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EFFECT OF HEAT SOURCE ON MHD FLOW OF VISCO-ELASTIC FLUID OF AN ARBITRARY INCLINED PLANE WITH HEAT TRANSFER

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ABSTRACT

The effect of heat source on the flow of visco-elastic conducting fluid of Rivlin – Ericksen type flowing down an inclined plane in presence of a transverse magnetic field with heat transfer is investigated. This type of problem finds application in many technological and engineering fields such as cosmical flight aerodynamics, plasma physics, Glass production and furnace engineering, rocket propulsion systems, space craft re-entry aerothermodynamics. Velocity of the fluid has been presented for various parameters. In this study velocity of fluid increases with the increase in H (Hartmann number), but it decreases with the increase in G_r (Grashof number) and S (Heat source parameter).

Key Words: *Rivlin-Ericksen fluid, Magnetic field, Heat transfer, An arbitrary inclined plane, Heat source.*

INTRODUCTION

Some hydromagnetic problems as investigated by Sengupta and Ghosh [1] and Sengupta and Bhattacharya [2] may be referred. Some fluids, some times exhibits various property of solids and viscous property of liquids are operation. Some problems associated with visco-elastic liquids have been considered by Sengupta and his research collabrators (Sengupta and Ghosh [3], Sengupta and Das [4], Sengupta and Kundu [5], Sengupta and Mukherjee [6]). Recently, Sultana and Ahmmed [8] have analysed MHD flow of visco-elastic fluid of an arbitrary inclined plane. Sharma and Varshney [7] have studied effect of heat transfer on MHD flow of visco-elastic fluid of arbitrary inclined plane.

In the present paper we consider the problem Sharma and Varshney [7] with heat source. The purpose of this study is to investigate the effect of heat source on the flow of visco-elastic conducting fluid of Rivlin – Ericksen type flowing down an inclined plane in presence of a transverse magnetic field with heat transfer.

MATERIALS AND METHODS

Let us consider electrical conducting visco-elastic Rivlin-Ericksen type fluid between two parallel inclined planes, the lower plane is at rest and the upper plane with heat transfer is in motion. A transverse uniform magnetic field B_o has been applied perpendicular to the time varying body force F(t′) taking the fluid initially at rest. The effect due to induced magnetic field and the perturbation of the magnetic field is neglected.

The equation of slow motion of a conducting visco-elastic Rivlin-Ericksen type fluid with heat transfer and heat source in two dimensional forms become

$$\frac{\partial u'}{\partial t'} = g'\sin\alpha + g'\beta'(T'-T_2) - \frac{1}{\rho}\frac{\partial p'}{\partial x'} + \overline{\upsilon}\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho}u' \quad .... \quad (1)$$

$$0 = g'\cos\alpha + \frac{1}{\rho}\frac{\partial p'}{\partial y'} \quad .... \quad (2)$$

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The equation of heat transfer is

$$\frac{\partial T'}{\partial t'} = D_T \frac{\partial^2 T'}{\partial y'^2} + S'(T' - T_2) \qquad \text{....} \qquad (3)$$

and the equation of continuity is

$$\frac{\partial u'}{\partial x'} = 0 \qquad \text{....} \qquad \text{....} \qquad (4)$$

where

$$\overline{\upsilon} = \frac{\mu(1 + \lambda' \frac{\partial}{\partial t'})}{\rho}$$

ρ is the density, g' is the acceleration due to gravity, σ is the electric conductivity, B_0 is the magnetic induction, α is the inclination of the plane to the horizontal and h is the height between two parallel plates, D_T is the thermal conductivity, S' is the coefficient of heat source, β' is the coefficient of temperature expansion. So, $g'\sin\alpha - \frac{1}{\rho}\frac{\partial p'}{\partial x'}$ is a function of t' alone and therefore, we can write

$$g'\sin\alpha - \frac{1}{\rho}\frac{\partial p'}{\partial x'} = -F(t') \qquad (5)$$

The value of F can be found out if the pressure is known at a given point (x_o, 0). Now the equation (1) reduces to

$$\frac{\partial u'}{\partial t'} = -F(t') + g'\beta'(T' - T_2) + \nu(1 + \lambda' \frac{\partial}{\partial t'})\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u' \qquad (6)$$

The boundary conditions are

(i) $u' = 0$, $T' = T_1$, $y' = 0$ for all t'
(ii) $u' = u'(t')$, $T' = T_2$, $y' = h$ for $\forall\, t'$

SOLUTION OF THE PROBLEM

We are now going to put the equation in a non-dimensional form by setting

$$u = \frac{u'}{U_0}, \qquad p = \frac{p'}{\rho U_0^2}$$

$$g = \frac{hg'}{U_0^2} \qquad t = \frac{t' U_o}{h},$$

$$y = \frac{y'}{h}, \qquad x = \frac{x'}{h}$$

$$\lambda = \frac{\lambda' U_0}{h}, \qquad \theta = \frac{T' - T_2}{T_1 - T_2}$$

$H = B_o h\sqrt{\frac{\sigma}{\nu\rho}}$ (Hartmann number)

$G_r = g\beta'(T_1 - T_2)$ (Grashof number)

$R = \frac{hU_0}{\nu}$ (Reynolds number)

$P_r = \frac{\nu}{D_T}$ (Prandtl number)

$S = \frac{h}{U_o}S'$ (Heat source parameter)

Thus the governing equation in non-dimensional form is

$$\frac{\partial u}{\partial t} = -F(t) + \frac{1}{R}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} - \frac{H^2}{R}u + G_r\theta \tag{7}$$

$$\frac{\partial\theta}{\partial t} = \frac{1}{RP_r}\frac{\partial^2\theta}{\partial y^2} + S\theta \quad .... \tag{8}$$

Now, the boundary conditions are
(i) $u = 0$, $\theta = 1$, $y = 0$ for all t
(ii) $u = u(t)$, $\theta = 0$, $y = 1$ for $\forall$ t

Fluid motion due to transient body force

Firstly, we suppose that a transient force $F_0 e^{-\frac{\omega U_0 t}{h}}$ is applied to the fluid and the velocity u and temperature θ of the fluid are considered as $u = \frac{\bar{u}}{U_o}e^{-\frac{\omega U_0 t}{h}}$,

$$\theta = \frac{\bar{\theta}}{U_o}e^{-\frac{\omega U_0 t}{h}}$$

Putting $u = \frac{\bar{u}}{U_o}e^{-\frac{\omega U_0 t}{h}}$, $\theta = \frac{\bar{\theta}}{U_o}e^{-\frac{\omega U_0 t}{h}}$ and $F(t) = F_o e^{-\frac{\omega U_0 t}{h}}$, the equations (7) and (8) take the form

$$\frac{d^2\bar{u}}{dy^2} + a_1^2\bar{u} = \beta - \frac{\beta G_r}{F_o}\bar{\theta} \quad .... \tag{9}$$

$$\frac{d^2\bar{\theta}}{dy^2} + a_2^2\bar{\theta} = 0 \quad .... \tag{10}$$

where

$$a_1^2 = \frac{N}{\left(1 - \frac{\lambda\omega U_o^2}{h}\right)}$$

$$a_2^2 = \frac{RP_r(\omega U_o + hS)}{h}$$

$$\beta = \frac{F_o U_o R}{\left(1 - \frac{\lambda\omega U_o^2}{h}\right)}$$

$$N = \frac{R\omega U_o}{h} - H^2$$

The solution of the equations (9) and (10) are

$$u = \frac{1}{U_o}\left[A\cos a_1 y + B\sin a_1 y + \frac{\beta}{a_1^2} - \frac{G_r\beta}{F_o(a_1^2 - a_2^2)}(\cos a_2 y - \cot a_2 . \sin a_2 y)\right] e^{-\frac{\omega U_0 t}{h}} \tag{11}$$

$$\theta = [\cos a_2 y - \cot a_2 . \sin a_2 y] e^{-\frac{\omega U_0 t}{h}} \quad .... \tag{12}$$

Applying boundary conditions

(i) u = 0, y = 0 for all t

(ii) $u = \frac{F_o U_o}{h} e^{-\frac{\omega U_0 t}{h}}$, y = 1, $\forall t$

We get

$$A = -\frac{\beta}{a_1^2} + \frac{G_r\beta}{F_o(a_1^2 - a_2^2)}$$

and $$B = \frac{1}{\sin a_1}\left[\frac{F_o U_o^2}{h} - \frac{G_r\beta\cos a_1}{F_o(a_1^2 - a_2^2)} + \frac{\beta}{a_1^2}(\cos a_1 - 1)\right]$$

RESULTS AND DISCUSSION

Fluid Velocity Profiles are tabulated in Table-1, 2 & 3 and plotted in Fig.-1, 2 & 3 for $U_o = 0.2$, $F_o = 1$, R = 15, ω = 25, h = 0.5, y = 0.5, λ = 0.05, $P_r = 0.3$ and different values of H (Hartmann number), G_r (Grashof number) and S (Heat source parameter).

It is observed from Fig.-1, 2 & 3 that all velocity Graphs are decreasing sharply up to t = 0.4, then after velocity in each Graphs begins to decrease and tends to zero with the increase in t. It is also observed from Fig.-1, 2 & 3 that velocity increases with the increase in H, but it decreases with the increase in G_r and S.

Temperature Profile is tabulated in Table-4 and plotted in Fig.-4 for $U_o = 0.2$, R = 15, ω = 25, h = 0.5, y = 0.5, $P_r = 0.3$ and different values of S (Heat source parameter). It is observed from Fig.-4 that all Graphs are increasing sharply up to t = 0.4, then after temperature in each Graphs begins to increase and tends to zero with the increase in t. It is also observed from Fig.-4 that temperature decreases with the increase in S.

CONCLUSION

The velocity of fluid and temperature decrease with the increase in S (Heat source parameter).

PARTICULAR CASE

When S (Heat source parameter) is equal to zero, this problem reduces to the problem of Sharma and Varshney [7].

Table-1: Values of velocity at $U_o = 0.2$, $F_o = 1$, $R = 15$, $\omega = 25$, $h = 0.5$, $y = 0.5$, $\lambda = 0.05$, $G_r = 5$, $P_r = 0.3$, $S = 2$ and different values of H.

t	Graph-1 (H = 6)	Graph-2 (H = 8)	Graph-3 (H = 10)
0	0.155000	0.166000	0.410000
0.2	0.020977	0.022466	0.055487
0.4	0.002839	0.003040	0.007509
0.6	0.000384	0.000411	0.001016
0.8	0.000052	0.000056	0.000138
1	0.000007	0.000008	0.000019

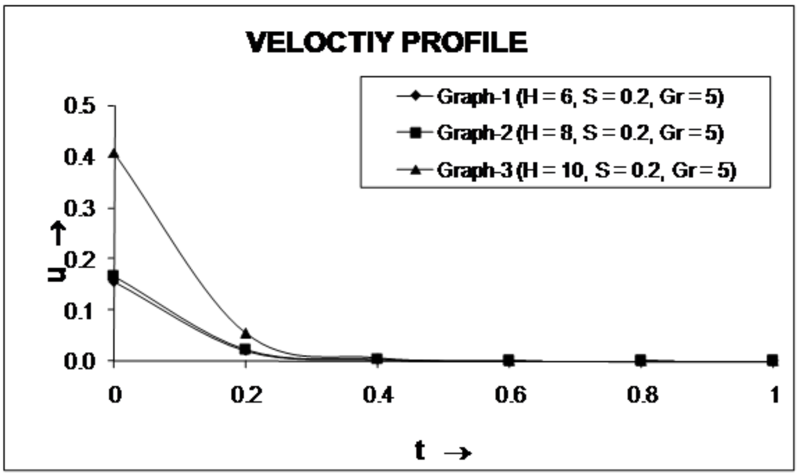

Fig.-1

Table-2: Values of velocity at $U_o = 0.2$, $F_o = 1$, $R = 15$, $\omega = 25$, $h = 0.5$, $y = 0.5$, $\lambda = 0.05$, $H = 6$, $P_r = 0.3$, $S = 0.2$ and different values of G_r.

t	Graph-1 ($G_r = 5$)	Graph-2 ($G_r = 10$)	Graph-3 ($G_r = 15$)
0	0.155000	0.091500	0.028350
0.2	0.020977	0.012383	0.003837
0.4	0.002839	0.001676	0.000519
0.6	0.000384	0.000227	0.000070
0.8	0.000052	0.000031	0.000010
1	0.000007	0.000004	0.000001

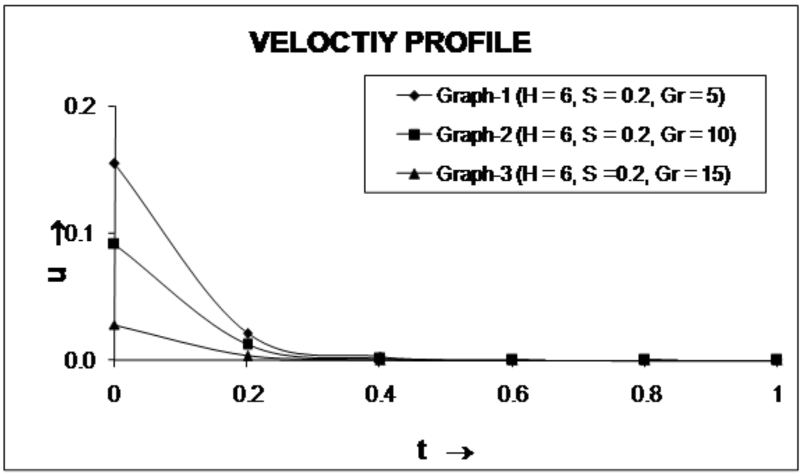

Fig.-2

Table-3: Values of velocity at $U_o = 0.2$, $F_o = 1$, $R = 15$, $\omega = 25$, $h = 0.5$, $y = 0.5$, $\lambda = 0.05$, $H = 6$, $G_r = 5$, $P_r = 0.3$ and different values of S.

t	Graph-1 (S = 0)	Graph-2 (S = 0.2)	Graph-3 (S = 0.4)
0	0.165000	0.155000	0.067200
0.2	0.022330	0.020977	0.009095
0.4	0.003022	0.002839	0.001231
0.6	0.000409	0.000384	0.000167
0.8	0.000055	0.000052	0.000023
1	0.000007	0.000007	0.000003

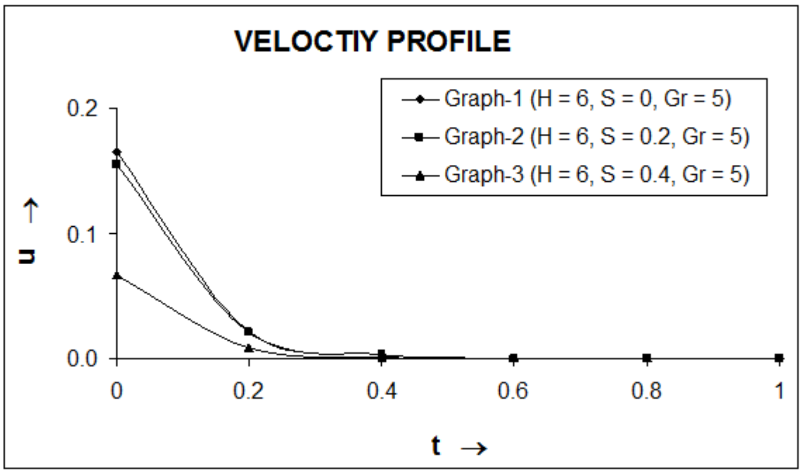

Fig.-3

Table-4: Values of temperature at $U_o = 0.2$, $R = 15$, $\omega = 25$, $h = 0.5$, $y = 0.5$, $P_r = 0.3$ and different values of S.

t	Graph-1 (S = 0)	Graph-2 (S = 0.2)	Graph-3 (S = 0.4)
0	-0.572000	-0.870400	-3.101000
0.2	-0.077412	-0.117796	-0.419675
0.4	-0.010477	-0.015942	-0.056797
0.6	-0.001418	-0.002158	-0.007687
0.8	-0.000192	-0.000292	-0.001040
1	-0.000026	-0.000040	-0.000141

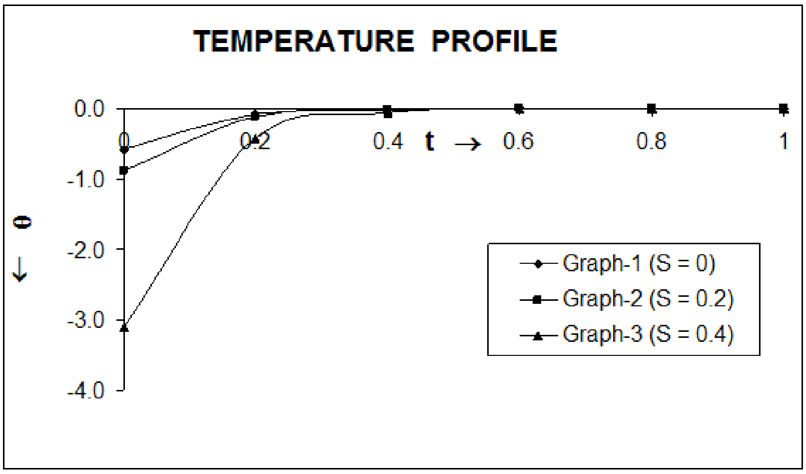

Fig.-4

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