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THREE SUMMATION FORMULAE USING CONTIGUOUS RELATION AND RELATED TO HYPERGEOMETRIC FUNCTION

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ABSTRACT

The main aim of this paper is to evaluate two summation formulae involving Contiguous Relation associated with Recurrence relation and Hypergeometric function.

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A. INTRODUCTION:

Generalized Gaussian Hypergeometric function of one variable

$${}_AF_B(a_1,a_2,\ldots,a_A;b_1,b_2,\ldots b_B;z) = \sum_{k=0}^{\infty} \frac{(a_1)_k(a_2)_k\ldots(a_A)_k\, z^k}{(b_1)_k(b_2)_k\ldots(b_B)_k\, k!} \qquad (1)$$

where the parameters b_1 , b_2 , ….,b_B are neither zero nor negative integers and A , B are non negative integers.

Contiguous Relations

[Andrews p.363(9.16) , E.D. p.51(10), H.T.F.I. p.103(32)]

$$(a-b)\, {}_2F_1(a, b ; c; z) = a\, {}_2F_1(a+1,b; c; z) - b\, {}_2F_1(a,b+1; c; z) \qquad (2)$$

[Abramowitz p.558 (15.2.19)]

$$(a-b)(1-z)\, {}_2F_1(a, b ; c; z) = (c-b)\, {}_2F_1(a,b-1; c; z) + (a-c)\, {}_2F_1(a-1,b; c; z) \qquad (3)$$

A New Summation Formula

[Ref. [2] p.337 (10)]

$${}_2F_1\left(a, b ; \frac{a+b-1}{2}; \frac{1}{2}\right) = 2^{b-1} \frac{\Gamma(\frac{a+b-1}{2})}{\Gamma(b)} \left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} \left\{ \frac{b+a-1}{a-1} \right\} + 2\, \frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})} \right] \qquad (4)$$

B. MAIN SUMMATION FORMULAE:

For all the formulae $a \neq b$

For $a<1$ and $a>8$

$${}_2F_1\left(a, b ; \frac{a+b-8}{2}; \frac{1}{2}\right) = 2^{b-1} \frac{\Gamma(\frac{a+b-8}{2})}{(a-b)\Gamma(b)} \left[\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-7}{2})} \left\{ \frac{384a-624a^2+396a^3-60a^4+9a^5-384b+224ab+348a^2b}{(a-7)(a-5)(a-3)(a-1)} + \right.\right.$$

$$\left. + \frac{-192a^3b+75a^4b+400b^2-604ab^2+72a^2b^2+42a^3b^2-140b^3+160ab^3-90a^2b^3+20b^4-35ab^4-b^5}{(a-7)(a-5)(a-3)(a-1)} \right\} +$$

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$$+\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-8}{2})}\{\frac{384a-400a^2+140a^3-20a^4+a^5-334b-224ab+604a^2b-160a^3b+35a^4b+624b^2-348ab^2-72a^2b^2}{(a-8)(a-6)(a-4)(a-2)}+$$

$$+\frac{90a^3b^2-396b^3+192ab^3-42a^2b^3+60b^4-75ab^4-9b^5}{(a-8)(a-6)(a-4)(a-2)}\}] \quad (5)$$

For a<1 and a>9

$$_2F_1(a, b ; \frac{a+b-9}{2}; \frac{1}{2}) = 2^{b-1}\frac{\Gamma(\frac{a+b-9}{2})}{(a-b)\Gamma(b)}[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-9}{2})}\{\frac{-945a+1689a^2-950a^3+230a^4-25a^5+a^6+945b-1870a^2b}{(a-9)(a-7)(a-5)(a-3)(a-1)}$$

$$+\frac{1540a^3b-275a^4b+44a^5b-1689b^2+1870ab^2-330a^3b^2+165a^4b^2+950b^3-1540ab^3+330a^2b^3-230b^4+275ab^4}{(a-9)(a-7)(a-5)(a-3)(a-1)}$$

$$+\frac{-165a^2b^4+25b^5-44ab^5-b^6}{(a-9)(a-7)(a-5)(a-3)(a-1)}\}+\frac{\Gamma(\frac{b-1}{2})}{\Gamma(\frac{a-8}{2})}\{\frac{1578a-1456a^2+716a^3-80a^4+10a^5-1578b+1276a^2b-352a^3b}{(a-8)(a-6)(a-4)(a-2)}$$

$$+\frac{110a^4b+1456b^2-1276ab^2+132a^3b^2-716b^3+352ab^3-132a^2b^3+80ab^4-110ab^4-10b^5}{(a-8)(a-6)(a-4)(a-2)}\}] \quad (6)$$

For a<1 and a>10

$$_2F_1(a, b ; \frac{a+b-10}{2}; \frac{1}{2}) = 2^{b-1}\frac{\Gamma(\frac{a+b-10}{2})}{(a-b)\Gamma(b)}[\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-9}{2})}\{\frac{-3840a+7392a^2-3784a^3+1276a^4-110a^5+11a^6+3840b}{(a-9)(a-7)(a-5)(a-3)(a-1)}$$

$$+\frac{-3008ab-3432a^2b+3872a^3b-682a^4b+154a^5b-4384b^2+5416ab^2-1672a^2b^2-396a^3b^2+297a^4b^2+1800b^3}{(a-9)(a-7)(a-5)(a-3)(a-1)}$$

$$+\frac{-3136ab^3+748a^2b^3-132a^3b^3-340b^4+410ab^4-275a^2b^4+30b^5-54ab^5-b^6}{(a-9)(a-7)(a-5)(a-3)(a-1)}\}+$$

$$\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-10}{2})}\{\frac{-3840a+4384a^2-1800a^3+340a^4-30a^5+a^6+3840b+3008ab-5416a^2b+3136a^3b-410a^4b+54a^5b}{(a-10)(a-8)(a-6)(a-4)(a-2)}$$

$$+\frac{-7392b^2+3432ab^2+1672a^2b^2-748a^3b^2+275a^4b^2+3784b^3-3872ab^3+396a^2b^3+132a^3b^3-1276b^4+682ab^4}{(a-10)(a-8)(a-6)(a-4)(a-2)}$$

$$+\frac{-297a^2b^4+110b^5-154ab^5-11b^6}{(a-10)(a-8)(a-6)(a-4)(a-2)}\}] \quad (7)$$

C. DERIVATIONS OF THE SUMMATION FORMULAE:

Derivation of (5): Substituting $c = \frac{a+b-8}{2}$ and $z = \frac{1}{2}$ in equation (3) , we get

$$(a-b)\ _2F_1(a, b; \frac{a+b-8}{2}; \frac{1}{2}) = (a-b-8)\ _2F_1(a , b-1 ; \frac{a+b-8}{2} ; \frac{1}{2}) + (a-b+8)\ _2F_1(a-1 , b ; \frac{a+b-8}{2} ; \frac{1}{2})$$

Now with the help of the derived result from equation (4) , we get

$$\text{L.H.S} = (a-b-8)\ 2^{b-2}\frac{\Gamma(\frac{a+b-8}{2})}{(a-b+1)\Gamma(b-1)}[\frac{\Gamma(\frac{b-1}{2})}{\Gamma(\frac{a-7}{2})}\{\frac{384-240a-356a^2+224a^3-43a^4+a^5-784b+340ab}{(a-7)(a-5)(a-3)(a-1)}$$

$$\frac{96a^2b-180a^3b+27a^4b+540b^2-672ab^2+126a^2b^2+42a^3b^2-160b^3+204ab^3-42a^2b^3+21b^4-27ab^4-b^5}{(a-7)(a-5)(a-3)(a-1)}\}$$

$$+\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-6}{2})}\{\frac{-384-32a+256a^2-88a^3+8a^4+672b-288ab-72a^2b+48a^3b-352b+216ab^2+72b^3-48ab^3-8b^4}{(a-6)(a-4)(a-2)}\}]$$

$$+ \ (a-b+8)\, 2^{b-1} \frac{\Gamma(\frac{a+b-8}{2})}{(a-b-1)\Gamma(b)} \Big[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-8}{2})} \Big\{ \frac{-384+784a-540a^2+160a^3-21a^4+a^5+240b-340ab+672a^2b}{(a-8)(a-6)(a-4)(a-2)}$$

$$+ \frac{-204a^3b+27a^4b+356b^2-96ab^2-126a^2b^2+42a^3b^2-224b^3+180ab^3-42a^2b^3+43b^4-27ab^4-b^5}{(a-8)(a-6)(a-4)(a-2)} \Big\}$$

$$+ \frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-7}{2})} \Big\{ \frac{384-672a+352a^2-72a^3+8a^4+32b+288ab-216a^2b+43a^3b-256b^2+72ab^2+88b^3-48ab^3-8b^4}{(a-7)(a-5)(a-3)} \Big\} \Big]$$

On simplification , we get the formula (5)

Similarly, we can prove the other formulae.

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