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VISCOUS DISSIPATION AND THERMAL RADIATION EFFECTS ON AN UNSTEADY MHD CONVECTION FLOW PAST A SEMI-INFINITE VERTICAL PERMEABLE MOVING POROUS PLATE

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ABSTRACT

An analytical study for the problem of unsteady two-dimensional hydromagnetic laminar mixed convection with thermal radiation, viscous dissipation and first-order chemical reaction on a boundary layer flow of viscous, electrically conducting fluid past a semi-infinite vertical permeable moving plate embedded in a uniform porous medium has been presented. The non-linear coupled partial differential equations are solved by perturbation technique. The results obtained show that the velocity, temperature and concentration fields are appreciably influenced by the presence of chemical reaction, thermal stratification and magnetic field. It is observed that the effect of chemical reaction parameter and schmid number decreases the velocity, temperature and concentration profiles in the boundary layer. Also, the effects of the various parameters on the skin-friction coefficient and the rate of heat transfer at the surface are discussed.

Key words: *MHD, Chemical reaction, Porous medium, Viscus dissipation, Thermal radiation.*

1. INTRODUCTION:

The research area of laminar flow is continuously growing, and it is the subject of intensive studies in recent years because of its application in engineering, particularly in aeronautical engineering. One of the most important applications of laminar flow is the calculation of friction drag of bodies in a flow, for example, the drag of a plate at zero incidences, the friction drag of ship, an airfoil. It is also important for heat transfer between a body and the fluid around it**.** The problem of mixed convection under the influence of magnetic field has attracted numerous researchers in view of its applications in geophysics and astrophysics. Soundalgekar et al. [1] investigated the problem of free convection effects on Stokes problem for a vertical plate with transverse applied magnetic field whereas Elbasheshy [2] studied MHD heat and mass transfer problem along a vertical plate under the combined buoyancy effects of thermal and spices diffusion. Magnetohydrodynamics (MHD) natural convection heat transfer flow in porous and non-porous media is of considerable interest in the technical field due to its frequent occurrence in industrial technology and geothermal applications, high-temperature plasmas applicable to nuclear fusion energy conversion, liquid metal fluids, and (MHD) power generation systems. Sparrow and Cess [3] studied the effect of magnetic field on the natural convection heat transfer. Takhar and Ram [4] studied the magneto hydrodynamic free convection flow of water through a porous medium. Damesh [5] studied the magnetohydrodynamics – mixed convection heat transfer problem from a vertical surface embedded in a porous media. El-Kabeir et al. [6] studied the unsteady MHD combined convection over a moving vertical sheet in a fluid saturated porous medium. Hayat et al. [7] examined the effects of Hall current and heat transfer on the rotating flow of a second grade fluid subject to a transverse applied magnetic field past a porous plate. Ali and Mahmood [8] studied the unsteady boundary layer flow of a viscous fluid through porous medium with uniform suction/injection at the wall.

The effects of radiation on MHD flow and heat transfer problem have become more important industrially. At high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. Bestman [9] examined the natural convection boundary layer with suction and mass transfer in a porous medium. His results confirmed the hypothesis that suction stabilizes the boundary layer and affords the most efficient method in boundary layer control yet know. Abdus Sattar and Hamid Kalim [10] investigated the unsteady free convection interaction with thermal radiation in a boundary layer flow past a vertical porous plate. Makinde [11] examined the transient free convection interaction with thermal radiation of an absorbing – emitting fluid

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along moving vertical permeable plate.

Muthucumaraswamy and Ganesan [12] studied effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate. Deka et al. [13] studied the effect of the first order homogeneous chemical reaction on the process of an unsteady flow past an infinite vertical plate with a constant heat and mass transfer. Soundalgekar and Patti [14] studied the problem of the flow past an impulsively started isothermal infinite vertical plate with mass transfer effects. The effect of foreign mass on the free-convection flow past a semi-infinite vertical plate was studied by Gebhart [15]. Chamkha [16] assumed that the plate is embedded in a uniform porous medium and moves with a constant velocity in the flow direction in the presence of a transverse magnetic field. Raptis [17] investigate the steady flow of a viscous fluid through a vary porous medium bounded by a porous plate subjected to a constant suction velocity by the presence of thermal radiation. Effects of the chemical reaction and radiation absorption on free convection flow through porous medium with variable suction in the presence of uniform magnetic field were studied by Sudheer Babu and Satyanarayana [18]. Recently Dulal Pal et al [19] studied Perturbation analysis of unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction.

In most of the studies mentioned above, viscous dissipation is neglected. Gebhart [20] has shown the importance of viscous dissipative heat in free convection flow in the case of isothermal and constant heat flux at the plate. Gebhart and Mollendorf [21] considered the effects of viscous dissipation for external natural convection flow over a surface. Soundalgekar [22] analyzed viscous dissipative heat on the two-dimensional unsteady free convective flow past an infinite vertical porous plate when the temperature oscillates in time and there is constant suction at the plate. Israel Cookey et al. [23] investigated the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction.

The objective of the present paper is to analyze the radiation and mass transfer effects on an unsteady two-dimensional laminar mixed convective boundary layer flow of a viscous, incompressible, electrically conducting chemically reacting fluid, along a vertical moving semi-infinite permeable plate with suction, embedded in a uniform porous medium, in the presence of transverse magnetic filed, by taking into account the effects of viscous dissipation. The equations of continuity, linear momentum, energy and diffusion, which govern the flow field, are solved by using a regular perturbation method. The behavior of the velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number has been discussed for variations in the governing parameters.

2. MATHEMATICAL ANALYSIS:

An unsteady two-dimensional hydromagnetic laminar mixed convective boundary layer flow of a viscous, incompressible, electrically conducting and chemically reacting fluid in an optically thin environment, past a semi-infinite vertical permeable moving plate embedded in a uniform porous medium, in the presence of thermal radiation is considered. The x' - axis is taken in the upward direction along the plate and y' - axis normal to it. A uniform magnetic field is applied in the direction perpendicular to the plate. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field is negligible. Also, it is assumed that the there is no applied voltage, so that the electric field is absent. The concentration of the diffusing species in the binary mixture is assumed to be very small in comparison with the other chemical species which are present, and hence the Soret and Dufour effects are negligible. Further due to the semi-infinite plane surface assumption, the flow variables are functions of normal distance y' and t' only. Now, under the usual Boussinesq's approximation, the governing boundary layer equations are

$$\frac{\partial v'}{\partial y'} = 0 \tag{2.1}$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + \nu\frac{\partial^2 u'}{\partial y'^2} + g\,\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) - \nu\frac{u'}{K'} - \frac{\sigma B_0^2 u'}{\rho} \tag{2.2}$$

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{1}{\rho C_p}\left[k\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q'}{\partial y'}\right] + \frac{\nu}{c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 - \frac{Q_0}{\rho C_p}\left(T' - T'_\infty\right) \tag{2.3}$$

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} - K'_r\left(C' - C'_\infty\right) \tag{2.4}$$

where u', v' are the velocity components in x', y' directions respectively, t' - the time, p' - the pressure, ρ - the fluid density, g - the acceleration due to gravity, β and β^*- the thermal and concentration expansion coefficients respectively, K' - the permeability of the porous medium, T' - the temperature of the fluid in the boundary layer, ν - the kinematic viscosity, σ - the electrical conductivity of the fluid, T'_∞ - the temperature of the fluid far away from the plate, C' - the species concentration in the boundary layer, C'_∞ - the species concentration in the fluid far away from the plate, B_0 - the magnetic induction, α - the fluid thermal diffusivity, k - the thermal conductivity, q' - the radiative heat flux, σ^* - the Stefan- Boltzmann constant and D - the mass diffusivity. The third and fourth terms on the right hand side of the momentum equation (2.2) denote the thermal and concentration buoyancy effects respectively. Also, the second and third terms on right hand side of the energy equation (2.3) represent the radiative heat flux and viscous dissipation respectively.

The boundary conditions for the velocity, temperature and concentration fields are

$$u' = u'_p,\ \ T' = T'_\infty + \varepsilon (T'_w - T'_\infty) e^{n't'},\ C' = C'_\infty + \varepsilon (C'_w - C'_\infty) e^{n't'} \quad at\ \ y' = 0 \tag{2.5}$$

$$u' = U'_\infty = U_0 (1 + \varepsilon\, e^{n't'}),\ \ T' \to T'_\infty,\ C' \to C'_\infty \qquad as \quad y' \to \infty$$

where u'_p is the plate velocity, T'_w and C'_w - the temperature and concentration of the plate respectively. U'_∞ - the free stream velocity, U_0 and n' - the constants. From Equation (2.1) it is clear that the suction velocity at the plate is either a constant or function of time only. Hence the suction velocity normal to the plate is assumed in the form

$$v' = -V_0 (1 + \varepsilon A e^{n't'}) \tag{2.6}$$

where A is a real positive constant, and ε is small such that $\varepsilon << 1$, $A << 1$, and V_0 is a non-zero positive constant, the negative sign indicates that the suction is towards the plate.

Outside the boundary layer, Equation (2.2) gives

$$-\frac{1}{\rho}\frac{\partial p'}{\partial x'} = \frac{dU'_\infty}{dt'} + \frac{\nu}{K'} U'_\infty + \frac{\sigma}{\rho} B_0^2 U'_\infty \tag{2.7}$$

Since the medium is optically thin with relatively low density, the radiative heat flux given by Equation (2.3), in the spirit of Cogley *et al.* [24], becomes

$$\frac{\partial q'}{\partial y'} = 4(T' - T'_\infty) I' \qquad I' = \frac{R\rho c_p V_0^2}{4\nu} \tag{2.8}$$

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$u = \frac{u'}{U_0},\ \ v = \frac{v'}{V_0},\ \ y = \frac{V_0 y'}{\nu},\ \ U_\infty = \frac{U'_\infty}{U_0}, U_p = \frac{u'_p}{U_0},\ \ t = \frac{t' V_0^2}{\nu} \tag{2.9}$$

$$\theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\ \ n = \frac{n'\nu}{V_0^2},\ \ K = \frac{K' V_0^2}{\nu^2}, \Pr = \frac{\nu \rho C_p}{k} = \frac{\nu}{\alpha}$$

$$Sc = \frac{\nu}{D},\ M = \frac{\sigma B_0^2 \nu}{\rho V_0^2}, Gr = \frac{\nu \beta g (T'_w - T'_\infty)}{U_0 V_0^2}, Gm = \frac{\nu \beta^* g (C'_w - C'_\infty)}{U_0 V_0^2}$$

$$Ec=\frac{U_0^2}{c_p(T'_w-T'_\infty)},\ R=\frac{4\nu I'}{\rho c_p V_0^2},\ K_r=\frac{K'_r\nu}{V_0^2},\ \ \phi=\frac{\nu Q_0}{\rho c_p \mathrm{V}_0^{2}}$$

In view of Equations (2.5), (2.6), (2.7), (2.8) and (2.9), Equations (2.1), (2.2), (2.3) and (2.4) reduce to the following dimensionless form.

$$\frac{\partial u}{\partial t}-(1+\varepsilon Ae^{nt})\frac{\partial u}{\partial y}=\frac{dU_\infty}{dt}+\frac{\partial^2 u}{\partial y^2}+Gr\theta+GmC+N(U_\infty-u) \tag{2.10}$$

$$\frac{\partial \theta}{\partial t}-(1+\varepsilon Ae^{nt})\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\left[\frac{\partial^2\theta}{\partial y^2}-R\theta\right]+Ec\left(\frac{\partial u}{\partial y}\right)^2-\phi\theta \tag{2.11}$$

$$\frac{\partial C}{\partial t}-(1+\varepsilon Ae^{nt})\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}-K_rC \tag{2.12}$$

where $N=M+\left(\frac{1}{K}\right)$ and *Gr, Gm, Pr, R, Ec , Sc,* K_r and ϕ are the thermal Grashof number, solutal Grashof Number, Prandtl Number, radiation parameter, Eckert number, Schmidt number ,chemical reaction parameter and porous permeability parameter respectively.

The corresponding dimensionless boundary conditions are

$$u=U_p,\quad \theta=1+\varepsilon e^{nt},\ C=1+\varepsilon e^{nt}\ \ at\ y=0$$
$$U\to U_\infty=1+\varepsilon e^{nt},\ \theta\to 0,\ C\to 0\ as\ y\to\infty \tag{2.13}$$

3. SOLUTION OF THE PROBLEM:

Equations (2.10) - (2.12) are coupled, non-linear partial differential equations and these cannot be solved in closed-form. However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighborhood of the plate as

$$\begin{aligned}u(y,t)&=u_0(y)+\varepsilon\ e^{nt}\,u_1(y)+o(\varepsilon^2)+\ldots\\ \theta(y,t)&=\theta_0(y)+\varepsilon e^{nt}\,\theta_1(y)+o(\varepsilon^2)+\ldots\\ C(y,t)&=C_0(y)+\varepsilon e^{nt}\,C_1(y)+o(\varepsilon^2)+\ldots\end{aligned} \tag{3.1}$$

Substituting (3.1) in Equations (2.10) - (2.12) and equating the harmonic and non- harmonic terms, and neglecting the higher-order terms of $o(\varepsilon^2)$, we obtain

$$u_0''+u_0'-Nu_0=-N-Gr\,\theta_0-Gm\,C_0 \tag{3.2}$$

$$u_1''+u_1'-(N+n)u_1=-(N+n)-A u_0'-Gr\,\theta_1-Gm\,C_1 \tag{3.3}$$

$$\theta_0''+\Pr\theta_0'-G_1\theta_0=-\Pr Ec\left(u_0'\right)^2 \tag{3.4}$$

$$\theta_1''+\Pr\theta_1'-N_1\theta_1=-\Pr A\theta_0'-2\Pr Ec\,u_0'u_1' \tag{3.5}$$

$$C_0''+Sc\,C_0'-K_rSc\,C_0=0 \tag{3.6}$$

$$C_1''+Sc\,C_1'-\left(K_r+n\right)Sc\,C_1=-A\,Sc\ C_0' \tag{3.7}$$

where $G_1 = (R + \Pr\phi)$ and $N_1 = (R + n\Pr + \phi)$
where prime denotes ordinary differentiation with respect to y.

The corresponding boundary conditions can be written as

$$u_0 = U_p, u_1 = 0,\ \theta_0 = 1,\ \theta_1 = 1,\ C_0 = 1,\ C_1 = 1\ at\ y = 0 \tag{3.8}$$
$$u_0 = 1,\ u_1 = 1,\ \theta_0 \to 0,\ \theta_1 \to 0,\ C_0 \to 0,\ C_1 \to 0\ as\ y \to \infty$$

The Equations (3.2) - (3.7) are still coupled and non-linear, whose exact solutions are not possible. So we expand $u_0, u_1, \theta_0, \theta_1, C_0, C_1$ in terms (f_0, f_1) of *Ec* in the following form, as the Eckert number is very small for incompressible flows.

$$f_0(y) = f_{01}(y) + Ecf_{02}(y) \tag{3.9}$$
$$f_1(y) = f_{11}(y) + Ec\ f_{12}(y)$$

Substituting (3.9) in Equations (3.2) - (3.7), equating the coefficients of *Ec* to zero and neglecting the terms in Ec^2 and higher order, we get the following equations.

The zeroth order equations are

$$u''_{01} + u'_{01} - Nu_{01} = -N - Gr\,\theta_{01} - Gm\,C_{01} \tag{3.10}$$

$$u''_{02} + u'_{02} - N\,u_{02} = -Gr\,\theta_{02} - Gm\,C_{02} \tag{3.11}$$

$$\theta''_{01} + \Pr\ \theta'_{01} - G_1\theta_{01} = 0 \tag{3.12}$$

$$\theta''_{02} + \Pr\theta'_{02} - G_1\theta_{02} = -\Pr\ {u'_{01}}^2 \tag{3.13}$$

$$C''_{01} + Sc\,C'_{01} - K_r C_{01} = 0 \tag{3.14}$$

$$C''_{02} + Sc\,C'_{02} - K_r C_{02} = 0 \tag{3.15}$$

and the respective boundary conditions are

$$u_{01} = U_p, u_{02} = 0, \theta_{01} = 1, \theta_{02} = 0, C_{01} = 1, C_{02} = 0 \ \text{ at } \ y = 0 \tag{3.16}$$

$$u_{01} \to 1, u_{02} \to 0, \theta_{01} \to 0, \theta_{02} \to 0, C_{01} \to 0, C_{02} \to 0 \ \text{ as } \ y \to \infty$$

The first order equations are

$$u''_1 + u'_{11} - (N+n)u_{11} = -(N+n) - Gr\,\theta_{11} - Gm\,C_{11} - Au'_{01} \tag{3.17}$$

$$u''_{12} + u'_{12} - (N+n)\,u_{12} = -Gr\,\theta_{12} - Gm\,C_{12} - Au'_{02} \tag{3.18}$$

$$\theta''_{11} + \Pr\theta'_{11} - N_1\,\theta_{11} = -A\Pr\theta'_{01} \tag{3.19}$$

$$\theta''_{12} + \Pr\theta'_{12} - N_1\theta_{12} = -\Pr A\,\theta'_{02} - 2\Pr u'_{01}u'_{11} \tag{3.20}$$

$$C''_{11} + Sc\,C'_{11} - (K_r + n)Sc\,C_{11} = -AScC'_{01} \tag{3.21}$$

$$C''_{12} + Sc\,C'_{12} - (K_r + n)Sc\,C_{12} = -AScC'_{02} \tag{3.22}$$

where the respective boundary conditions are

$$u_{11} = 0, u_{12} = 0, \theta_{11} = 1, \theta_{12} = 0, C_{11} = 1, C_{12} = 0 \text{ at } y = 0 \qquad (3.23)$$
$$u_{11} \to 1, u_{12} \to 0, \theta_{11} \to 0, \theta_{12} \to 0, C_{11} \to 0, C_{12} \to 0 \text{ as } y \to \infty$$

Solving Equations (3.10) - (3.15) under the boundary conditions (3.16) and Equations (3.17)-(3.22) under the boundary conditions (3.23), and using Equations (3.9) and (3.1), we obtain the velocity, temperature and concentration distributions in the boundary layer as

$$\begin{aligned} u(y,t) = {} & B_3\, e^{-m_4 y} + B_1\, e^{-m_1 y} + B_2\, e^{-m_2 y} + 1 + Ec\,\{D_8\, e^{-m_4 y} + D_1\, e^{-m_1 y} + D_2\, e^{-2m_4 y} \\ & + D_3 e^{-2m_1 y} + D_4\, e^{-2m_2 y} + D_5\, e^{-(m_1+m_4)y} + D_6\, e^{-(m_1+m_2)y} + D_7\, e^{-(m_1+m_2)y}\} \\ & + \varepsilon e^{nt}\,[\{J_8\, e^{-m_6 y} + J_1\, e^{-m_5 y} + J_2\, e^{-m_1 y} + J_3\, e^{-m_3 y} + J_4\, e^{-m_2 y} + J_5\, e^{-m_4 y} + J_6\, e^{-2m_1 y} \\ & + J_7\, e^{-m_2 y} + 1\} + Ec\,\{L_{41}\, e^{-m_6 y} + L_1\, e^{-m_5 y} + L_2\, e^{-m_1 y} + L_3\, e^{-2m_4 y} + L_4\, e^{-2m_1 y} + L_5\, e^{-2m_2 y} \\ & + L_6\, e^{-(m_1+m_4)y} + L_7\, e^{-(m_1+m_2)y} + L_8\, e^{-(m_2+m_4)y} + L_9\, e^{-(m_4+m_6)y} + L_{10}\, e^{-(m_4+m_5)y} \\ & + L_{11}\, e^{-(m_1+m_4)y} + L_{12}\, e^{-(m_3+m_4)y} + L_{13}\, e^{-(m_2+m_4)y} + L_{14}\, e^{-2m_4 y} + L_{15}\, e^{-(m_1+m_4)y} + L_{16}\, e^{-(m_2+m_4)y} \\ & + L_{17}\, e^{-(m_1+m_6)y} + L_{18}\, e^{-(m_1+m_5)y} + L_{19}\, e^{-2m_1 y} + L_{20}\, e^{-(m_1+m_3)y} + L_{21}\, e^{-(m_1+m_2)y} + L_{22}\, e^{-(m_1+m_4)y} \\ & + L_{23}\, e^{-2m_1 y} + L_{24}\, e^{-(m_1+m_2)y} + L_{25}\, e^{-(m_2+m_6)y} + L_{26}\, e^{-(m_2+m_5)y} + L_{27}\, e^{-(m_1+m_2)y} + L_{28}\, e^{-(m_2+m_3)y} \\ & + L_{29}\, e^{-2m_2 y} + L_{30}\, e^{-(m_2+m_4)y} + L_{31}\, e^{-(m_1+m_2)y} + L_{32}\, e^{-2m_2 y} + L_{33}\, e^{-m_4 y} + L_{34}\, e^{-m_1 y} + L_{35}\, e^{-2m_4 y} \\ & + L_{36}\, e^{-2m_1 y} + L_{37}\, e^{-2m_2 y} + L_{38}\, e^{-(m_1+m_4))y} + L_{39}\, e^{-(m_1+m_2)y} + L_{40}\, e^{-(m_1+m_2)y}\}] \end{aligned}$$

$$\begin{aligned} \theta(y,t) = {} & e^{-m_1 y} + Ec\,\{Z_7\, e^{-m_1 y} + Z_1\, e^{-2m_4 y} + Z_2\, e^{-m_1 y} + Z_3\, e^{-2m_2 y} + Z_4\, e^{-(m_1+m_4)y} + Z_5\, e^{-(m_1+m_2)y} \\ & + Z_6\, e^{-(m_2+m_4)y}\} + \varepsilon\, e^{nt}\,[W_2 e^{-m_5 y} + W_1\, e^{-m_1 y} + E_C\,\{H_{32} e^{-m_5 y} + H_1 e^{-m_1 y} + H_2 e^{-2m_4 y} \\ & + H_3 e^{-2m_1 y} + H_4 e^{-2m_2 y} + H_5 e^{-(m_1+m_4)y} + H_6 e^{-(m_1+m_2)y} + H_7 e^{-(m_2+m_4)y} + H_8 e^{-(m_4+m_6)y} + H_9 e^{-(m_4+m_5)y} \\ & + H_{10} e^{-(m_1+m_4)y} + H_{11} e^{-(m_3+m_4)y} + H_{12} e^{-(m_2+m_4)y} + H_{14} e^{-(m_1+m_4)y} + H_{15} e^{-(m_2+m_4)y} + H_{16} e^{-(m_1+m_6)y} \\ & + H_{17} e^{-(m_1+m_5)y} + H_{18} e^{-2m_1 y} + H_{19} e^{-(m_1+m_3)y} + H_{20} e^{-(m_1+m_2)y} + H_{21} e^{-(m_1+m_4)y} + H_{22} e^{-2m_1 y} \\ & + H_{23} e^{-(m_1+m_2)y} + H_{24} e^{-(m_2+m_6)y} + H_{25} e^{-(m_2+m_5)y} + H_{26} e^{-(m_1+m_2)y} + H_{27} e^{-(m_2+m_3)y} \\ & + H_{28} e^{-2m_2 y} + H_{29} e^{-(m_2+m_4)y} + H_{30} e^{-(m_2+m_1)y} + H_{31} e^{-2m_2 y}\}] \end{aligned}$$

$$C(y,t) = e^{-m_2 y} + \varepsilon\, e^{nt}\,\{F_2 e^{-m_3 y} + F_1 e^{-m_2 y}\}$$

The skin-friction, Nusselt number and Sherwood number are important physical parameters for this type of boundary layer flow.

Knowing the velocity field, the skin-friction at the plate can be obtained, which in non-dimensional form is given by

$$C_f = \frac{\tau'_w}{\rho U_0 V_0} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left(\frac{\partial u_0}{\partial y} + \varepsilon\, e^{nt}\frac{\partial u_1}{\partial y}\right)_{y=0}$$

Knowing the temperature field, the rate of heat transfer coefficient can be obtained, which in the non-dimensional form, in terms of the Nusselt number, is given by

$$Nu = -x\frac{\left(\partial T/\partial y'\right)_{y'=0}}{T'_w - T'_\infty} \Rightarrow Nu\,\mathrm{Re}_x^{-1} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = -\left(\frac{\partial \theta_0}{\partial y} + \varepsilon e^{nt}\frac{\partial \theta_1}{\partial y}\right)_{y=0}$$

where $Re_x = \dfrac{V_0 x}{\nu}$ is the local Reynolds number.

Knowing the concentration field, the rate of mass transfer coefficient can be obtained, which in the non-dimensional form, in terms of the Sherwood number, is given by

$$Sh = -x\frac{\left(\partial C'/\partial y'\right)_{y'=0}}{C'_w - C'_\infty} \Rightarrow Sh\,\mathrm{Re}_x^{-1} = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = -\left(\frac{\partial C_0}{\partial y} + \varepsilon e^{nt}\frac{\partial C_1}{\partial y}\right)_{y=0}$$

Where

$$m_1 = \frac{\Pr + \sqrt{\Pr^2 + 4G_1}}{2},\ m_2 = \frac{Sc + \sqrt{Sc^2 + 4ScKr}}{2},\ m_3 = \frac{Sc + \sqrt{Sc^2 + 4Sc\left(n + Kr\right)}}{2}\quad m_4 = \frac{1 + \sqrt{1 + 4N}}{2},$$

$$m_5 = \frac{\Pr + \sqrt{\Pr^2 + 4N_1}}{2},\ m_6 = \frac{1 + \sqrt{1 + 4(N + n)}}{2}$$

$$G_1 = \left(R + \phi \Pr\right),\quad N_1 = \left(R + n\Pr + \phi\right),\quad F_1 = \frac{-AScm_2}{m_2^2 - m_2 - \left(n + K_r\right)Sc},$$

The other constants are not given here to save space.

4. RESULTS AND DISCUSSION:

The formulation of the problem that accounts for the effects of radiation, permeable pameter and viscous dissipation on the flow of an incompressible viscous chemically reacting fluid along a semi-infinite, vertical moving porous plate embedded in a porous medium in the presence of transverse magnetic field was accomplished in the preceding sections. Following Cogley et al. [24] approximation for the radiative heat flux in the optically thin environment, the governing equations of the flow field were solved analytically, using a perturbation method, and the expressions for the velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number were obtained. In order to get a physical insight of the problem, the above physical quantities are computed numerically for different values of the governing parameters viz., Thermal Grashof number *Gr*, the solutal Grashof number *Gm*, Prandtl number *Pr*, Schmidt number *Sc*, the plate velocity U_p, the radiation parameter *R,* and the Eckert number *Ec*. In order to assess the accuracy of this method, we have compared our results with accepted data for the velocity and temperature profiles for a stationary vertical porous plate corresponding to the case computed by Helmy [25] and to the case of moving vertical porous plate as computed by Kim [26]. The results of these comparisons are found to be in very good agreement [See table 4].

Fig.1(a) presents the typical velocity profiles in the boundary layer for various values of the thermal Grashof number. It is observed that an increase in *Gr*, leads to a decrease in the values of velocity due to enhancement in buoyancy force. Here, the positive values of *Gr* correspond to cooling of the plate. In addition, it is observed that the velocity decreases rapidly near the wall of the porous plate as Grashof number increases and then decays to the free stream velocity. Fig 1(b) shows if *Gr* increases then the temperature also increases. For the case of different values of the solutal Grashof number, the velocity and temperature profiles in the boundary layer are shown in Fig. 2(a) and 2(b) the velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach a free stream value. As expected, the fluid velocity increases and the peak value becomes more distinctive due to increase in the buoyancy force represented by *Gm*. Where as *Gm* increases, the velocity increases, however the temperature decreases.

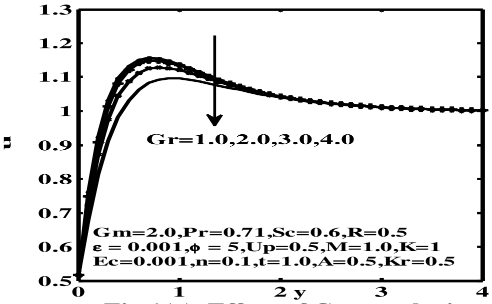


Fig 1(a). Effect of Gr on velocity

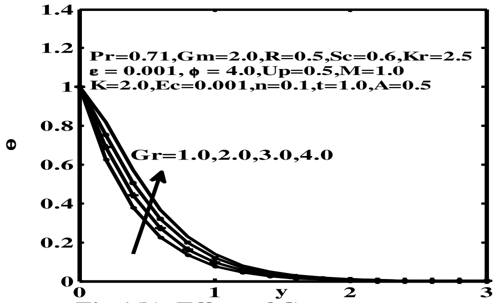


Fig 1(b). Effect of Gr on temperature

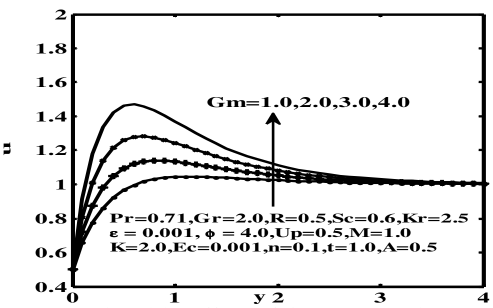


Fig 2(a). Effect of Gm on velocity

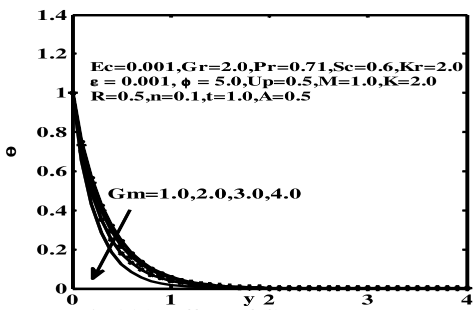


Fig 2(b). Effect of Gm on temperature

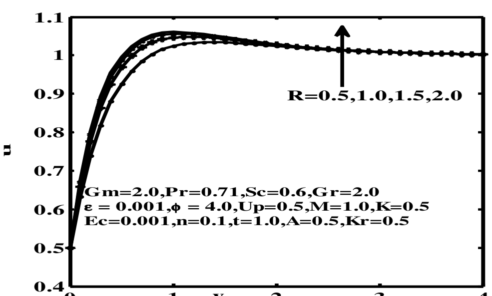


Fig 3(a). Effect of radiation parameter on velocity

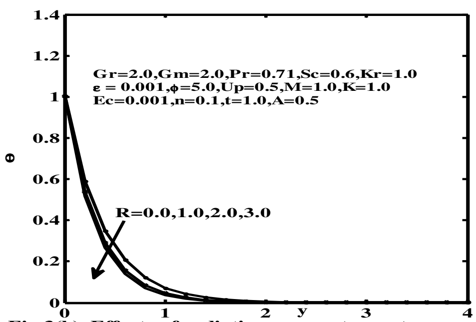


Fig 3(b). Effects of radiation parameter on temperature

For different values of the radiation parameter *R*, the velocity and temperature profiles are plotted in Figs.3 (a) and 3 (b). It is noticed that an increase in the radiation parameter results increase in the velocity and decrease in the temperature within the boundary layer, as well as increased the thickness of the velocity and decrease the temperature boundary layers.

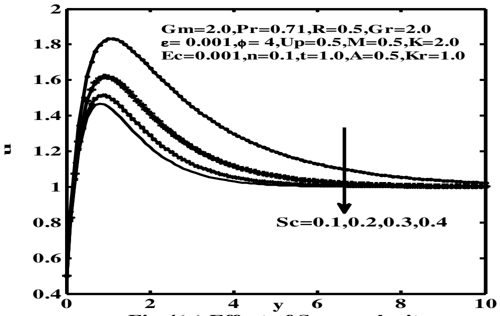


Fig 4(a) Effect of Sc on velocity

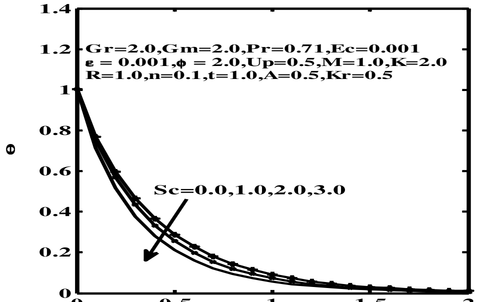


Fig 4(b). FiEffect of Sc on temperature

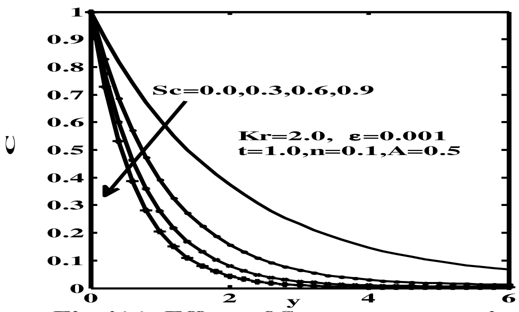


Fig 4(c). Effect of Sc on concentration

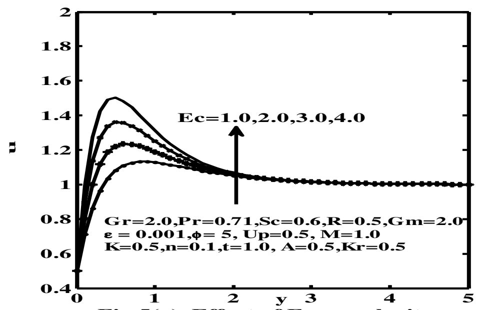


Fig 5(a). Effect of Ec on velocity

Figs. 4(a), 4(b) and 4(c) display the effects of Schmidt number on the velocity, temperature and concentration respectively. As the Schmidt number increases, the velocity, temperature and concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. Reductions in the velocity and concentration distributions are accompanied by simultaneous reductions in the velocity and concentration boundary layers. The effects of the viscous dissipation parameter i.e., the Eckert number on the velocity and temperature are shown in Figs. 5(a) and 5(b). Greater viscous dissipative heat causes a rise in the velocity and decreases in the temperature.

Figs.6 (a) and 6(b) illustrate the behavior velocity and temperature for different values of Prandtl number. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity. From Fig.6 (b), it is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature with in the boundary layer. The reason is that smaller values of *Pr* are equivalent to increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface more rapidly for higher values of *Pr*. Hence in the case of smaller Prandtl numbers as the thermal boundary layer is thicker and the rate of heat transfer is reduced. The effects of the chemical reaction parameter Kr on the velocity, temperature and concentration are shown in Figs. 7(a), 7(b) and 7(c). It is noticed that an increase in the chemical reaction parameter results a decrease in the velocity, temperature and concentration within the boundary layer.

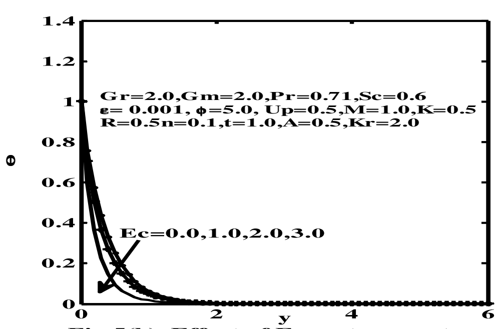


Fig 5(b). Effect of Ec on temperature

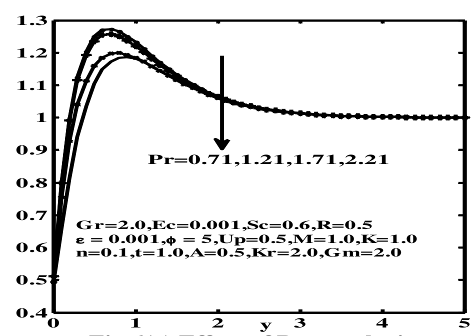


Fig 6(a) Effect of Pr on velocity

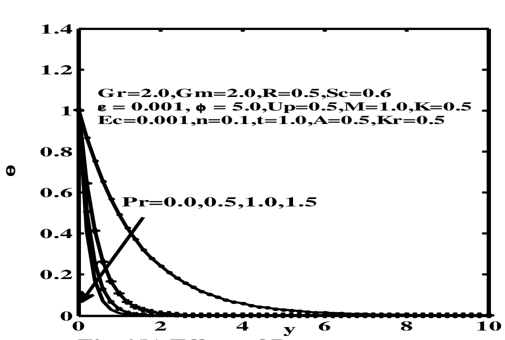


Fig 6(b).Effect of Pr on temperature

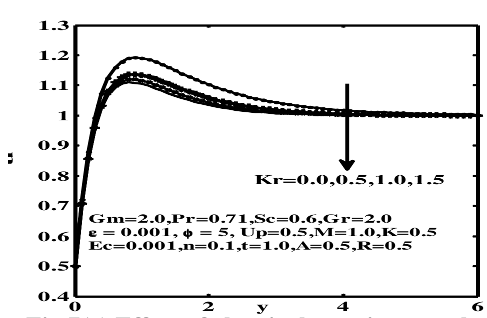
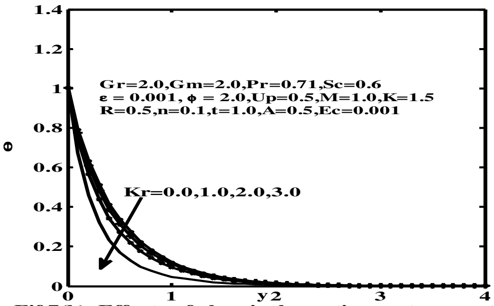


Fig 7(a) Effect of chemical reaction on velocity



Fif 7(b). Effects of chemical reaction on temperature

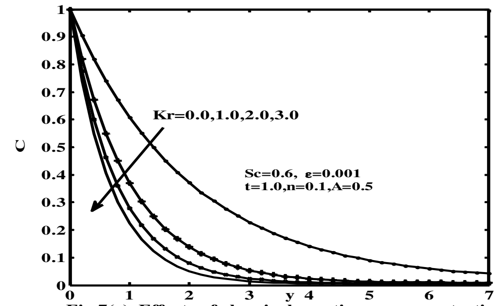


Fig 7(c). Effects of chemical reaction on concentration

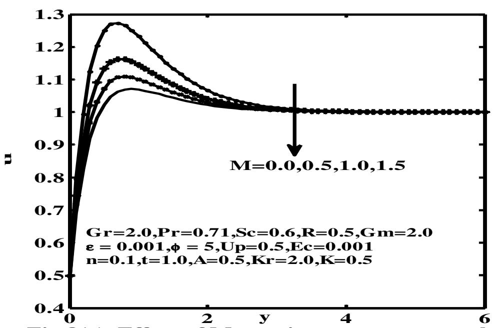


Fig 8(a). Effect of Magnetic parameter on velocity

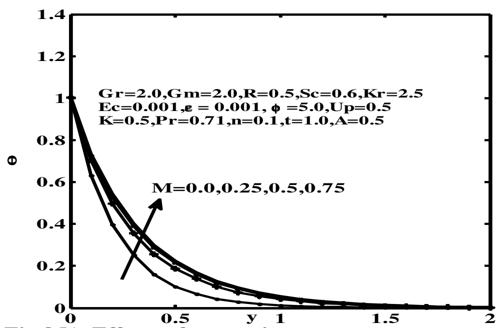


Fig 8(b). Effects of magnetic parameter on temperature

For various values of the magnetic parameter *M*, the velocity and temperature profiles are plotted in Fig.8 (a) and 8(b). It is obvious that existence of the magnetic field decreases the velocity and increases in the temperature.

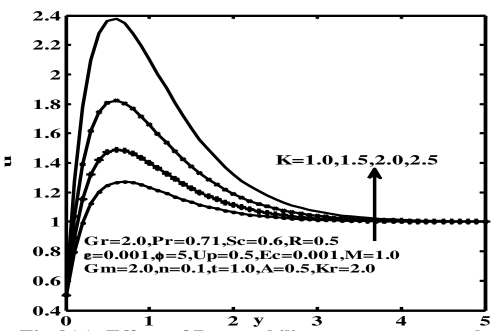


Fig 9(a). Effect of Permeability parameter on velocity

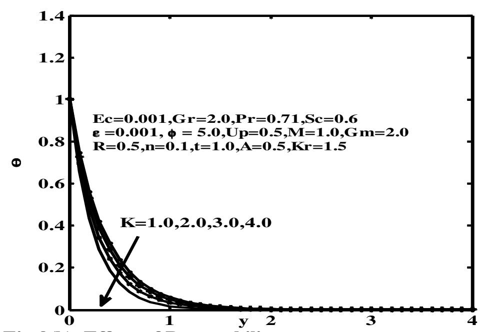


Fig 9(b). Effect of Permeability parameter on temperature

Fig.9 (a) and 9(b) shows the velocity and temperature profiles for different values of the permeability parameter. Clearly, as *K* increases the peak values of the velocity tends to increase and decreases in the temperature.

Tables 1-3 show the effects of the radiation parameter, Eckert number and chemical reaction parameter on the skin-friction C_f, Nusselt number Nu, and Sherwood number Sh. From Table 1, it can be seen that as the radiation parameter increases, the skin-friction increases and the Nusselt number decreases. However, from Table 2, it is noticed that, an increase in the chemical reaction parameter reduces the skin-friction, decreases the Sherwood number and Nusselt number stable. Finally, from Table 3, it is observed that as Eckert number increases the skin-friction decreases, and the Nusselt number stable.

Table 1. Effects of radiation on C_f and $Nu\,\mathrm{Re}_x^{-1}$. Reference values as in Fig.3 (a) and 3(b).

R	C_f	$Nu\,\mathrm{Re}_x^{-1}$
0.00	2.743	-3.854
0.25	2.062	-3.932
0.50	1.650	-3.966
0.75	1.451	-3.993

Table 2. Effects of K_r on C_f and $Sh\,\mathrm{Re}_x^{-1}$. Reference values as in Fig.4 (a), 4(b) and 4(c).

K_r	C_f	$Nu\,\mathrm{Re}_x^{-1}$	$Sh\,\mathrm{Re}_x^{-1}$
1.0	2.490	-3.545	-1.008
2.0	2.608	-3.966	-1.290
3.0	2.921	-5.357	-1.510
4.0	4.332	-16.04	-1.697

Table 3. Effects of *Ec* on C_f and $Nu\,\mathrm{Re}_x^{-1}$ Reference values as in Fig.5 (a) and 5(b).

Ec	C_f	$Nu\,\mathrm{Re}_x^{-1}$
0.01	2.196	-2.782
0.03	2.173	-2.830
0.05	2.151	-2.878
0.07	2.129	-2.927

Table 4. Comparison of present result with those of Kim [26] with different values M for C_f, $N_u\,\mathrm{Re}_x^{-1}$

Kim[31](G=2,K=∞,Up=0)			Present results for M (Sc=0.6,Pr=0.71, ε=0.001, ϕ=0,Up=0,Gr=2.0,K=∞,Gm=2.0,n=0.1,t=1.0,A=0.5,Kr=0.01, Ec=0.001, R=1.0)	
M	C_f	$N_u\,\mathrm{Re}_x^{-1}$	C_f	$N_u\,\mathrm{Re}_x^{-1}$
0.0	4.5383	-0.9430	4.5384	-0.9430
2.0	3.9234	-0.9430	3.9235	-0.9430
5.0	4.4457	-0.9430	4.4458	-0.9430
10.0	5.2976	-0.9430	5.2975	-0.9430

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