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HYDROMAGNTIC STABILITY OF SELF GRAVITATIONAL OSCILLATING STREAMING FLUID JET PERVADED BY AZIMUTHAL VARYING MAGNETIC FIELD

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ABSTRACT

The hydromagnetic stability of self gravitational oscillating streaming fluid jet has been discussed. The basic equations are solved in the perturbed and unperturbed states. A total second order integrodiffere--ntial equation in the amplitude of perturbation has been obtained. The oscillating streaming of the fluid has stabilizing effect in all modes of perturbation. The self gravitating force is only destabilizing in axisymmetric domain 0<x<1.0667. mWhile it is stabilizing in the other axisymmetric domains $1.0667 \le x < \infty$ *. The transverse magnetic field is purely destabilizing in the axisymmetric mode. While it is stabilizing or destabilizing in all non-axisymmetric according to restrictions. The axial magnetic field interior the fluid has strong stabilizing effect and this effect is independent of the kind of perturbation. Under some adjustable values of the transverse parameter, the instability of the model could be suppre -ssed and then stability sets in. Such kind of studies has astrophysical applications e.g. stability of spiral arm of galaxy, sunspots ……. etc.*

1. INTRODUCTION:

The responsible about the self gravitating instability of a fluid cylinder is due to Chandrasekhar and Fermi (1953). They are the first to discuss such kind of stability which are interesting for astrophysical applications e.g. the stability of spiral arm of galaxy, sunspot ….etc. In fact they (1953) used a technique valid only in axisysmmetric mode of perturbation, in which a solenoidal vector could be represented by poloidal and toroidal quantities. Later, the winner Nobil prize (1986), Chandrasekhar (1981) studied the stability of such model for axisymmetric and non-axisymmetric perturbation modes by utilizing the normal mode analysis. Also he (1981) made several extensions for stability analysis of different models under the action of couple of external forces. Chandrasekhar (1981) examined the effect of the magnetic field on the self gravitating force also he identified the influence of the magnetic force on the capillary instability of fluid cylinder. These studies have been carried out upon using technique valid for axisymmeric perturbation as the fluid at rest in the unperturbed state and pervaded by constant axial magnetic field. For more details and studies of MHD instability of different modes whether the fluids at rest or uniformly streaming in the unperturbed state we may refer to the published works of Radwan et.al.(1998), (2002),(2003) ,(2007), (2008) and (2009). Hasan (2010) discussed the stability of oscillating streaming fluid cylinder subject to combined effect of the capillary, selfgravitating and electrodynamic forces for all axisymmetric and non-axisymmetric perturbation modes.
Here we investigate the hydromagntic stability of self gravitational oscillating streaming fluid jet pervaded by azimuthal varying magnetic field for all axisymmetric and non-axisymmetric modes of perturbation. Such kind of study has several applications in wide domains of science.

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2. FORMULATION OF THE PROBLEM:

Consider a fluid jet of uniform cross–section of (radius R_o) density .The fluid is assumed to be incompressible, non-viscous and non-dissipative of permeability coefficient .The fluid jet is surrounded by a tenuous medium of negligible motion. Interior the fluid there is a uniform axial magnetic field

$$\underline{H}_o^{(in)} = (0,0,H_o) \tag{1}$$

While the surrounding region exterior the fluid is pervaded by the transverse varying magnetic field

$$\underline{H}_o^{(ex)} = (0,\frac{\beta H_o R_o}{r},0) \tag{2}$$

where H_o is the intensity of the magnetic field in the fluid and β is parameter. The fluid is assumed to be streaming with the oscillating velocity.

$$\underline{u}_o = (0,0,U\cos\Omega t) \tag{3}$$

In the unperturbed state where U is (the amplitude of the velocity) constant and Ω is the oscillation frequency of the velocity. The components of $\underline{H}_o^{(in)}, H_o^{(ex)}$ and $\underline{u}_o$ are considered along the cylindrical coordinates (r,φ,z) with z-axis coinciding with the axis of the cylinder. The fluid is acted by the combination of the inertia; self gravitating, magnetodynamic and pressure gradient forces. The required basic equations for studying the stability of the problem under consideration are the pure hydrodynamic equations, self gravitating equations and Maxwell's equations concerning the electromagnetism theory. These equations are given as follows interior the fluid

$$\rho\left(\frac{\partial\underline{u}}{\partial t}+(\underline{u}.\nabla)\underline{u}\right)=\rho\nabla V+\mu(\nabla\wedge\underline{H})\wedge\underline{H}-\nabla p \tag{4}$$

$$\nabla\underline{u}=0\,\nabla^2 V=-4\pi G\rho, \qquad (5), (6)$$

$$\frac{\partial\underline{H}}{\partial t}=\nabla\wedge(\underline{u}\wedge\underline{H}) \quad , \quad \nabla.\underline{H} \ = 0 \qquad (7), (8)$$

exterior the fluid cylinder in the tenuous medium

$$\nabla\wedge\underline{H}^{(ex)}=0 \qquad ,\nabla.\underline{H}^{(ex)}=0 \qquad (9), (10)$$

$$\nabla^2 V^{(ex)}=0 \tag{11}$$

where $\underline{u}$ and p are the fluid velocity vector and kinetic pressure, $\underline{H}$ the magnetic field intensity, V and G are the self gravitating potential and constant.

Equation (4), could be rewritten in the form

$$\rho\left(\frac{\partial\underline{u}}{\partial t}+(\underline{u}.\nabla)\underline{u}\right)+\mu(\nabla\wedge\underline{H})\wedge\underline{H}=-\nabla\Pi \tag{12}$$

with

$$\Pi = p+\frac{\mu}{2}(\underline{H}.\underline{H})-\rho V \tag{13}$$

where $p+\frac{\mu}{2}(\underline{H}.\underline{H})=\Pi+\rho V$ represented the total magneto dynamic pressure which is the sum of kinetic fluid pressure and magnetic pressure.

The basic equations (4)-(11) are solved, with taking into account equations (1)-(3), in the unperturbed state and upon applying the boundary conditions at r =R$_o$, we get

$$\Pi_o = p_o + \pi G\rho^2 r^2 + (\mu/2)H_o^2 = const \tag{14}$$

$$V_o^{(in)} = -\pi G\rho r^2 \tag{15}$$

$$V_o^{(ex)} = 2\pi G\rho R_o^2 \ln(R_o/r) - \pi G\rho R_o^2 \tag{16}$$

Upon applying the balance of the pressure, we finally obtain

$$p_o = \pi G\rho^2 (R_o^2 - r^2) + (\mu/2)(\beta^2 - 1)H_o^2 \tag{17}$$

As $r = R_o$, we see that the self gravitational forces have no contribution to p_o, also as we see that the magnetic forces have no contribution to p_o, uiz.,

$$p_o = \pi G\rho^2 (R_o^2 - r^2) \qquad \text{as } \beta = 1 \text{ or } H_o = 0 \tag{18}$$

$$p_o = (\mu/2)(\beta^2 - 1)H_o^2 \qquad \text{as G} = 0 \tag{19}$$

3. FORMULATION OF THE PROBLEM:

For small departures from the unperturbed state, we consider a small wave disturbance across the boundary surface of the fluid cylinder propagated in the positive z-direction. At time t, the surface deflection along the fluid cylinder interface is assumed to be of the form

$$r = R_o + \hat{\eta}(\varphi, z, t) \tag{20}$$

with

$$\hat{\eta} = \eta(t)\exp(i(kz + m\varphi)) \tag{21}$$

Here $\hat{\eta}(\varphi, z, t)$ is the elevation of the surface wave measured from the unperturbed position, $\eta(t)$ is (a function of time) the amplitude of the deflection, *k* the axial wave number and *m* is the azimuthal wave number. Based on the perturbation (20) and (21) along the cylindrical fluid surface, every physical quantity $Q(r,\varphi,z;t)$, may be written as

$$Q(r,\varphi,z,t) = Q_o(r) + \eta(t)Q_1(r,\varphi,z) + \ldots\ldots \tag{22}$$

Here $Q(r,\varphi,z;t)$ stands for *p*, $\underline{u}$, $\mathrm{V}^{(in)}$, $\mathrm{V}^{(ex)}$, $\underline{H}^{(in)}$ and $\underline{H}^{(ex)}$ with subscripts 1 is pertaining the perturbed quantities while those with index o are being the unperturbed quantities.
By an appeal to the expansion (22), based on the perturbation technique, the linearized perturbation equations are given from:

Interior the fluid

$$\rho(\frac{\partial \underline{u}_1}{\partial t} + (\underline{u}_o.\nabla)\underline{u}_1) + \mu(\underline{H}_o.\nabla)\underline{H}_1 = -\nabla\Pi_1 \tag{23}$$

$$\Pi_1 = p_1 - \rho V_1 + \mu(\underline{H}_o.\underline{H}_1) \tag{24}$$

$$\nabla.\underline{u}_1 = 0, \quad \nabla^2 V_1^{(in)} = 0 \qquad (25), (26)$$

$$\frac{\partial \underline{H}}{\partial t} = (\underline{u}_o.\nabla)\underline{H}_1 + (\underline{H}_o.\nabla)\underline{u}_1, \qquad \nabla.H_1^{(in)} = 0 \qquad (27), (28)$$

Exterior the fluid

$$\nabla \wedge \underline{H}_1^{(ex)} = 0, \;\; \nabla.H_1^{(ex)} = 0, \qquad (29)\text{-}(30)$$

$$\nabla^2 V_1^{(ex)} = 0 \tag{31}$$

In order to solve this system of partial differential equations, we follow the following steps.
Acting by divergence operator an equation (23), yields

$$\nabla^2 \Pi_1 = 0 \tag{32}$$

Equation (29) means that could be derived from a scalar function

$$\underline{H}_1^{(ex)} = \nabla \psi_1^{(ex)} \tag{33}$$

Combining equations (30) and (33), we get

$$\nabla^2 \psi_1^{(ex)} = 0 \tag{34}$$

Since the fluid is non-dissipative and irrotational, so we could write

$$\underline{u}_1 = \nabla \phi_1 \tag{35}$$

Substituting from (35) into equation (25), we have

$$\nabla^2 \phi_1 = 0 \tag{36}$$

Now, substituting from equation (1) and (3) and using the dependence of space (equation (21)) into equation (27), we get

$$\underline{H}_1 = \frac{ikH_o}{\frac{d\eta}{dt} + ikU\eta\cos\Omega t}\underline{u}_1 \tag{37}$$

The solution of the partial differential equations (23)-(31) could be determined upon identifying the solution of Laplace's equations (26), (31), (34) and (36) in the cylindrical coordinates (r, φ, z). Based on the normal mode analysis, the functions Π_1 , $\psi_1^{(ex)}$ and ϕ_1 may be expanded as

$$\Pi_1(r,\varphi,z;t) = \Pi_1(r)\eta(t)\exp(i(kz + m\varphi)) \tag{38}$$

$$\psi_1^{(ex)}(r,\varphi,z;t) = \psi_1^{(ex)}(r)\eta(t)\exp(i(kz+m\varphi)) \tag{39}$$

$$\phi_1(r,\varphi,z;t) = \phi_1(r)\eta(t)\exp(i(kz+m\varphi)) \tag{40}$$

$$V_1^{(in)}(r,\varphi,z;t) = V_1^{(in)}(r)\eta(t)\exp(i(kz+m\varphi)) \tag{41}$$

$$V_1^{(ex)}(r,\varphi,z;t) = V_1^{(ex)}(r)\eta(t)\exp(i(kz+m\varphi)) \tag{42}$$

Substituting from (38)-(42) into equations (26), (31), (32), (34) and (36), we get Bessel equations in $\Pi_1(r)$, $\psi_1^{(ex)}(r)$, $\phi_1(r)$, $V_1^{(in)}$ and $V_1^{(ex)}$ .Therfore, the non-singular solutions of equations (32), (34) and (36), are given as

$$\Pi_1(r,\varphi,z;t) = c_1 I_m(kr)\eta(t)\exp(i(kz+m\varphi)) \tag{43}$$

$$\psi_1^{(ex)}(r,\varphi,z;t) = c_2 K_m(kr)\eta(t)\exp(i(kz+m\varphi)) \tag{44}$$

$$\phi_1(r,\varphi,z;t) = c_3 I_m(kr)\eta(t)\exp(i(kz+m\varphi)) \tag{45}$$

$$V_1^{(in)}(r,\varphi,z;t) = c_4 I_m(kr)\eta(t)\exp(i(kz+m\varphi)) \tag{46}$$

$$V_1^{(ex)}(r,\varphi,z;t) = c_5 K_m(kr)\eta(t)\exp(i(kz+m\varphi)) \tag{47}$$

Where c_1, c_2, c_3, c_4 and c_5 are constants while $I_m(kr)$ and $K_m(kr)$ are the modified Bessel functions of the first and second kind, respectively, of order *m*. By means of solution (41)-(43) we could determine $\underline{u}_1$, $\underline{H}_1^{(in)}$, $\underline{H}_1^{(ex)}$, $V_1^{(in)}$ and $V_1^{(ex)}$.

4. BOUNDARY CONDITIONS:

The solution of the basic equations (4)-(11) in the unperturbed state given by equations (1)-(3) and (14)-(17) and in the perturbed state given by (33), (35), (37) and (43)-(47) must satisfy certain boundary conditions across the fluid interface (20) at r = R_o .Under the present circumstances these appropriate boundary conditions are given as follows:

(I) Self gravitating conditions:

(i) The self gravitating potential $V(=V_o+\eta V_1+..........)$ must be continuous across the fluid interface (20) at r = R_o . This condition reads

$$V_1^{(in)} + \hat{\eta}\frac{\partial V_o^{(in)}}{\partial r} = V_1^{(ex)} + \hat{\eta}\frac{\partial V_o^{(ex)}}{\partial r}, \text{ at } r = R_o \tag{48}$$

(ii) The derivative of the self gravitating potential must be continuous across the interface (20) at r = R_o . Mathematically, this condition reads

$$\frac{\partial V_1^{(in)}}{\partial r} + \hat{\eta}\frac{\partial^2 V_o^{(in)}}{\partial r^2} = \frac{\partial V_1^{(ex)}}{\partial r} + \hat{\eta}\frac{\partial^2 V_o^{(ex)}}{\partial r^2}, \quad r = R_o \tag{49}$$

Substituting from equations (15), (16), (21), (46) and (47) into the conditions (48) and (49), we get

$$c_4 = 4\pi G\rho(xK_m(x)/k \tag{50}$$

and

$$c_5 = 4\pi G\rho(xI_m(x)/k \tag{51}$$

Where use has been made of the Wronskian relation (cf. Abramwitz and stegan (1970))

$$I_m(x)K_m^{'}(x) - I_m^{'}(x)K_m = -x^{-1} \tag{52}$$

and where x = (kR_o) is the dimensionless longitudinal wave number.

(II) The magnetodynamic condition:

This condition states that "the normal component of the magnetic field must be continuous across the cylindrical fluid jet interface (20) at r = R_o. This condition is given as

$$\underline{n}_o.\underline{H}_1^{(in)} + \underline{n}_1.\underline{H}_o^{(in)} = \underline{n}_o.\underline{H}_1^{(ex)} + \underline{n}_1.\underline{H}_o^{(ex)} \tag{53}$$

with

$$\underline{n}_o = (1,0,0), \quad n_1 = (0,-\frac{im}{R_o},-ik)\eta \tag{54}$$

Substituting from equations (1), (2), (21), (23), (33), (37), (43), (44) and (54) into the condition (53), we get

$$c_2 = im\beta H_o/(xK_m^{'}(x)) \tag{55}$$

(III) Kinematic condition:

"The normal component of the velocity vector of the fluid must be compatible with the velocity of the perturbed boundary fluid interface (20) at r = R_o". This condition reads

$$\frac{df(r,\varphi,z;t)}{dt} = 0, \quad \text{or} \quad \frac{\partial f}{\partial t} + (\underline{u}.\nabla)f = 0 \tag{56}$$

with

$$f(r,\varphi,z;t) = r - R_o - \eta(t)\exp(i(kz + m\varphi)) \tag{57}$$

from which

$$u_{1r} = \frac{d\eta}{dt} + ikU\eta\cos\Omega t \tag{58}$$

Combining equations (58) and

$$u_{1r} = \frac{\partial\phi_1^{(in)}}{\partial r} = c_3 kI_m^{'}(kr)\exp(i(kz + m\varphi)) \tag{59}$$

we get

$$c_3 = \frac{1}{k\eta I_m^{'}(x)}(\frac{d\eta}{dt} + ikU\eta\cos\Omega t) \tag{60}$$

We may obtain upon using equations (23), (37), (38) and (59), the relation

$$\rho\left(\frac{\partial u_{1r}}{\partial t}+U\cos\Omega t\frac{\partial u_{1r}}{\partial t}\right)+\frac{\mu k^2H_o^2}{(\frac{d\eta}{dt}+ikU\cos\Omega t)}u_{1r}=-\frac{\partial\Pi_1}{\partial r} \tag{61}$$

Equation (61) gives

$$\rho\frac{d^2\eta}{dt^2}+2ikU\rho\cos\Omega t\frac{d\eta}{dt}-(ikU\Omega\sin\Omega t)\eta-(k^2U^2\rho\cos\Omega t)\eta$$
$$+\mu k^2H_o^2=-c_1k\eta I_m^{'}(kr) \tag{62}$$

from which

$$c_1=\frac{-\rho}{k\eta I_m^{'}(x)}\left[\frac{d^2\eta}{dt^2}+2ikU\cos\Omega t\frac{d\eta}{dt}+\frac{\mu H_o^2k^2}{\rho}\eta-(ikU\sin\Omega t+k^2U^2\cos^2\Omega t)\eta\right] \tag{63}$$

5. THE BALANCES OF PRESSURE:

As it is obvious from the foregoing calculation we have explicit expressions for every variable of the problem. For the stability theory we have to go one more step to obtain the relation from which we may identify the instability domains of the present model. We have to apply the stress condition. This condition states "the normal component of the stresses in the interior and exterior the fluid cylinder must be continuous across the cylindrical fluid interface (20) at r = R_o. This condition reads

$$\Pi_1^{(in)}+\hat{\eta}\frac{\partial\Pi_o^{(in)}}{\partial r}=\mu(\underline{H}_o.\underline{H}_1)+(\mu/2)\hat{\eta}\frac{\partial}{\partial r}(2\underline{H}_o^{(ex)}.\underline{H}_o^{(ex)}) \tag{64}$$

Substituting for Π_1, Π_o, $\hat{\eta}$, $\underline{H}_o^{(ex)}$ and $\underline{H}_1^{(ex)}$ in the condition (64), we finally obtain

$$\frac{d^2\eta}{dt^2}+2ikU\cos\Omega t\frac{d\eta}{dt}-(ikU\Omega\sin\Omega t)\eta-(k^2U^2\rho\cos\Omega t)\eta$$
$$=4\pi G\rho(xI_m^{'}(x)/I_m(x)(I_m(x)K_m(x)-1/2)\eta \tag{65}$$
$$+\frac{\mu H_o^2}{\rho R_o^2}\left[-x^2+m^2\beta^2\frac{I_m^{'}(x)K_m(x)}{I_m(x)K_m^{'}(x)}+\beta^2(\frac{xI_m^{'}(x)}{I_m(x)}\right]\eta$$

6. DISCUSSIONS:

Equation (65) is a total second order integro-differential equation in the deflection amplitude η of the perturbation. It relates η with the modified Bessel functions of the first and second kind of order *m*, the wave numbers *x* and *m*, the oscillation frequency of the streaming , the uniform streaming velocity *U*, the transverse varying magnetic field parameter β and with the other parameter ρ, G, μ, H_o and R_o of the problem. Equation (65) contains the quantity $(\mu H_o^2/(\rho R_o^2))^{1/2}$ as well as $(4\pi G\rho)^{1/2}$ as a unit of time, which each of them is benefit in using them to write the dispersion relation in a dimensionless form. Assuming that $\eta\approx\exp(\sigma t)$ where σ is the growth rate, equation (65) gives the general dispersion relation

$$\sigma^2+(2ikU\cos\Omega t)\sigma-(ikU\Omega\sin\Omega t+k^2U^2\rho\cos\Omega t)$$
$$=4\pi G\rho\left(\frac{I_m^{'}(x)}{I_m(x)}\right)(I_m(x)K_m(x)-1/2)+\frac{\mu H_o^2}{\rho R_o^2}\left[-x^2+m^2\beta^2\frac{I_m^{'}(x)K_m(x)}{I_m(x)K_m^{'}(x)}+\beta^2(\frac{xI_m^{'}(x)}{I_m(x)}\right]$$

If we assume that U=0 in the relation (66), we get

$$\sigma^2 = 4\pi G\rho\left(\frac{I'_m(x)}{I_m(x)}\right)(I_m(x)K_m(x)-1/2) \tag{67}$$
$$+\frac{\mu H_o^2}{\rho R_o^2}\left[-x^2+m^2\beta^2\frac{I'_m(x)K_m(x)}{I_m(x)K'_m(x)}+\beta^2(\frac{xI'_m(x)}{I_m(x)}\right]$$

The discussion of equation (67) shows that the model at hand is stable if and only if

$$\left(\frac{H_o}{H_G}\right)^2 \geq \frac{xI'_m(x)K'_m(x)(I_m(x)K_m(x)-\frac{1}{2})}{x^2 I_m(x)K'_m(x)-\beta^2 I'_m(x)(xK'_m(x)+m^2K_m(x)} \tag{68}$$

with

$$H_G = 2\rho R_o\sqrt{\pi G/\mu} \tag{69}$$

has a unit of magnetic field.

If we assume $U\neq 0$ while $\Omega = 0$, the dispersion relation (66) yields

$$(\sigma+ikU)^2 = 4\pi G\rho(xI'_m(x)/I_m(x)(I_m(x)K_m(x)-1/2) \tag{70}$$
$$+\frac{\mu H_o^2}{\rho R_o^2}\left[-x^2+m^2\beta^2\frac{I'_m(x)K_m(x)}{I_m(x)K'_m(x)}+\beta^2(\frac{xI'_m(x)}{I_m(x)}\right]$$

The discussion of equation (70) shows that m uniform streaming has strong destabilizing effect and this effect does not depend on the kind (axisymmetric or non-axisymmetric) of perturbations. See also Sengar (1981) for other model.

If we neglect the secular term in the general equation (65) and transform equation (65) to Mathieu type equation (cf. Mclachlan (1947) and Kelly (1965)), we may prove that the oscillating streaming of the fluid has stabilizing tendency.

As U=0, $\Omega = 0$, $H_o = 0$ and $m = 0$, the general dispersion (66) yields

$$\sigma^2 = 4\pi G\rho(xI'_1(x)/I_0(x)(I_0(x)K_0(x)-1/2), \;\; I_o(x) = I_1(x) \tag{71}$$

As U=0, $\Omega = 0$, $H_o = 0$ and $m \geq 0$, the general dispersion (66) yields

$$\sigma^2 = 4\pi G\rho\left(\frac{I'_m(x)}{I_m(x)}\right)(I_m(x)K_m(x)-1/2), \tag{72}$$

The relation (71) has been derived for first time by Chandrasekhar and Fermi (1953) upon using technique valid only to axisymmetric mode (m=0) of perturbation in which a solenoidal vector could be represented as toroidal and poloidal quantites. The discussion of equations (71) and (72) for determining the stable and unstable domains it is found that

$$\left(\frac{\sigma^2}{4\pi G\rho}\right) > 0 \quad \text{in the domain} \quad 0 < x < 1.0667 \ \text{ for } m = 0 \text{ mode} \tag{73}$$

$$\left(\frac{\sigma^2}{4\pi G\rho}\right) \leq 0 \ \text{in the domain} \ (1.0667 < x < \infty \ \text{ as } m = 0) \text{ and} \tag{74}$$

$$0 < x < \infty \ \text{ as } m \geq 1$$

This means that the fluid cylinder is self gravitating unstable in the axisymmetric small domain $0 < x < 1.0667$ .While it is self gravitating stable in all non-axisymmetric modes $m \geq 1$ and in the axisymmetric domains $1.0667 < x < \infty$.

As U=0, $\Omega = 0$ and G=0, the relation (66) degenerates to

$$\sigma^2 = \frac{\mu H_o^2}{\rho R_o^2}\left[-x^2 + m^2\beta^2 \frac{I_m^{'}(x) K_m(x)}{I_m(x) K_m^{'}(x)} + \beta^2 \left(\frac{x I_m^{'}(x)}{I_m(x)}\right) \right] \tag{75}$$

The influence of the magnetic field pervaded interior the fluid cylinder is represented by the term $\frac{\mu H_o^2}{\rho R_o^2}(-x^2)$. It has strong stabilizing influence and this influence is independent on the (axisymmetric m=0 and non-axisymmetric perturbation modes. This influence of the transverse varying magnetic field pervaded in the tenuous surrounding medium of the fluid cylinder is represented by the terms including β^2. The term $\frac{\mu H_o^2}{\rho R_o^2}\left(m^2\beta^2 \frac{I_m^{'}(x) K_m(x)}{I_m(x) K_m^{'}(x)}\right)$ has no effect on the stability of the fluid cylinder in axisymmetric mode m=0, while it is stabilizing in the non-axisymmetric modes m taking into account $K_m^{'}(x)$ is always negative for every value of $x \neq 0$ .The term $\beta^2\left(\frac{x I_m^{'}(x)}{I_m(x)}\right)$ is destabilizing in the all axisymmetric mode m = 0 and non-axisymmetric modes $m \geq 1$ .Therefore, the magnetodynamic forces are stabilizing or destabilizing according to restrictions.
We conclude that the instability character of the model could be reduced, shrinked and suppressed under certain modifications and then stability sets in.

7. NUMERICAL ANALYSIS:

The numerical investigation of the dispersion relation (70) has been carried out in the symmetric mode m=0 for all short and long wave lengths in the range 0<x<3.0. The corresponding values of the amplification σ and oscillation frequency ω in the normal unit $(4\pi G\rho)^{1/2}$ are collected and tabulated. For β =2 and H_o/H_g = 0.0, 0.5, and 1.0. We get the unstable domains and stable domains; the axial magnetic field has strong stabilizing effect, the transverse magnetic field in the region surrounding the cylinder has strong destabilizing effect. For very high intensity of magnetic field the destabilizing character of the model could be suppressed completely and stability sets in. See figures (1-2)

(i) For β = corresponding to H_o/H_g = 0.0, 0.5, and 1.0., it is found that the electrogravitational unstable domains are $0 < x < 2.62$, $0 < x < 2.88$ and $0 < x < 2.9$ .While the neighboring stable domains are $2.62 \leq x < \infty$, $2.88 \leq x < \infty$,and $2.9 \leq x < \infty$ where the equalities correspond to the marginal stability states. See Fig. (1).

(ii) For β = corresponding to H_o/H_g = 0.0, 0.5, and 1.0., it is found that the electrogravitational unstable domains are $0 < x < 1.104$, $0 < x < 1.453$ and $0 < x < 2.1$.While the neighboring stable domains are $1.104 \le x < \infty$, $1.453 \le x < \infty$,and $2.1 \le x < \infty$ where the equalities correspond to the marginal stability states. See Fig. (2).

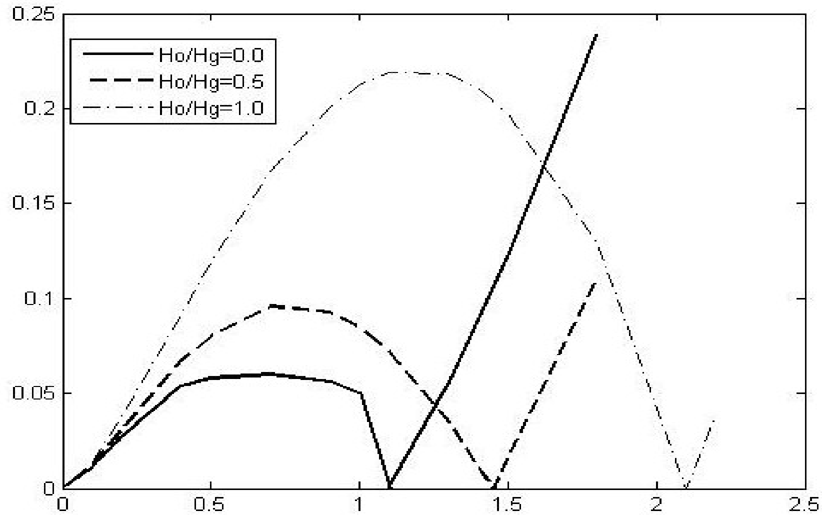

Figure (1)

Magnetogravitodynamic stable and unstable domains for β=2

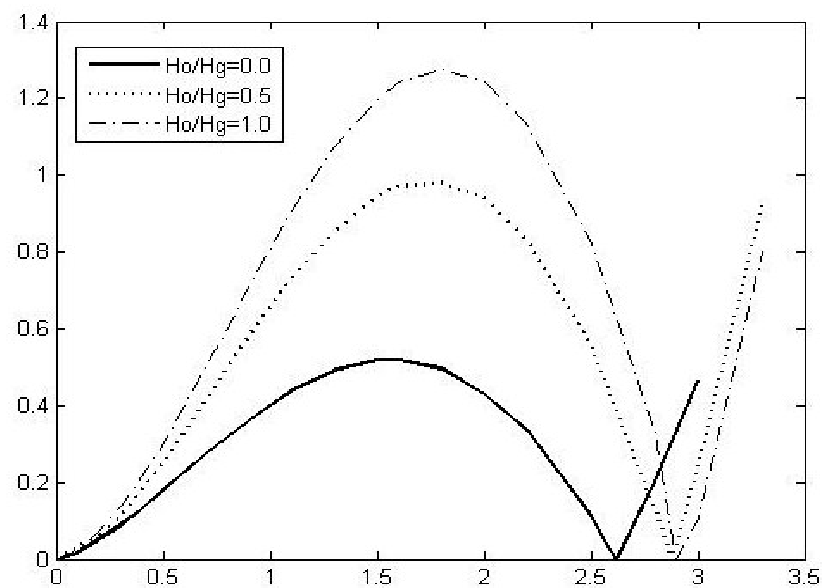

Figure (2)

Magnetogravitodynamic stable and unstable domains for β=2

REFERENCES:

[1] M. Abramowitz, I. Stegun, "Handbook of Mathematical Function" (Dover pubal, New York, U. S. A) 1970.

[2] S. Chandrasekhar, and E .Fermi, Astrophys J., 116 (1953) 118.

[3] S. Chandrasekhar, "Hydrodynamic and Hydro magnetic Stability" (Dover pubal, New York, U. S. A) 1981.

[4] R.E. Kelly, J. Fluid Mech., 22 (1965) 547.

[5] N.W. Mclachlan, "Theory and Application of Mathieu Functions" Oxford Univ-press, New York, (1947).

[6] R.S. Sengar, Proc. Nat. Acad. Scie., India, 51A (1981) 44.

[7] Radwan, A., Int. Engng. Sci., 36 (1998) 87.

[8] Radwan, A., Nuovo Cimento, 1178(2002)257.

[9] Radwan, A. ,Kyungpook Math, J., 47(2007)455.

[10] Radwan, A. E. and Ogail, M. A., Nuovo Cimento, 118 B (2003) 713.

[11] Radwan, A. E. and Hasan, A.A., Applied Mathematical Modelling, 33(4) (2009) 2121.

[12] Radwan, A. E. and Hasan, A.A., Int. J. Applied Math. (IAENG), 38(3) (2008)113.

[13] Hasan, A. A., Physica B, Accepted Manuscript, On line 27 October 2010.

[12] J.W. Rayleigh, "The Theory of Sound" (Dover Publ., New York, USA) (1945).

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