is continuous function of *t, x* and $\lambda_k x(t_k)$. Therefore by Example 6.1 of Kaleva [8], for each $k = 0,1,2,...$ the fuzzy IVP

$$\left.\begin{array}{l} x'(t) = x(t) + m(t)\lambda_k x(t_k),\ t \in [t_k, t_{k+1}],\ t_k = k, \\ x(t_k) = x_{t_k}, \end{array}\right\}$$

has a unique solution $[t_k, t_{k+1}]$. To numerically solve the hybrid fuzzy IVP (12) we will apply the STHWS technique for hybrid fuzzy differential equation with N = 2 to obtain $y_{1,2}(r)$ approximating $x(2.0;r)$.The Exact and Approximate solutions by Fourth order Runge-Kutta Method, Runge-Kutta Fehlberg method and STHWS are compared at *t* = 2 (see Table 1 and Figure 1). Error in comparing Fourth Order Runge-Kutta method, Runge-Kutta Fehlberg method and STHWS are shown in table 2, Figure 2 and Figure 3.

Table 1: Discrete Solutions

r	Exact		RK-Fourth Order		RK-Fehlberge		STHWS	
	$y_1(t_i;r)$	$y_2(t_i;r)$	$y_1(t_i;r)$	$y_2(t_i;r)$	$y_1(t_i;r)$	$y_2(t_i;r)$	$y_1(t_i;r)$	$y_2(t_i;r)$
0.1	7.49966	10.76564	7.49355	10.75688	7.49931	10.76514	7.49966	10.76564
0.2	7.74158	10.64446	7.73528	10.63601	7.74123	10.64419	7.74158	10.64446
0.3	7.98350	10.52371	7.97701	10.51515	7.98314	10.52323	7.98350	10.52371
0.4	8.22543	10.40275	8.21874	10.39428	8.22505	10.40227	8.22543	10.40275
0.5	8.46735	10.28179	8.46046	10.27342	8.46697	10.28132	8.46735	10.28179
0.6	8.70928	10.16082	8.70219	10.15256	8.70888	10.16036	8.70928	10.16082
0.7	8.95120	10.03986	8.94392	10.03169	8.95079	10.03940	8.95120	10.03986
0.8	9.19313	9.91890	9.18565	9.91083	9.19271	9.91845	9.19313	9.91890
0.9	9.43505	9.79794	9.42737	9.78997	9.43462	9.79749	9.43505	9.79794
1.0	9.67698	9.67698	9.66910	9.66910	9.67653	9.67653	9.67698	9.67698

Table 2: Error between exact and discrete solutions

r	RK-Fourth Order		RK-Fehlberge		STHWS	
	$y_1(t_i;r)$	$y_2(t_i;r)$	$y_1(t_i;r)$	$y_2(t_i;r)$	$y_1(t_i;r)$	$y_2(t_i;r)$
0.1	0.00611	0.00876	0.00035	0.00027	1E-07	1E-07
0.2	0.0063	0.00845	0.00035	0.00048	2E-07	2E-07
0.3	0.00649	0.00856	0.00036	0.00048	3E-07	3E-07
0.4	0.00669	0.00847	0.00038	0.00047	4E-07	4E-07
0.5	0.00689	0.00837	0.00038	0.00046	5E-07	5E-07
0.6	0.00709	0.00826	0.0004	0.00046	6E-07	6E-07
0.7	0.00728	0.00817	0.00041	0.00045	7E-07	7E-07
0.8	0.00748	0.00807	0.00042	0.00045	8E-07	8E-07
0.9	0.00768	0.00797	0.00043	0.00045	9E-07	9E-07
1.0	0.00788	0.00788	0.00045	0	1E-06	1E-06

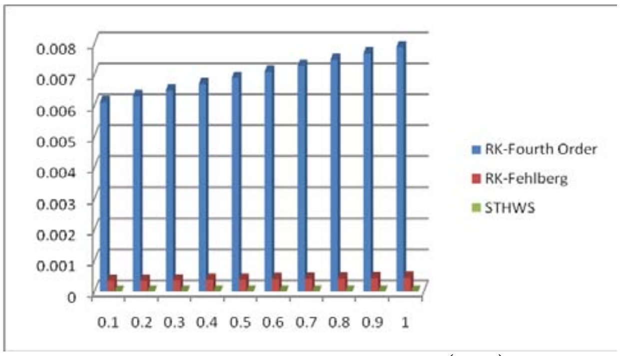

Figure 1. Error estimation of $y_1(t_i;r)$

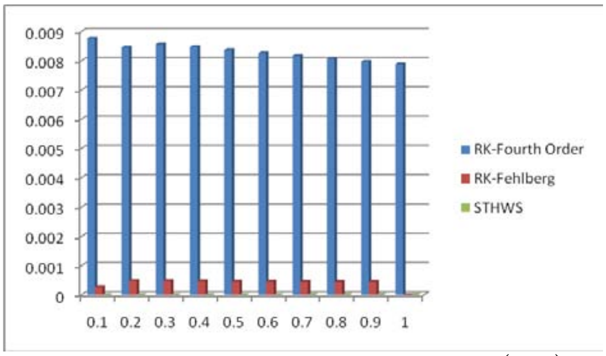

Figure 2. Error estimation of $y_2(t_i;r)$

From the graphical representation is given for the fuzzy hybrid differential equations shows that STHWS method approximate solutions have less error compare to Fourth Order Runge-Kutta method and Runge-Kutta Fehlberg method solutions [ ] in the all the stages.

6. CONCLUSIONS

In this paper, the single-term Haar wavelet series (STHWS) technique has been successfully employed to obtain the approximate analytical solutions of the fuzzy hybrid differential equation. Compare to Fourth Order Runge-Kutta method and Runge-Kutta Fehlberg method, STHW technique gives accurate results from the Table 1, Table 2. Also it is clear that from Table 2, Figure 1 and Figure 2 the STHWS method introduced in Section 4 performs better than Runge-Kutta method of Order Four and Runge-Kutta Fehlberg method.

REFERENCES

[1] Abbasbandy, S., Allah Viranloo, T., Numerical solution of fuzzy differential equation by Talor method, Journal of Computational Methods in Applied mathematics, 2 (2002), 113-124.

[2] Abbasbandy, S., Allah Viranloo, T., Numerical solution of fuzzy differential equation by Runge-Kutta method, Nonlinear Studies, 11 (2004), 117-129.

[3] Bukley, J.J., Feuring, T., Fuzzy Differential Equations, Fuzzy Sets and Systems, 110 (2000), 43-54.

[4] Goetschel, R., Woxman, Elementary fuzzy calculus, Fuzzy Sets and Systems, 18 (1986), 31-43.
[5] Hsiao, C. F., and Wang, W. J., Haar wavelet approach to nonlinear stiff systems, Mathematics and Computation in Simulation, 57 (1998) 347-353.
[6] Jayakumar, T., Kanakarajan, K., Indrakumar, S., Numerical Solution of *Nth*-Order Fuzzy Differential Equation by Runge-Kutta Nystrom Method, International Journal of Mathematical Engineering and Science, 1(5) (2012), 1-13.
[7] Jayakumar, T., Kanagarajan, K., Numerical Solution for Hybrid Fuzzy System by Runge-Kutta Method of Order Five, International Journal of Applied Mathematical Science, 6 (2012), 3591-3606.
[8] Kaleva, O., Fuzzy Differential Equations, Fuzzy Sets and Systems, 24 (1987), 301-317.
[9] Kanagarajan, K., Sambath, M., Numerical Solution for Fuzzy Differential Equations by Third Order Runge-Kutta Method, International Journal of Applied Mathematics and Computation, 2(4) (2010), 1-10.
[10] Lakshmikantham, V., Liu, X.Z., Impulsive Hybrid systems and stability theory, International Journal of Nonlinear Differential Equations, 5 (1999), 9-17.
[11] Lakshmikantham, V., Mohapatra, R.N., Theory of Fuzzy Differential Equations and Inclusions, Taylor and Francis, United Kingdom, 2003.
[12] Ma, M., Friedman, M., Kandel, A., Numerical solutions of fuzzy differential equations, Fuzzy Sets and Systems, 105 (1999), 133-138.
[13] Pederson, S., Sambandham, M., The Runge-Kutta method for Hybrid Fuzzy Differential Equations, Nonlinear Analysis Hybrid Systems, 2 (2008), 626-634.
[14] Sambandham, M., Perturbed Lyapunov-like functions and Hybrid Fuzzy Differential Equations, International Journal of Hybrid Systems, 2 (2002), 23-34.
[15] Sekar, S., Manonmani, A., A study on time-varying singular nonlinear systems via single-term Haar wavelet series, International Review of Pure and Applied Mathematics, 5(2009), 435-441.
[16] Sekar, S., Balaji, G., Analysis of the differential equations on the sphere using single-term Haar wavelet series, International Journal of Computer, Mathematical Sciences and Applications, 4(2010), 387-393.
[17] Sekar, S., Duraisamy, M., A study on CNN based hole-filler template design using single-term Haar wavelet series, International Journal of Logic Based Intelligent Systems, 4(2010), 17-26.
[18] Sekar, S., Jaganathan, K., Analysis of the singular and stiff delay systems using single-term Haar wavelet series, International Review of Pure and Applied Mathematics, 6(2010), 135-142.
[19] Sekar, S., Kumar, R., Numerical investigation of nonlinear volterra-hammerstein integral equations via single-term Haar wavelet series, International Journal of Current Research, 3(2011), 099-103.
[20] Sekar, S., Paramanathan, E., A study on periodic and oscillatory problems using single-term Haar wavelet series, International Journal of Current Research, 2(2011), 097-105.
[21] Sekar, S., Vijayarakavan, M., Analysis of the non-linear singular systems from fluid dynamics using single-term Haar wavelet series, International Journal of Computer, Mathematical Sciences and Applications, 4(2010), 15-29.
[22] Seikkala, S., On the fuzzy initial value problem, Fuzzy Sets and Systems, 24 (1987), 319-330.
[23] Wu, C.X., Ma, M., Embedding problem of fuzzy number space Part I, Fuzzy Sets and Systems, 44 (1991), 33-38.

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