International Journal of Mathematical Archive-2(4), Apr. - 2011, Page: 424-428
Available online through www.ijma.info ISSN 2229 – 5046

$\ddot{g}$-REGULAR AND $\ddot{g}$-NORMAL SPACES

[1]O. Ravi* and [2]S. Ganesan

[1]*Department of Mathematics, P. M. Thevar College, Usilampatti, Madurai District, Tamil Nadu, India*
[2]*Department of Mathematics, N. M. S. S. V. N College, Nagamalai, Madurai, Tamil Nadu, India*

E-mail: siingam@yahoo.com, sgsgsgsgsg77@yahoo.com

(Received on: 04-02-2011; Accepted on: 21-03-2011)

ABSTRACT

The concept of $\ddot{g}$-closed sets was introduced by Ravi and Ganesan [9]. The aim of this paper is to introduce and characterize $\ddot{g}$-regular spaces and $\ddot{g}$-normal spaces via the concept of $\ddot{g}$-closed sets.

2000 Mathematics Subject Classification: *54C10, 54C08, 54C05*

Keywords and Phrases: *$\ddot{g}$-closed set, $\ddot{g}$-open set, $\ddot{g}$-regular space, $\ddot{g}$-normal space, $\ddot{g}$-continuous function, $\ddot{g}$-irresolute function.*

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1. INTRODUCTION:

As a generalization of closed sets, in 1970, Levine [6] initiated the study of so called g-closed sets. As the strong forms of g-closed sets, the notion of ĝ-closed sets (= ω-closed sets) were introduced and studied by Veerakumar [18] (Sheik John [16]). Using g-closed sets, Munshi [8] introduced g-regular and g-normal spaces in topological spaces. In a similar way, Sheik John [16] introduced ω-regular and ω-normal spaces using ω-closed sets in topological spaces.

In this paper, we introduce $\ddot{g}$-regular spaces and $\ddot{g}$-normal spaces in topological spaces. We obtain several characterizations of $\ddot{g}$-regular and $\ddot{g}$-normal spaces and some preservation theorems for $\ddot{g}$-regular and $\ddot{g}$-normal spaces.

2. PRELIMINARIES:

Throughout this paper, (X, τ) and (Y, σ) (or X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For any subset A of a space (X, τ), the closure of A, the interior of A and the complement of A are denoted by cl(A), int(A) and A^c respectively.

We recall the following definitions which are useful in the sequel.

Definition: 2.1
A subset A of a space (X, τ) is called: semi-open set [5]

if $A \subseteq cl(int(A))$

The complement of semi-open set is semi-closed.

The semi-closure [3] of a subset A of X, denoted by scl(A), is defined to be the intersection of all semi-closed sets of (X, τ) containing A. It is known that scl(A) is a semi-closed set. For any subset A of an arbitrarily chosen topological space, the semi-interior [3] of A, denoted by sint(A), is defined to be the union of all semi-open sets of (X, τ) contained in A.

Definition: 2.2
A subset A of a space (X, τ) is called:

(i) a ĝ-closed set [18] (= ω-closed set [16]) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ). The complement of ĝ-closed set is called ĝ-open set;

(ii) a semi-generalized closed (briefly sg-closed) set [1] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ). The complement of sg-closed set is called sg-open set;

(iii) a $\ddot{g}$-closed set [9] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg-open in (X, τ). The complement of $\ddot{g}$-closed set is called $\ddot{g}$-open set.

The collection of all $\ddot{g}$-closed sets of X is denoted by $\ddot{G}C(X)$.

Definition: 2.3
A function f : (X, τ) → (Y, σ) is called:

(i) a $\ddot{g}$-continuous [11] if $f^{-1}(V)$ is $\ddot{g}$-closed in (X, τ) for every closed set V in (Y, σ).

(ii) a $\ddot{g}$-irresolute [11] if $f^{-1}(V)$ is $\ddot{g}$-closed in (X, τ) for every $\ddot{g}$-closed set V in (Y, σ).

Corresponding author: O. Ravi
***E-mail: siingam@yahoo.com**

(iii) a pre-sg-open [12] if f(V) is sg-open in (Y, σ) for every sg-open set V of (X, τ).

(iv) an sg-irresolute [2, 17] if $f^{-1}(V)$ is sg-closed in (X, τ) for each sg-closed set V of (Y, σ).

(v) a $\ddot{g}$ -closed [13] if the image of every closed set in (X, τ) is $\ddot{g}$ -closed in (Y, σ).

(vi) Weakly continuous [7] if for each point x ∈ X and each open set V in (Y, σ) containing f(x), there exists an open set U containing x such that f(U) ⊆ cl(V).

Definition: 2.4 [14]
Let (X, τ) be a topological space. Let x be a point of X and G be a subset of X. Then G is called an $\ddot{g}$ -neighbourhood of x (briefly, $\ddot{g}$ -nbhd of x) in X if there exists an $\ddot{g}$ -open set U of X such that x ∈ U ⊂ G.

Definition: 2.5 [15]
A space (X, τ) is called a gT $\ddot{g}$ -space if every g-closed set in it is $\ddot{g}$ -closed.

Definition: 2.6 [6]
A topological space (X, τ) will be termed symmetric if and only if for x and y in (X, τ), x ∈ cl(y) implies that y ∈ cl(x).

Definition: 2.7 [10]
For every set A ⊆ X, we define the $\ddot{g}$ -closure of A to be the intersection of all $\ddot{g}$ -closed sets containing A.

In symbols, $\ddot{g}$ -cl(A) = ∩ {F : A ⊆ F ∈ $\ddot{G}$ C(X)}.

Definition: 2.8 [19]
For a subset A of a topological space (X, τ), $cl_\theta(A)$ = {x ∈ X: cl(U) ∩ A ≠ ϕ, U ∈ τ and x ∈ U}.

Theorem: 2.9 [11]
A set A is $\ddot{g}$ -open if and only if F ⊆ int(A) whenever F is sg-closed and F ⊆ A.

Theorem: 2.10 [6]
The space (X, τ) is symmetric if and only if {x} is g-closed in (X, τ) for each point x of (X, τ).

Theorem 2.11 [11]
If f: (X,τ) → (Y,σ) is bijective, pre-sg-open and $\ddot{g}$ -continuous, then f is $\ddot{g}$ -irresolute.

Theorem: 2.12 [13]
If f : (X, τ) → (Y, σ) is sg-irresolute $\ddot{g}$ -closed and A is a $\ddot{g}$ -closed subset of (X, τ), then f(A) is $\ddot{g}$ -closed.

3. $\ddot{g}$ -REGULAR AND $\ddot{g}$ -NORMAL SPACES:

We introduce the following definition.

Definition: 3.1
A space (X, τ) is said to be $\ddot{g}$ -regular if for every $\ddot{g}$ -closed set F and each point x ∉ F, there exist disjoint open sets U and V such that F ⊆ U and x ∈ V.

Theorem: 3.2
Let (X, τ) be a topological space. Then the following statements are equivalent:

(i) (X, τ) is a $\ddot{g}$ -regular space.

(ii) For each x ∈ X and $\ddot{g}$ -neighbourhood W of x there exists an open neighbourhood V of x such that cl(V) ⊆ W.

Proof: (i) ⇒ (ii). Let W be any $\ddot{g}$ -neighbourhood of x. Then there exist a $\ddot{g}$ -open set G such that x ∈ G ⊆ W. Since G^c is $\ddot{g}$ -closed and x ∉ G^c, by hypothesis there exist open sets U and V such that G^c ⊆ U, x ∈ V and U ∩ V = ϕ and so V ⊆ U^c. Now, cl(V) ⊆ cl(U^c) = U^c and G^c ⊆ U implies U^c ⊆ G ⊆ W. Therefore cl(V) ⊆ W.

(ii) ⇒ (i). Let F be any $\ddot{g}$ -closed set and x ∉ F. Then x ∈ F^c and F^c is $\ddot{g}$ -open and so F^c is a $\ddot{g}$ -neighbourhood of x. By hypothesis, there exists an open neighbourhood V of x such that x ∈ V and cl(V) ⊆ F^c, which implies F ⊆ $(cl(V))^c$. Then $(cl(V))^c$ is an open set containing F and V ∩ $(cl(V))^c$ = ϕ. Therefore, X is $\ddot{g}$ -regular.

Theorem: 3.3
For a space (X, τ) the following are equivalent:

(i) (X, τ) is normal.

(ii) For every pair of disjoint closed sets A and B, there exist $\ddot{g}$ -open sets U and V such that A ⊆ U, B ⊆ V and U ∩ V = ϕ.

Proof: (i) ⇒ (ii). Let A and B be disjoint closed subsets of (X, τ). By hypothesis, there exist disjoint open sets (and hence $\ddot{g}$ -open sets) U and V such that A ⊆ U and B ⊆ V.

(ii) ⇒ (i). Let A and B be closed subsets of (X, τ). Then by assumption, A ⊆ G, B ⊆ H and G ∩ H = ϕ, where G and H are disjoint $\ddot{g}$ -open sets. Since A and B are sg-closed, by Theorem 2.9, A ⊆ int(G) and B ⊆ int(H). Further, int(G) ∩ int(H) = int(G ∩ H) = ϕ.

Theorem: 3.4
A gT $\ddot{g}$ -space (X, τ) is symmetric if and only if {x} is $\ddot{g}$ -closed in (X, τ) for each point x of (X, τ).

Proof: Follows from Definitions 2.5., 2.6 and Theorem 2.10.

Theorem: 3.5

A topological space (X, τ) is $\ddot{g}$-regular if and only if for each $\ddot{g}$-closed set F of (X, τ) and each $x \in F^c$ there exist open sets U and V of (X, τ) such that $x \in U$, $F \subseteq V$ and $cl(U) \cap cl(V) = \phi$.

Proof: Let F be a $\ddot{g}$-closed set of (X, τ) and $x \notin F$. Then there exist open sets U_0 and V of (X, τ) such that $x \in U_0$, $F \subseteq V$ and $U_0 \cap V = \phi$, which implies $U_0 \cap cl(V) = \phi$. Since cl(V) is closed, it is $\ddot{g}$-closed and $x \notin cl(V)$. Since (X, τ) is $\ddot{g}$-regular, there exist open sets G and H of (X, τ) such that $x \in G$, $cl(V) \subseteq H$ and $G \cap H = \phi$, which implies $cl(G) \cap H = \phi$. Let $U = U_0 \cap G$, then U and V are open sets of (X, τ) such that $x \in U$, $F \subseteq V$ and $cl(U) \cap cl(V) = \phi$.
Converse part is trivial.

Corollary: 3.6

If a space (X, τ) is $\ddot{g}$-regular, symmetric and gT$\ddot{g}$-space, then it is Urysohn.

Proof: Let x and y be any two distinct points of (X, τ). Since (X, τ) is symmetric and gT$\ddot{g}$-space, $\{x\}$ is $\ddot{g}$-closed by Theorem 3.4. Therefore, by Theorem 3.5, there exist open sets U and V such that $x \in U$, $y \in V$ and $cl(U) \cap cl(V) = \phi$.

Theorem: 3.7

Let (X, τ) be a topological space. Then the following statements are equivalent:

(i) (X,τ) is $\ddot{g}$-regular.

(ii) For each point $x \in X$ and for each $\ddot{g}$-neighbourhood W of x, there exists an open neighbourhood V of x such that $cl(V) \subseteq W$.

(iii) For each point $x \in X$ and for each $\ddot{g}$-closed set F not containing x, there exists an open neighbourhood V of x such that $cl(V) \cap F = \phi$.

Proof: (i) $\Leftrightarrow$ (ii). It is obvious from Theorem 3.2.
(ii) $\Rightarrow$ (iii). Let $x \in X$ and F be a $\ddot{g}$-closed set such that $x \notin F$. Then F^c is a $\ddot{g}$-neighbourhood of x and by hypothesis, there exists an open neighbourhood V of x such that $cl(V) \subseteq F^c$ and hence $cl(V) \cap F = \phi$.

(iii) $\Rightarrow$ (ii). Let $x \in X$ and W be a $\ddot{g}$-neighbourhood of x. Then there exists a $\ddot{g}$-open set G such that $x \in G \subseteq W$. Since G^c is $\ddot{g}$-closed and $x \notin G^c$, by hypothesis there exists an open neighbourhood V of x such that $cl(V) \cap G^c = \phi$. Therefore, $cl(V) \subseteq G \subseteq W$.

Theorem: 3.8

The following are equivalent for a space (X, τ).

(i) (X, τ) is $\ddot{g}$-regular.

(ii) $cl_\theta(A) = \ddot{g}\text{-}cl(A)$ for each subset A of (X, τ).

(iii) $cl_\theta(A) = A$ for each $\ddot{g}$-closed set A.

Proof: (i) $\Rightarrow$ (ii). For any subset A of (X, τ), we have always $A \subseteq \ddot{g}\text{-}cl(A) \subseteq cl_\theta(A)$. Let $x \in (\ddot{g}\text{-}cl(A))^c$. Then there exists a $\ddot{g}$-closed set F such that $x \in F^c$ and $A \subseteq F$. By assumption, there exist disjoint open sets U and V such that $x \in U$ and $F \subseteq V$. Now, $x \in U \subseteq cl(U) \subseteq V^c \subseteq F^c \subseteq A^c$ and therefore $cl(U) \cap A = \phi$. Thus, $x \in (cl_\theta(A))^c$ and hence $cl_\theta(A) = \ddot{g}\text{-}cl(A)$.
(ii) $\Rightarrow$ (iii). It is trivial.
(iii) $\Rightarrow$ (i). Let F be any $\ddot{g}$-closed set and $x \in F^c$. Since F is $\ddot{g}$-closed, by assumption $x \in (cl_\theta(F))^c$ and so there exists an open set U such that $x \in U$ and $cl(U) \cap F = \phi$. Then $F \subseteq (cl(U))^c$. Let $V = (cl(U))^c$. Then V is an open such that $F \subseteq V$. Also, the sets U and V are disjoint and hence (X, τ) are $\ddot{g}$-regular.

Theorem: 3.9

If (X, τ) is a $\ddot{g}$-regular space and $f : (X, \tau) \rightarrow (Y, \sigma)$ is bijective, pre-sg-open, $\ddot{g}$-continuous and open, then (Y, σ) is $\ddot{g}$-regular.

Proof: Let F be any $\ddot{g}$-closed subset of (Y, σ) and $y \notin F$. Since the map f is $\ddot{g}$-irresolute by Theorem 2.11, we have $f^{-1}(F)$ is $\ddot{g}$-closed in (X, τ). Since f is bijective, let f(x) =y, then $x \notin f^{-1}(F)$. By hypothesis, there exist disjoint open sets U and V such that $x \in U$ and $f^{-1}(F) \subseteq V$. Since f is open and bijective, we have $y \in f(U)$, $F \subseteq f(V)$ and $f(U) \cap f(V) = \phi$. This shows that the space (Y, σ) is also $\ddot{g}$-regular.

Theorem: 3.10

If $f : (X, \tau) \rightarrow (Y, \sigma)$ is sg-irresolute $\ddot{g}$-closed continuous injection and (Y, σ) is $\ddot{g}$-regular, then (X, τ) is $\ddot{g}$-regular.

Proof: Let F be any $\ddot{g}$-closed set of (X, τ) and $x \notin F$. Since f is sg-irresolute $\ddot{g}$-closed, by Theorem 2.12, f(F) is $\ddot{g}$-closed in (Y, σ) and $f(x) \notin f(F)$. Since (Y, σ) is $\ddot{g}$-regular and so there exist disjoint open sets U and V in (Y, σ) such that $f(x) \in U$ and $f(F) \subseteq V$. i.e., $x \in f^{-1}(U)$, $F \subseteq f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \phi$. Therefore, (X, τ) is $\ddot{g}$-regular.

Theorem: 3.11

If $f : (X, \tau) \rightarrow (Y, \sigma)$ is weakly continuous $\ddot{g}$-closed injection and (Y, σ) is $\ddot{g}$-regular, then (X, τ) is regular.

Proof: Let F be any closed set of (X, τ) and $x \notin F$. Since f is $\ddot{g}$-closed, f(F) is $\ddot{g}$-closed in (Y, σ) and $f(x) \notin f(F)$. Since (Y, σ) is $\ddot{g}$-regular by Theorem 3.5 there exist open sets U and V such that $f(x) \in U$, $f(F) \subseteq V$ and $cl(U) \cap cl(V) = \phi$. Since f is weakly continuous it follows that [7, Theorem 1], $x \in f^{-1}(U) \subseteq int(f^{-1}(cl(U)))$, $F \subseteq f^{-1}(V) \subseteq int(f^{-1}(cl(V)))$ and $int(f^{-1}(cl(U))) \cap int(f^{-1}(cl(V))) = \phi$. Therefore, (X, τ) is regular.

We conclude this section with the introduction of $\ddot{g}$-normal space in topological spaces.

Definition: 3.12

A topological space (X, τ) is said to be $\ddot{g}$-normal if for any pair of disjoint $\ddot{g}$-closed sets A and B, there exist disjoint open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

We characterize $\ddot{g}$-normal space.

Theorem: 3.13

Let (X, τ) be a topological space. Then the following statements are equivalent:

(i) (X, τ) is $\ddot{g}$-normal.

(ii) For each $\ddot{g}$-closed set F and for each $\ddot{g}$-open set U containing F, there exists an open set V containing F such that $cl(V) \subseteq U$.

(iii) For each pair of disjoint $\ddot{g}$-closed sets A and B in (X, τ), there exists an open set U containing A such that $cl(U) \cap B = \phi$.

(iv) For each pair of disjoint $\ddot{g}$-closed sets A and B in (X, τ), there exist open sets U containing A and V containing B such that $cl(U) \cap cl(V) = \phi$.

Proof: (i) ⇒ (ii). Let F be a $\ddot{g}$-closed set and U be a $\ddot{g}$-open set such that $F \subseteq U$. Then $F \cap U^c = \phi$. By assumption, there exist open sets V and W such that $F \subseteq V$, $U^c \subseteq W$ and $V \cap W = \phi$, which implies $cl(V) \cap W = \phi$. Now, $cl(V) \cap U^c \subseteq cl(V) \cap W = \phi$ and so $cl(V) \subseteq U$.

(ii) ⇒ (iii). Let A and B be disjoint $\ddot{g}$-closed sets of (X, τ). Since $A \cap B = \phi$, $A \subseteq B^c$ and B^c is $\ddot{g}$-open. By assumption, there exists an open set U containing A such that $cl(U) \subseteq B^c$ and so $cl(U) \cap B = \phi$.

(iii) ⇒ (iv). Let A and B be any two disjoint $\ddot{g}$-closed sets of (X, τ). Then by assumption, there exists an open set U containing A such that $cl(U) \cap B = \phi$. Since cl(U) is closed, it is $\ddot{g}$-closed and so B and cl(U) are disjoint $\ddot{g}$-closed sets in (X, τ). Therefore again by assumption, there exists an open set V containing B such that $cl(V) \cap cl(U) = \phi$.

(iv) ⇒ (i). Let A and B be any two disjoint $\ddot{g}$-closed sets of (X, τ). By assumption, there exist open sets U containing A and V containing B such that $cl(U) \cap cl(V) = \phi$, we have $U \cap V = \phi$ and thus (X, τ) is $\ddot{g}$-normal.

Theorem: 3.14

If $f : (X, \tau) \to (Y, \sigma)$ is bijective, pre-sg-open, $\ddot{g}$-continuous and open and (X, τ) is $\ddot{g}$-normal, then (Y, σ) is $\ddot{g}$-normal.

Proof: Let A and B be any disjoint $\ddot{g}$-closed sets of (Y, σ). The map f is $\ddot{g}$-irresolute by Theorem 2.11 and so $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint $\ddot{g}$-closed sets of (X, τ). Since (X, τ) is $\ddot{g}$-normal, there exist disjoint open sets U and V such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$. Since f is open and bijective, we have f(U) and f(V) are open in (Y, σ) such that $A \subseteq f(U)$, $B \subseteq f(V)$ and $f(U) \cap f(V) = \phi$. Therefore, (Y, σ) is $\ddot{g}$-normal.

Theorem: 3.15

If $f : (X, \tau) \to (Y, \sigma)$ is sg-irresolute $\ddot{g}$-closed continuous injection and (Y, σ) is $\ddot{g}$-normal, then (X, τ) is $\ddot{g}$-normal.

Proof: Let A and B be any disjoint $\ddot{g}$-closed subsets of (X,τ). Since f is sg-irresolute $\ddot{g}$-closed, f(A) and f(B) are disjoint $\ddot{g}$-closed sets of (Y, σ) by Theorem 2.12. Since (Y, σ) is $\ddot{g}$-normal, there exist disjoint open sets U and V such that $f(A) \subseteq U$ and $f(B) \subseteq V$. i.e., $A \subseteq f^{-1}(U)$, $B \subseteq f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \phi$. Since f is continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are open in (X, τ), we have (X, τ) is $\ddot{g}$-normal.

Theorem: 3.16

If $f : (X, \tau) \to (Y, \sigma)$ is weakly continuous $\ddot{g}$-closed injection and (Y, σ) is $\ddot{g}$-normal, then (X, τ) is normal.

Proof: Let A and B be any two disjoint closed sets of (X, τ). Since f is injective and $\ddot{g}$-closed, f(A) and f(B) are disjoint $\ddot{g}$-closed sets of (Y, σ). Since (Y, σ) is $\ddot{g}$-normal, by Theorem 3.13, there exist open sets U and V such that $f(A) \subseteq U$, $f(B) \subseteq V$ and $cl(U) \cap cl(V) = \phi$. Since f is weakly continuous, it follows that [7, Theorem 1], $A \subseteq f^{-1}(U) \subseteq int(f^{-1}(cl(U)))$, $B \subseteq f^{-1}(V) \subseteq int(f^{-1}(cl(V)))$ and $int(f^{-1}(cl(U))) \cap int(f^{-1}(cl(V))) = \phi$. Therefore, (X, τ) is normal.

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