



REMARKS ON BITOPOLOGICAL $(1, 2)^*$ - ω -HOMEOMORPHISMS

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ABSTRACT

The aim of this paper is to introduce the concepts of $(1, 2)^*$ - ω - continuous mappings and study some of its properties. Their corresponding $(1,2)^*$ - ω -irresolute mappings and $(1,2)^*$ - ω -homeomorphisms are also defined and investigated in this paper.

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1. INTRODUCTION:

Regular open sets have been introduced and investigated by Stone [32]. Levine [16, 17], Cameron [3], Sundaram and Sheik John [34], and Gnanambal [7] introduced and investigated semi-open sets, regular semiopen sets, weakly closed sets and generalized pre-regular closed sets respectively. Regular ω -closed sets have been introduced and investigated by Benchalli and Wali [2] respectively, which is properly placed in between the class of ω -closed sets [33] and the class of regular-generalized closed sets [19]. Recently Ravi, Lellis Thivagar, Ekici and Many others [20-31] defined different weak forms of semi-open sets, preopen sets, regular open sets, regular semi open sets etc., in bitopological spaces.

In this paper, we introduce the notions of $(1,2)^*$ - ω -continuous mappings, $(1,2)^*$ - ω -irresolute mappings and $(1,2)^*$ - ω -homeomorphisms in bitopological spaces and study some of their basic properties. In most of the occasions our ideas are illustrated and substantiated by some suitable examples.

2. PRELIMINARIES:

Throughout this paper, X, Y and Z denote bitopological spaces (X, τ_1, τ_2) , (Y, σ_1, σ_2) and (Z, η_1, η_2) respectively.

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Definition: 2.1

Let S be a subset of a bitopological space X . Then S is called $\tau_{1,2}$ -open [11] if $S = A \cup B$, where $A \in \tau_1$ $B \in \tau_2$. The complement of $\tau_{1,2}$ -open set is called $\tau_{1,2}$ -closed. The family of all $\tau_{1,2}$ -open sets of X is denoted by $(1,2)^*$ - $O(X)$.

Definition: 2.2

Let S be a subset of a bitopological space X . Then

- (i) the $\tau_{1,2}$ -closure of S [11], denoted by $\tau_{1,2}\text{-cl}(S)$, is defined by $\cap \{U: S \subseteq U \text{ and } U \text{ is } \tau_{1,2}\text{-closed}\}$;
- (ii) the $\tau_{1,2}$ -interior of S [11], denoted by $\tau_{1,2}\text{-int}(S)$, is defined by $\cup \{U: U \subseteq S \text{ and } U \text{ is } \tau_{1,2}\text{-open}\}$.

Remark: 2.3

Notice that $\tau_{1,2}$ -open subsets of X need not necessarily form a topology.

Now we recall some definitions and results, which are used in this paper.

Definition: 2.4

A subset S of a bitopological space X is said to be

- (i) $(1, 2)^*$ - α -open [13] if $S \subseteq \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(S)))$;
- (ii) $(1, 2)^*$ -semi-open [13] if $S \subseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(S))$;
- (iii) regular $(1,2)^*$ -open [20] if $S = \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(S))$;
- (iv) $(1, 2)^*$ -preopen [13] if $S \subseteq \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(S))$;
- (v) $(1, 2)^*$ - π -open [15] if S is the finite union of regular $(1, 2)^*$ -open sets.

The complements of all the above mentioned open sets are called their respective closed sets.

The family of all $(1,2)^*$ - α -open (resp. $(1,2)^*$ -semi-open, $(1,2)^*$ -preopen, regular $(1,2)^*$ -open, $(1,2)^*$ - π -open) sets of X will be denoted by $(1,2)^*$ - $\alpha O(X)$ (resp. $(1,2)^*$ - $SO(X)$, $(1,2)^*$ - $PO(X)$, $(1,2)^*$ - $RO(X)$, $(1,2)^*$ - $\pi O(X)$).

The $(1,2)^*$ -semi-closure [25](resp. $(1,2)^*$ -preclosure [21], $(1,2)^*$ - α -closure [21]) of a subset S of X is, denoted by $(1,2)^*\text{-scl}(S)$ (resp. $(1,2)^*\text{-pcl}(S)$, $(1,2)^*\text{-}\alpha\text{cl}(S)$), defined as the intersection of all $(1,2)^*$ -semi-closed (resp. $(1,2)^*$ -preclosed, $(1,2)^*$ - α -closed) sets containing S .

Definition: 2.5

A subset S of a bitopological space X is said to be

- (i) a regular $(1,2)^*$ -generalized closed (briefly, $(1,2)^*$ -rg-closed [22]) if $\tau_{1,2}\text{-cl}(S) \subset U$ whenever $S \subset U$ and $U \in (1,2)^*\text{-RO}(X)$.
- (ii) a $(1,2)^*$ - ω -closed or $(1,2)^*$ - \hat{g} -closed [9] if $\tau_{1,2}\text{-cl}(S) \subset U$ whenever $S \subset U$ and $U \in (1,2)^*\text{-SO}(X)$.
- (iii) a $(1,2)^*$ -gpr-closed [28] if $(1,2)^*\text{-pcl}(S) \subset U$ whenever $S \subset U$ and $U \in (1,2)^*\text{-RO}(X)$.
- (iv) a $(1,2)^*$ -generalized closed (briefly, $(1,2)^*$ -g-closed [23]) if $\tau_{1,2}\text{-cl}(S) \subset U$ whenever $S \subset U$ and $U \in (1,2)^*\text{-O}(X)$.
- (v) a weakly $(1,2)^*$ -generalized closed (briefly, $(1,2)^*$ -wg-closed [29]) if $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(S)) \subset U$ whenever $S \subset U$ and $U \in (1,2)^*\text{-O}(X)$.
- (vi) a $(1,2)^*$ - π g-closed [15] if $\tau_{1,2}\text{-cl}(S) \subset U$ whenever $S \subset U$ and $U \in (1,2)^*\text{-}\pi O(X)$.

The complements of all the above mentioned closed sets are called their respective open sets.

Definition: 2.6[27]

A subset S of a bitopological space X is called

- (i) regular (1,2)*-semiopen if there is a regular (1,2)*-open set U such that $U \subset S \subset \tau_{1,2}\text{-cl}(U)$.
- (ii) regular (1,2)*- α -open (briefly, (1,2)*-r α -open) if there is a regular (1,2)*-open set U such that $U \subset S \subset (1,2)^*\text{-}\alpha\text{cl}(U)$.

The family of all regular (1, 2)*-semiopen(resp. regular (1,2)*- α -open) sets of X is denoted by (1,2)*-RSO(X) (resp. R α O(X)).

Definition: 2.7 [31]

A subset A of a bitopological space X is called a regular generalized-(1,2)*- α -closed set (briefly, (1,2)*-rg α -closed) if $(1,2)^*\text{-}\alpha\text{-cl}(A) \subset U$ whenever $A \subset U$ and $U \in \text{R}\alpha\text{O}(X)$.

We denote the set of all (1,2)*-rg α -closed sets in X by (1,2)*-RG α C(X).

Definition: 2.8 [27]

A subset S of a bitopological space X is called regular (1, 2)*- ω -closed (briefly (1,2)*-r ω -closed) if $\tau_{1,2}\text{-cl}(S) \subset U$ whenever $S \subset U$ and $U \in (1,2)^*\text{-RSO}(X)$.

The complement of regular (1,2)*- ω -closed set is called regular (1,2)*- ω -open (briefly (1,2)*-r ω -open).

We denote the family of all (1,2)*-r ω -closed (resp. (1,2)*-r ω -open) sets in X by (1,2)*-R ω C(X)(resp. (1,2)*-R ω O(X)).

Definition: 2.9

A map $f : X \rightarrow Y$ is said to be

- (i) (1, 2)*-continuous [20] if $f^{-1}(V)$ is $\tau_{1,2}$ -closed in X, for every $\sigma_{1,2}$ -closed set V in Y.
- (ii) (1, 2)*-semi-continuous [25] if $f^{-1}(V)$ is (1, 2)*-semi-closed in X, for every $\sigma_{1,2}$ -closed set V in Y.
- (iii) (1, 2)*- ω -continuous [28] if $f^{-1}(V)$ is (1, 2)*- ω -closed in X, for every $\sigma_{1,2}$ -closed set V in Y.
- (iv) (1, 2)*-rg-continuous [22] if $f^{-1}(V)$ is (1, 2)*-rg-closed in X, for every $\sigma_{1,2}$ -closed set V in Y.
- (v) (1, 2)*- π g-continuous [15] if $f^{-1}(V)$ is (1, 2)*- π g-closed in X, for every $\sigma_{1,2}$ -closed set V in Y.
- (vi) (1, 2)*-g-continuous [24] if $f^{-1}(V)$ is (1, 2)*-g-closed in X, for every $\sigma_{1,2}$ -closed set V in Y.
- (vii) (1, 2)*-gpr-continuous [12] if $f^{-1}(V)$ is (1, 2)*-gpr-closed in X, for every $\sigma_{1,2}$ -closed set V in Y.
- (viii) (1, 2)*-wg-continuous [29] if $f^{-1}(V)$ is (1, 2)*-wg-closed in X, for every $\sigma_{1,2}$ -closed set V in Y.

Definition: 2.10

A map $f : X \rightarrow Y$ is said to be

- (i) (1, 2)*-semi-irresolute [13] if $f^{-1}(V)$ is (1, 2)*-semi-open in X, for every (1, 2)*-semi-open V in Y.
- (ii) (1, 2)*- ω -irresolute [28] if $f^{-1}(V)$ is (1, 2)*- ω -closed in X, for every (1, 2)*- ω -closed V in Y.

Definition 2.11

A bijective $f : X \rightarrow Y$ is said to be

- (i) (1, 2)*-g-homeomorphism [26] if both f and f^{-1} are (1,2)*-g continuous
- (ii) (1, 2)*- ω -homeomorphism [28] if both f and f^{-1} are (1,2)*- ω -continuous
- (iii) (1, 2)*-homeomorphism [26] if both f and f^{-1} are (1,2)*- continuous

Remark: 2.12

Every $\tau_{1,2}$ -closed set is $(1,2)^*$ - $r\omega$ -closed set but not conversely [27].

3. $(1, 2)^*$ - $r\omega$ -CONTINUOUS MAPPINGS:

Definition: 3.1

A subset A of a bitopological space X is called a regular weakly generalized- $(1,2)^*$ -closed set (briefly, $(1,2)^*$ -rwg-closed) if $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular $(1,2)^*$ -open in X .

Definition: 3.2

A map $f : X \rightarrow Y$ is said to be

- (i) $(1, 2)^*$ - $r\omega$ -continuous if $f^{-1}(V)$ is $(1,2)^*$ - $r\omega$ -closed in X , for every $\sigma_{1,2}$ -closed set V in Y .
- (ii) $(1, 2)^*$ -rwg-continuous if $f^{-1}(V)$ is $(1,2)^*$ -rwg-closed in X , for every $\sigma_{1,2}$ -closed set V in Y .

Theorem: 3.3

Every $(1, 2)^*$ -continuous map is $(1,2)^*$ - $r\omega$ -continuous.

Proof:

Let $f : X \rightarrow Y$ be $(1,2)^*$ -continuous and V be any $\sigma_{1,2}$ -closed set in Y . Then $f^{-1}(V)$ is $\tau_{1,2}$ -closed set in X . By Remark 2.12, $f^{-1}(V)$ is $(1, 2)^*$ - $r\omega$ -closed in X . Therefore, f is $(1, 2)^*$ - $r\omega$ -continuous.

Remark: 3.4

The converse of Theorem 3.3 need not be true as shown in the following example.

Example: 3.5

Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{b\}\}$ and $\tau_2 = \{\emptyset, X, \{a, c\}\}$. Then the sets in $\{\emptyset, X, \{b\}, \{a, c\}\}$ are called $\tau_{1,2}$ -open and $\tau_{1,2}$ -closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\emptyset, Y, \{b, c\}\}$ and $\sigma_2 = \{\emptyset, Y, \{c\}\}$. Then the sets in $\{\emptyset, Y, \{c\}, \{b, c\}\}$ are called $\sigma_{1,2}$ -open and the sets in $\{\emptyset, Y, \{a\}, \{a, b\}\}$ are called $\sigma_{1,2}$ -closed. The map $f : X \rightarrow Y$ defined by $f(a) = a$, $f(b) = b$, $f(c) = c$ is $(1,2)^*$ - $r\omega$ -continuous but not $(1,2)^*$ -continuous.

Theorem: 3.6

If $f : X \rightarrow Y$ is $(1,2)^*$ - ω -continuous map then it is $(1,2)^*$ - $r\omega$ -continuous.

Proof:

Let V be any $\sigma_{1,2}$ -closed set of Y . Then by hypothesis $f^{-1}(V)$ is $(1, 2)^*$ - ω -closed set in X . But every $(1, 2)^*$ - ω -closed set is $(1, 2)^*$ - $r\omega$ -closed, by Theorem 3.3[27]. Therefore, f is $(1, 2)^*$ - $r\omega$ -continuous.

Remark: 3.7

The converse of Theorem 3.6 need not be true as shown in the following example.

Example: 3.8

Let $X = \{a, b, c, d\}$, $\tau_1 = \{\emptyset, X, \{b\}\}$ and $\tau_2 = \{\emptyset, X, \{c\}, \{b, c, d\}\}$. Then the sets in $\{\emptyset, X, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{a\}, \{a, d\}, \{a, b, d\}, \{a, c, d\}\}$ are called $\tau_{1,2}$ -closed. Let $Y = \{a, b, c, d\}$, $\sigma_1 = \{\emptyset, Y, \{a, b, c\}\}$ and $\sigma_2 = \{\emptyset, Y, \{a, c\}\}$. Then the sets in $\{\emptyset, Y, \{a, c\}, \{a, b, c\}\}$ are called $\sigma_{1,2}$ -open and the sets in $\{\emptyset, Y, \{d\}, \{b, d\}\}$ are called $\sigma_{1,2}$ -closed. The map $f : X \rightarrow Y$ defined by $f(a) = a$, $f(b) = b$, $f(c) = c$ and $f(d) = d$ is $(1,2)^*$ - $r\omega$ -continuous but not $(1,2)^*$ - ω -continuous.

Theorem: 3.9

If $f : X \rightarrow Y$ is $(1,2)^*$ - $r\omega$ -continuous map then it is $(1,2)^*$ -rg-continuous.

Proof:

Let V be any $\sigma_{1,2}$ -closed set of Y . Then by hypothesis $f^{-1}(V)$ is $(1, 2)^*$ - $r\omega$ -closed set in X . But every $(1, 2)^*$ - $r\omega$ -closed set is $(1, 2)^*$ -rg-closed, by Theorem 3.6[27]. Therefore f is $(1, 2)^*$ -rg-continuous.

Remark: 3.10:

The converse of Theorem 3.9 need not be true as shown in the following example.

Example: 3.11

Let $X = \{a, b, c, d\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{b\}, \{a, b, c\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ are called $\tau_{1,2}$ -closed. Let $Y = \{a, b, c, d\}$, $\sigma_1 = \{\emptyset, Y, \{a, b, c\}\}$ and $\sigma_2 = \{\emptyset, Y\}$. Then the sets in $\{\emptyset, Y, \{a, b, c\}\}$ are called $\sigma_{1,2}$ -open and the sets in $\{\emptyset, Y, \{d\}\}$ are called $\sigma_{1,2}$ -closed. The map $f : X \rightarrow Y$ defined by $f(a) = b, f(b) = a, f(c) = c$ and $f(d) = d$ is $(1,2)^*$ -rg-continuous but not $(1,2)^*$ -r ω -continuous.

Theorem: 3.12

If $f : X \rightarrow Y$ is $(1,2)^*$ -r ω -continuous map then it is $(1,2)^*$ -gpr-continuous.

Proof:

Let V be any $\sigma_{1,2}$ -closed set of Y . Then by hypothesis $f^{-1}(V)$ is $(1, 2)^*$ -r ω -closed set in X . But every $(1, 2)^*$ -r ω -closed set is $(1, 2)^*$ -gpr-closed, by Theorem 3.12[27]. Therefore, f is $(1, 2)^*$ -gpr-continuous.

Remark: 3.13

The converse of Theorem 3.12 need not be true as shown in the following example.

Example: 3.14

Let $X = \{a, b, c, d\}$, $\tau_1 = \{\emptyset, X, \{b\}\}$ and $\tau_2 = \{\emptyset, X, \{c\}, \{b, c, d\}\}$. Then the sets in $\{\emptyset, X, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{a\}, \{a, d\}, \{a, b, d\}, \{a, c, d\}\}$ are called $\tau_{1,2}$ -closed. Let $Y = \{a, b, c, d\}$, $\sigma_1 = \{\emptyset, Y, \{a, b, c\}\}$ and $\sigma_2 = \{\emptyset, Y, \{a, c\}\}$. Then the sets in $\{\emptyset, Y, \{a, c\}, \{a, b, c\}\}$ are called $\sigma_{1,2}$ -open and the sets in $\{\emptyset, Y, \{d\}, \{b, d\}\}$ are called $\sigma_{1,2}$ -closed. The map $f : X \rightarrow Y$ defined by $f(a) = a, f(b) = b, f(c) = c$ and $f(d) = d$ is $(1,2)^*$ -gpr-continuous but not $(1,2)^*$ -r ω -continuous.

Remark: 3.15

The concepts of

- (i) $(1, 2)^*$ -r ω -continuous and $(1, 2)^*$ -g-continuous are independent.
- (ii) $(1, 2)^*$ -r ω -continuous and $(1, 2)^*$ -semi-continuous are independent.
- (iii) $(1, 2)^*$ -r ω -continuous and $(1, 2)^*$ -wg-continuous are independent.
- (iv) $(1, 2)^*$ -r ω -continuous and $(1, 2)^*$ - π g-continuous are independent.

Example: 3.16

Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{b\}\}$ and $\tau_2 = \{\emptyset, X, \{c\}\}$. Then the sets in $\{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$ are called $\tau_{1,2}$ -closed. Let $\sigma_1 = \{\emptyset, Y, \{a\}\}$ and $\sigma_2 = \{\emptyset, Y, \{b, c\}\}$. Then the sets in $\{\emptyset, Y, \{a\}, \{b, c\}\}$ are called $\sigma_{1,2}$ -open and the sets in $\{\emptyset, Y, \{a\}, \{b, c\}\}$ are called $\sigma_{1,2}$ -closed. Also, the sets in $\{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ are called $(1,2)^*$ -r ω -closed and the sets in $\{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$ are called $(1,2)^*$ -g-closed in X . Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by $f(a) = a, f(b) = b$ and $f(c) = c$. Then f is $(1, 2)^*$ -r ω -continuous but not $(1, 2)^*$ -g-continuous.

Example: 3.17

Let $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\emptyset, X\}$ and $\tau_2 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ are called $\tau_{1,2}$ -closed. Let $\sigma_1 = \{\emptyset, Y\}$ and $\sigma_2 = \{\emptyset, Y, \{c\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$. Then the sets in $\{\emptyset, Y, \{c\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ are called $\sigma_{1,2}$ -open and the sets in $\{\emptyset, Y, \{d\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$ are called $\sigma_{1,2}$ -closed. Also, the sets in $\{\emptyset, X, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ are called $(1, 2)^*$ -r ω -closed and the sets in $\{\emptyset, X, \{d\}, \{b, d\}, \{c, d\}, \{a, d\}, \{b, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ are called $(1, 2)^*$ -g-closed in X . Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by $f(a) = a, f(b) = b, f(c) = c$ and $f(d) = d$. Then f is $(1, 2)^*$ -g-continuous but not $(1, 2)^*$ -r ω -continuous.

Example: 3.18

Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{b\}\}$ and $\tau_2 = \{\emptyset, X, \{c\}\}$. Then the sets in $\{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$ are called $\tau_{1,2}$ -closed. Let $\sigma_1 = \{\emptyset, Y, \{a\}\}$ and $\sigma_2 = \{\emptyset, Y, \{b, c\}\}$. Then the sets in $\{\emptyset, Y, \{a\}, \{b, c\}\}$ are called $\sigma_{1,2}$ -open and the sets in $\{\emptyset, Y, \{a\}, \{b, c\}\}$ are called $\sigma_{1,2}$ -closed. Also, the sets in $\{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ are called $(1,2)^*$ -r ω -closed and the sets in $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$ are called $(1,2)^*$ -semi-closed in X . Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by $f(a) = a, f(b) = b$ and $f(c) = c$. Then f is $(1, 2)^*$ -r ω -continuous but not $(1, 2)^*$ -semi-continuous.

Example: 3.19

Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{b\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{c\}, \{a, c\}, \{b, c\}\}$ are called $\tau_{1,2}$ -closed. Let $\sigma_1 = \{\emptyset, Y\}$ and $\sigma_2 = \{\emptyset, Y, \{b\}, \{a, b\}, \{b, c\}\}$. Then the sets in $\{\emptyset, Y, \{b\}, \{a, b\}, \{b, c\}\}$ are called $\sigma_{1,2}$ -open and the sets in $\{\emptyset, Y, \{a\}, \{c\}, \{a, c\}\}$ are called $\sigma_{1,2}$ -closed. Also, the sets in $\{\emptyset, X, \{c\}, \{a, c\}, \{b, c\}\}$ are called $(1, 2)^*$ - $r\omega$ -closed and the sets in $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$ are called $(1, 2)^*$ -semi-closed in X . Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then f is $(1, 2)^*$ -semi-continuous but not $(1, 2)^*$ - $r\omega$ -continuous.

Example: 3.20

Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{b\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{c\}, \{a, c\}, \{b, c\}\}$ are called $\tau_{1,2}$ -closed. Let $\sigma_1 = \{\emptyset, Y, \{c\}\}$ and $\sigma_2 = \{\emptyset, Y, \{a, b\}\}$. Then the sets in $\{\emptyset, Y, \{c\}, \{a, b\}\}$ are called $\sigma_{1,2}$ -open and $\sigma_{1,2}$ -closed. Also, the sets in $\{\emptyset, X, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ are called $(1, 2)^*$ - $r\omega$ -closed and the sets in $\{\emptyset, X, \{c\}, \{a, c\}, \{b, c\}\}$ are called $(1, 2)^*$ -wg-closed in X . Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then f is $(1, 2)^*$ - $r\omega$ -continuous but not $(1, 2)^*$ -wg-continuous.

Example: 3.21

Let $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{b\}, \{a, b, c\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ are called $\tau_{1,2}$ -closed. Let $\sigma_1 = \{\emptyset, Y, \{b, c\}\}$ and $\sigma_2 = \{\emptyset, Y, \{c\}, \{a, c\}, \{a, b, c\}\}$. Then the sets in $\{\emptyset, Y, \{c\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ are called $\sigma_{1,2}$ -open and the sets in $\{\emptyset, Y, \{d\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$ are called $\sigma_{1,2}$ -closed. Also, the sets in $\{\emptyset, X, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ are called $(1, 2)^*$ - $r\omega$ -closed and the sets in $\{\emptyset, X, \{c\}, \{d\}, \{c, d\}, \{a, d\}, \{b, d\}, \{a, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ are called $(1, 2)^*$ -wg-closed in X . Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by $f(a) = a$, $f(b) = b$, $f(c) = c$ and $f(d) = d$. Then f is $(1, 2)^*$ -wg-continuous but not $(1, 2)^*$ - $r\omega$ -continuous.

Example: 3.22

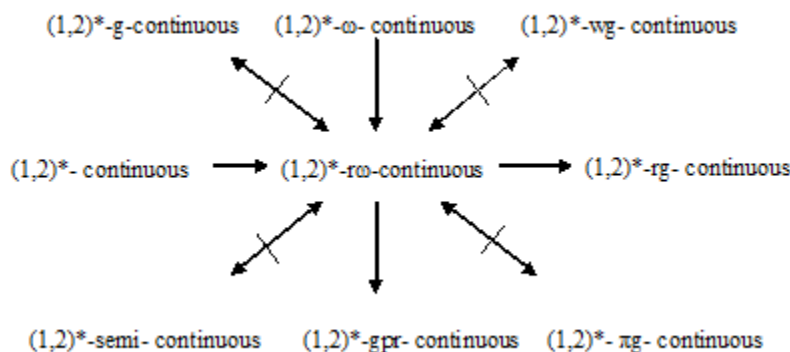
Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{b\}\}$ and $\tau_2 = \{\emptyset, X, \{c\}\}$. Then the sets in $\{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$ are called $\tau_{1,2}$ -closed. Let $\sigma_1 = \{\emptyset, Y, \{a\}\}$ and $\sigma_2 = \{\emptyset, Y, \{b, c\}\}$. Then the sets in $\{\emptyset, Y, \{a\}, \{b, c\}\}$ are called $\sigma_{1,2}$ -open and the sets in $\{\emptyset, Y, \{a\}, \{b, c\}\}$ are called $\sigma_{1,2}$ -closed. Also, the sets in $\{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ are called $(1, 2)^*$ - $r\omega$ -closed and the sets in $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ are called $(1, 2)^*$ - πg -closed in X . Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then f is $(1, 2)^*$ - $r\omega$ -continuous but not $(1, 2)^*$ - πg -continuous.

Example: 3.23

Let $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\emptyset, X, \{b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}, \{a, b, c\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ are called $\tau_{1,2}$ -closed. Let $\sigma_1 = \{\emptyset, Y, \{a, d\}\}$ and $\sigma_2 = \{\emptyset, Y, \{a, b, d\}\}$. Then the sets in $\{\emptyset, Y, \{a, d\}, \{a, b, d\}\}$ are called $\sigma_{1,2}$ -open and the sets in $\{\emptyset, Y, \{c\}, \{b, c\}\}$ are called $\sigma_{1,2}$ -closed. Also, the sets in $\{\emptyset, X, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ are called $(1, 2)^*$ - $r\omega$ -closed and the sets in $\{\emptyset, X, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ are called $(1, 2)^*$ - πg -closed in X . Define $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ by $f(a) = a$, $f(b) = b$, $f(c) = c$ and $f(d) = d$. Then f is $(1, 2)^*$ - πg -continuous but not $(1, 2)^*$ - $r\omega$ -continuous.

Remark: 3.24

The following diagram summarizes the above discussions.



Remark: 3.25

The following Example shows that the composition of two (1, 2)*-r ω -continuous maps need not be a (1, 2)*-r ω -continuous.

Example: 3.26

Let $X = Y = Z = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{b\}\}$, $\tau_2 = \{\emptyset, X, \{c\}\}$, $\sigma_1 = \{\emptyset, Y, \{b\}\}$, $\sigma_2 = \{\emptyset, Y, \{a, c\}\}$, $\eta_1 = \{\emptyset, Z, \{a\}\}$ and $\eta_2 = \{\emptyset, Z, \{a, b\}\}$. The map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ defined as $f(a) = b$, $f(b) = a$, $f(c) = c$ and the map $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be an identity map are (1,2)*-r ω -continuous but $g \circ f$ is not (1,2)*-r ω -continuous, since $(g \circ f)^{-1}(\{c\}) = \{c\}$ is not (1,2)*-r ω -closed set in X.

Result: 3.27 [31]

Every (1, 2)*-r ω -continuous map is (1,2)*-rg α -continuous

4. (1,2)*- r ω -IRRESOLUTE MAPPINGS:

Definition: 4.1

A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called

(i) (1, 2)*-r ω -irresolute if the inverse image of every (1, 2)*-r ω -closed set in Y is (1, 2)*-r ω -closed in X.

(ii) (1, 2)*-rg α -irresolute if $f^{-1}(V)$ is (1, 2)*-rg α -closed in X, for every (1, 2)*-rg α -closed V in Y.

Theorem: 4.2

Every (1, 2)*-r ω -irresolute function is (1, 2)*-r ω -continuous but not conversely.

Proof:

Assume that $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is (1,2)*-r ω -irresolute and V is $\sigma_{1,2}$ -closed set in Y. So it is (1, 2)*-r ω -closed set in Y by Remark 2.12. By our assumption inverse image of V is a (1, 2)*-r ω -closed set in X. Therefore, f is (1, 2)*-r ω -continuous.

Example: 4.3

Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{b\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{c\}, \{a, c\}, \{b, c\}\}$ are called $\tau_{1,2}$ -closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\emptyset, Y, \{a\}, \{a, b\}\}$ and $\sigma_2 = \{\emptyset, Y\}$. Then the sets in $\{\emptyset, X, \{a\}, \{a, b\}\}$ are called $\sigma_{1,2}$ -open and the sets in $\{\emptyset, X, \{c\}, \{b, c\}\}$ are called $\sigma_{1,2}$ -closed. Then the map $f : X \rightarrow Y$ defined as $f(a) = b$, $f(b) = a$ and $f(c) = c$ is (1,2)*-r ω -continuous but not (1,2)*-r ω -irresolute because $f^{-1}(\{b\}) = \{a\}$ is not an (1,2)*-r ω -closed set in X.

Theorem: 4.4

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be any two maps. Then $g \circ f$ is (1, 2)*-r ω -continuous if g is (1, 2)*-continuous and f is (1,2)*-r ω -continuous.

Proof:

Let V be any $\eta_{1,2}$ -closed set in Z. Then $g^{-1}(V)$ is $\sigma_{1,2}$ -closed in Y, since g is (1,2)*-continuous. Then $f^{-1}(g^{-1}(V))$ is (1, 2)*-r ω -closed in X, as f is (1, 2)*-r ω -continuous. That is, $(g \circ f)^{-1}(V)$ is (1, 2)*-r ω -closed in X. Hence $g \circ f$ is (1, 2)*-r ω -continuous.

Theorem: 4.5

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be any two maps. Then $g \circ f$ is (1, 2)*-r ω -irresolute if g is (1, 2)*-r ω -irresolute and f is (1,2)*-r ω -irresolute.

Proof:

Let V be any (1, 2)*-r ω -closed set in Z. Since g is (1, 2)*-r ω -irresolute, $g^{-1}(V)$ is (1,2)*-r ω -closed in Y. Then $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is (1, 2)*-r ω -closed in X, as f is (1, 2)*-r ω -irresolute. Therefore, $g \circ f$ is (1, 2)*-r ω -irresolute.

Theorem: 4.6

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be any two maps. Then $g \circ f$ is (1, 2)*-r ω -continuous if g is (1, 2)*-r ω -continuous and f is (1, 2)*-r ω -irresolute.

Proof:

Let V be any $\eta_{1,2}$ -closed set in Z. Since g is (1, 2)*-r ω -continuous, $g^{-1}(V)$ is (1, 2)*-r ω -closed in Y. Then $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is (1, 2)*-r ω -closed in X, as f is (1,2)*-r ω -irresolute. Therefore, $g \circ f$ is (1, 2)*-r ω -continuous.

5. (1,2)*-r ω -HOMEOMORPHISMS:

We introduce the following definitions.

Definition: 5.1

A bijection $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called (1,2)*-r ω -homeomorphism if both f and f^{-1} are (1,2)*-r ω -continuous.

Definition: 5.2

A bijection $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called

(i) (1,2)*-regular generalized α -homeomorphism (briefly, (1, 2)*-rg α -homeomorphism) if f and f^{-1} are (1,2)*-rg α -continuous.

(ii) (1, 2)*-r ω g-homeomorphism if both f and f^{-1} are (1,2)*- r ω g-continuous.

Definition: 5.3

A bijection $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called (1,2)*- ω *-homeomorphism if both f and f^{-1} are (1,2)*- ω -irresolute.

Definition: 5.4

(i) A bijection $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called (1, 2)*-rg α c-homeomorphism if both f and f^{-1} are (1,2)*-rg α -irresolute.

(ii) A bijection $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called (1, 2)*-r ω c-homeomorphism if both f and f^{-1} are (1,2)*-r ω -irresolute.

Example 5.5

Let $X = \{a, b, c, d\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{b\}, \{a, b, c\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ are called $\tau_{1,2}$ -closed. Let $Y = \{a, b, c, d\}$, $\sigma_1 = \{\emptyset, Y, \{a\}\}$ and $\sigma_2 = \{\emptyset, Y, \{b\}\}$. Then the sets in $\{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$ are called $\sigma_{1,2}$ -open and the sets in $\{\emptyset, Y, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ are called $\sigma_{1,2}$ -closed. If the map $f : X \rightarrow Y$ is an identity map, then f is bijective, (1,2)*-r ω -continuous and f^{-1} is (1,2)*-r ω -continuous. Therefore f is (1,2)*-r ω -homeomorphism.

Theorem: 5.6

Every (1, 2)*-homeomorphism is an (1, 2)*-r ω -homeomorphism.

Proof:

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a (1,2)*-homeomorphism. Then f and f^{-1} are (1, 2)*-continuous and f is bijection. As every (1, 2)*-continuous map is (1, 2)*-r ω -continuous, we have f and f^{-1} are (1, 2)*-r ω -continuous. Therefore f is (1, 2)*-r ω -homeomorphism.

Remark: 5.7

The converse of Theorem 5.6 need not be true as shown in the following example.

Example: 5.8

Let $X = \{a, b, c, d\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{b\}, \{a, b, c\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ are called $\tau_{1,2}$ -closed. Let $Y = \{a, b, c, d\}$, $\sigma_1 = \{\emptyset, Y, \{a\}\}$ and $\sigma_2 = \{\emptyset, Y, \{b\}\}$. Then the sets in $\{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$ are called $\sigma_{1,2}$ -open and the sets in $\{\emptyset, Y, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ are called $\sigma_{1,2}$ -closed. If the map $f : X \rightarrow Y$ is an identity map, then f is (1,2)*-r ω -homeomorphism but it is not (1,2)*-homeomorphism.

Theorem: 5.9

Every (1, 2)*- ω -homeomorphism is an (1, 2)*-r ω -homeomorphism.

Proof:

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a (1,2)*- ω -homeomorphism. Then f and f^{-1} are (1, 2)*- ω -continuous and f is bijection. As every (1, 2)*- ω -continuous map is (1, 2)*-r ω -continuous, we have f and f^{-1} are (1, 2)*-r ω -continuous. Therefore f is (1, 2)*-r ω -homeomorphism.

Remark: 5.10

The converse of Theorem 5.9 need not be true as shown in the following example.

Example: 5.11

Let $X = \{a, b, c, d\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{b\}, \{a, b, c\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ are called $\tau_{1,2}$ -closed. Let $Y = \{a, b, c, d\}$, $\sigma_1 = \{\emptyset, Y, \{a\}\}$ and $\sigma_2 = \{\emptyset, Y, \{b\}\}$. Then the sets in $\{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$ are called $\sigma_{1,2}$ -open and the sets in $\{\emptyset, Y, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ are called $\sigma_{1,2}$ -closed. If the map $f : X \rightarrow Y$ is the identity map, then this map is $(1,2)^*$ - $r\omega$ -homeomorphism but it is not $(1,2)^*$ - ω -homeomorphism.

Theorem 5.12

Every $(1, 2)^*$ - $r\omega$ -homeomorphism is an $(1, 2)^*$ - $rg\alpha$ -homeomorphism.

Proof:

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a $(1,2)^*$ - $r\omega$ -homeomorphism. Then f and f^{-1} are $(1, 2)^*$ - $r\omega$ -continuous and f is bijection. As every $(1, 2)^*$ - $r\omega$ -continuous map is $(1,2)^*$ - $rg\alpha$ -continuous, we have f and f^{-1} are $(1,2)^*$ - $rg\alpha$ -continuous. Therefore f is $(1, 2)^*$ - $rg\alpha$ -homeomorphism.

Remark: 5.13

The converse of Theorem 5.12 need not be true as shown in the following example.

Example: 5.14

Let $X = \{a, b, c, d\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{b\}, \{a, b, c\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ are called $\tau_{1,2}$ -closed. Let $Y = \{a, b, c, d\}$, $\sigma_1 = \{\emptyset, Y, \{a\}\}$ and $\sigma_2 = \{\emptyset, Y, \{b\}\}$. Then the sets in $\{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$ are called $\sigma_{1,2}$ -open and the sets in $\{\emptyset, Y, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ are called $\sigma_{1,2}$ -closed. If the map $f : X \rightarrow Y$ is the identity map, then the map is $(1,2)^*$ - $rg\alpha$ -homeomorphism but it is not $(1,2)^*$ - $r\omega$ -homeomorphism.

Corollary: 5.15

Every $(1, 2)^*$ - ω^* -homeomorphism is an $(1, 2)^*$ - $r\omega$ -homeomorphism.

Proof:

It is evident that every $(1,2)^*$ - ω^* -homeomorphism is a $(1,2)^*$ - ω -homeomorphism. By Theorems 5.9 and 5.12., every $(1,2)^*$ - ω -homeomorphism is a $(1,2)^*$ - $rg\alpha$ -homeomorphism and hence $(1,2)^*$ - ω^* -homeomorphism is a $(1,2)^*$ - $rg\alpha$ -homeomorphism.

Theorem: 5.16

Every $(1, 2)^*$ - $r\omega$ -homeomorphism is an $(1, 2)^*$ - rwg -homeomorphism.

Proof:

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(1,2)^*$ - $r\omega$ -homeomorphism. Then f and f^{-1} are $(1, 2)^*$ - $r\omega$ -continuous and f is bijection. Since every $(1,2)^*$ - $r\omega$ -continuous map is $(1,2)^*$ - rwg -continuous, we have f and f^{-1} and $(1,2)^*$ - rwg -continuous. Therefore f is $(1, 2)^*$ - rwg -homeomorphism.

Remark: 5.17

The converse of Theorem 5.16 need not be true as shown in the following example.

Example: 5.18

Let $X = \{a, b, c, d\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{b\}, \{a, b, c\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ are called $\tau_{1,2}$ -open and the sets in $\{\emptyset, X, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ are called $\tau_{1,2}$ -closed. Let $Y = \{a, b, c, d\}$, $\sigma_1 = \{\emptyset, Y, \{a\}\}$ and $\sigma_2 = \{\emptyset, Y, \{b\}\}$. Then the sets in $\{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$ are called $\sigma_{1,2}$ -open and the sets in $\{\emptyset, Y, \{c\}, \{a, c\}, \{b, c\}\}$ are called $\sigma_{1,2}$ -closed. If the map $f : X \rightarrow Y$ is defined as $f(a) = c$, $f(b) = b$, $f(c) = a$ and $f(d) = d$, then f is rwg -homeomorphism but it is not $r\omega$ -homeomorphism.

Remark: 5.19

The concepts of

- (i) $(1, 2)^*$ - $r\omega c$ -homeomorphisms and ω^* -homeomorphisms are independent.
- (ii) $(1, 2)^*$ - $r\omega$ -homeomorphisms and $(1, 2)^*$ - g -homeomorphisms are independent.

Example: 5.20

In Example 5.18, if the map $f : X \rightarrow Y$ is an identity map, then f is $(1,2)^*$ - ω^* -homeomorphism but it is not $(1,2)^*$ - $r\omega$ -homeomorphism, since f is not $(1,2)^*$ - $r\omega$ -irresolute.

Example: 5.21

In Example 5.18, if the map $f : X \rightarrow Y$ is an identity map, then f is $(1,2)^*$ - $r\omega$ -homeomorphism but it is not $(1,2)^*$ - ω^* -homeomorphism, since f is not $(1,2)^*$ - ω -irresolute.

Example: 5.22

In Example 5.18, if the map $f : X \rightarrow Y$ is defined as $f(a) = c$, $f(b) = b$, $f(c) = a$ and $f(d) = d$, then f is $(1,2)^*$ - $r\omega$ -homeomorphism but it is not $(1,2)^*$ - g -homeomorphism.

Example: 5.23

In Example 5.18, if the map $f : X \rightarrow Y$ is defined as $f(a) = c$, $f(b) = b$, $f(c) = a$ and $f(d) = d$, then f is $(1,2)^*$ - g -homeomorphism but it is not $(1,2)^*$ - $r\omega$ -homeomorphism.

Theorem: 5.24

Every $(1, 2)^*$ - $r\omega$ -homeomorphism is an $(1, 2)^*$ - $r\omega$ -homeomorphism.

Proof:

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be an $(1,2)^*$ - $r\omega$ -homeomorphism. Then f and f^{-1} are $(1, 2)^*$ - $r\omega$ -irresolute and f is bijection. By Theorem 4.2 f and f^{-1} are $(1, 2)^*$ - $r\omega$ -continuous. Therefore f is $(1, 2)^*$ - $r\omega$ -homeomorphism.

Remark: 5.25

The converse of Theorem 5.24 need not be true as shown in the following example.

Example: 5.26

In Example 5.18, if the map $f : X \rightarrow Y$ is the identity map, then f is $(1,2)^*$ - $r\omega$ -homeomorphism but it is not $(1,2)^*$ - $r\omega$ -homeomorphism, since f is not $(1,2)^*$ - $r\omega$ -irresolute.

Theorem: 5.27

Every $(1,2)^*$ - $r\omega$ -homeomorphism is $(1,2)^*$ - rwg -homeomorphism.

Proof:

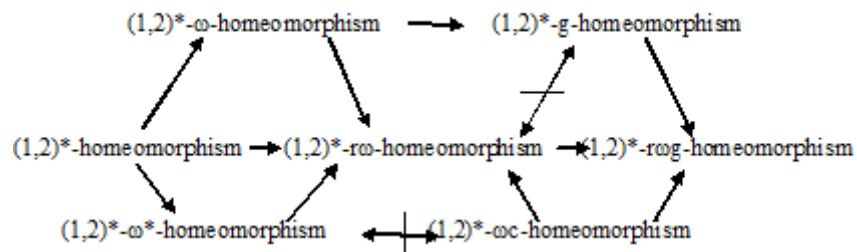
Proof follows from Theorems 5.16. and 5.24.

Example: 5.28

In Example 5.18, if the map $f : X \rightarrow Y$ is an identity map, then f is $(1,2)^*$ - rwg -homeomorphism but it is not $(1,2)^*$ - $r\omega$ -homeomorphism, since f is not $(1,2)^*$ - $r\omega$ -irresolute.

Remark: 5.29

The following diagram summarizes the above discussions.



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