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STATISTICAL QUALITY CONTROL FOR CONSTRAINED MULTI-ITEM INVENTORY LOT-SIZE MODEL WITH INCREASING VARYING HOLDING COST VIA GEOMETRIC PROGRAMMING

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ABSTRACT

In this study, a multi-item economic production quantity model under available limited storage space and setup cost constraints is considered. The model having varying holding cost, which is considered to be continuous function of order quantity. The model policy is discussed statistical quality control using subgroup ranges and modified geometric programming methods.

Keywords: *Quality control, Inventory, Storage space, Setup and holding costs, Geometric programming.*

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1. INTRODUCTION:

Many researchers have studied inventory models assuming the holding cost to be constant and independent of the order quantity using geometric programming an Lagrangian methods without quality control concept. Teng and Yang [9] proposed a deterministic inventory lot-size models with time-varying demand and cost under generalized holding costs. Kotb [5] and Abou-El-Ata and Kotb [1] applied geometric programming approach to solve some inventory models with variable inventory costs. Shawky and Abou-El-Ata [8] solved a constrained production lot size model with trade policy by geometric programming and Lagrangian methods. Other related inventory models were written by Cheng [2], Jang and Klein [4] and Mandal et al. [7]. Recently, Kotb and Fergany [6] discussed multi-item EOQ model with varying holding cost: a geometric programming approach.

This paper examines statistical quality control inventory lot-size model with increasing varying holding cost under linear and non-linear constraints which are assumed binding. The optimal order quantity of each item using geometric programming is the objective and it is used to confirm that the production process is in control. Also, it is used to obtain the optimal cost.

2. NOTATIONS AND ASSUMPTIONS:

The following notations are adopted for developing the model:

$C_{hi}(Q_i)$ = Varying holding cost for the i^{th} item of inventory.
C_{oi} = Order cost for the i^{th} item of inventory.
C_{pi} = Purchase (production) cost for the i^{th} item of inventory.
CL = Control limit.
d_i = Annual rate of production for the i^{th} item.
D_i = Uniform demand rate for the i^{th} item of inventory.
K_1 = Limitation on maximum inventory space (Storage limitation).
K_2 = Limitation on the total setup cost.
LCL = Lower control limit.
n = Number of different items carried in inventory.
Q_i = Order (production) quantity batch (decision variable).
R_r = Subgroup ranges.
$\overline{R}$ = Average of the subgroup ranges (CL).
S_i = Storage space required per unit of inventory.

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S_R^2 = Variance of the subgroup ranges.

$TC(Q_i)$ = Annual total cost, $i = 1, 2, 3, \cdots n$.

UCL = Upper control limit.

In addition, The following assumptions are made for developing the model:

1. Demand rate is uniform over time.

2. Shortages are not allowed.

3. Time horizon is infinite.

4. Holding cost $C_{hi}(Q_i) = \alpha Q_i^{\beta}$, $i = 1, 2, 3, \cdots n$, $\alpha > 0$, $0 \le \beta < 1$ for i^{th} item is increasing continuous function of the production quantity Q_i. Where α **and** β are real constants selected to provide the best fit of the estimated cost function.

5. Production rate for each product is finite and constant.

6. Minimize annual relevant total cost and quality control (QC) are our objective.

3. MATHEMATICAL MODEL:

The annual relevant total cost (sum of production, setup and inventory carrying costs) which, according to the basic notations and assumptions of the inventory lot-size model, is:

$$TC(Q_i) = \sum_{i=1}^{n} \left[D_i C_{pi} + \frac{D_i}{Q_i} C_{oi} + \frac{1}{2}(1 - \frac{D_i}{d_i}) Q_i C_{hi}(Q_i) \right] \tag{1}$$

Substituting $C_{hi}(Q_i)$ into (1) yields:

$$TC(Q_i) = \sum_{i=1}^{n} D_i C_{pi} + \sum_{i=1}^{n} \left[D_i Q_i^{-1} C_{oi} + \frac{\alpha}{2}(1 - \frac{D_i}{d_i}) Q_i^{1+\beta} \right] \tag{2}$$

The constraint set can be stated as:

$$\sum_{i=1}^{n} S_i Q_i \le K_1 \quad \text{and} \quad \sum_{i=1}^{n} \frac{D_i}{Q_i} C_{oi} \le K_2 \quad \text{and} \quad Q_i > 0 \text{ for all } i \tag{3}$$

Where K_1 and K_2 set limits on maximum inventory space and total setup cost respectively.

The term $\sum_{i=1}^{n} D_i C_{pi}$ is constant and hence can be ignored.

To solve this primal function which is a convex programming problem, we can write it in the following simplified version of equation (2):

$$\min TC = \sum_{i=1}^{n} \left[D_i Q_i^{-1} C_{oi} + \frac{1}{2} \alpha D_i' Q_i^{\beta+1} \right] , \quad D_i' = \left(1 - \frac{D_i}{d_i} \right) \tag{4}$$

Subject to:

$$\sum_{i=1}^{n} \frac{Q_i S_i}{K_1} \le 1 \quad \text{and} \quad \sum_{i=1}^{n} \frac{D_i C_{oi}}{K_2 Q_i} \le 1 \tag{5}$$

Applying the geometric programming technique to relations (4) and (5), the enlarged predual function could be written as:

$$G(\underline{W})=\prod_{i=1}^{n}\left(\frac{D_iC_{oi}}{Q_iW_{1i}}\right)^{W_{1i}}\left(\frac{\alpha D_i' Q_i^{\beta+1}}{2W_{2i}}\right)^{W_{2i}}\left(\frac{Q_i S_i}{K_1W_{3i}}\right)^{W_{3i}}\left(\frac{D_i C_{oi}}{K_2 Q_i W_{4i}}\right)^{W_{4i}}$$

$$=\prod_{i=1}^{n}\left(\frac{D_iC_{oi}}{W_{1i}}\right)^{W_{1i}}\left(\frac{\alpha D_i'}{2W_{2i}}\right)^{W_{2i}}\left(\frac{S_i}{K_1W_{3i}}\right)^{W_{3i}}\left(\frac{D_i C_{oi}}{K_2 W_{4i}}\right)^{W_{4i}}\times \times Q_i^{-W_{1i}+(\beta+1)W_{2i}+W_{3i}-W_{4i}} \tag{6}$$

where the dual variable vector $\underline{W}=W_{ji}$, $0<W_{ji}<1$, $j=1,2,3,4$, $i=1,2,3,\ldots,n$ is arbitrary and can be chosen according to convenience subject to the normality condition:

$$W_{1i}+W_{2i}=1 \tag{7}$$

We choose $\underline{W}$ such that the exponent of Q_i is zero, thus making the right hand side of (6) independent of the decision variable. To do this we require:

$$-W_{1i}+(\beta+1)W_{2i}+W_{3i}-W_{4i}=0 \tag{8}$$

This is called the orthogonality condition, which together with (7) are two linear equations in four unknowns having infinite number of solutions. However the problem is to select the optimal solution of the weights W_{ji}^*.
Solving equations (7) and (8), we get:

$$W_{1i}=\frac{1}{\beta+2}(\beta+1+W_{3i}-W_{4i}) \text{ and } W_{2i}=\frac{1}{\beta+2}(1-W_{3i}+W_{4i}) \tag{9}$$

Substituting W_{1i} and W_{2i} in equation (6), then the dual function is given by:

$$g(W_{3i},W_{4i})=\prod_{i=1}^{n}\left(\frac{(\beta+2)D_iC_{oi}}{\beta+1+W_{3i}-W_{4i}}\right)^{\frac{1}{\beta+2}(\beta+1+W_{3i}-W_{4i})}\left(\frac{(\beta+2)\alpha D_i'}{2(1-W_{3i}+W_{4i})}\right)^{\frac{1}{\beta+2}(1-W_{3i}+W_{4i})}\times\left(\frac{S_i}{K_1W_{3i}}\right)^{W_{3i}}\left(\frac{D_iC_{oi}}{K_2W_{4i}}\right)^{W_{4i}} \tag{10}$$

To find W_{3i} and W_{4i} which maximize $g(W_{3i},W_{4i})$, the logarithm of both sides of (10), and the partial derivatives were taken with respect to W_{3i} and W_{4i}, respectively. Setting each of them to equal zero and simplifying, we get:

$$\left(\frac{S_i}{K_1e}\right)\left(\frac{2D_iC_{oi}}{\alpha D_i'}\right)^{\frac{1}{\beta+2}}\left(\frac{1-W_{3i}+W_{4i}}{\beta+1+W_{3i}-W_{4i}}\right)^{\frac{1}{\beta+2}}\frac{1}{W_{3i}}=1 \tag{11}$$

and

$$\left(\frac{D_iC_{oi}}{K_2e}\right)\left(\frac{\alpha D_i'}{2D_iC_{oi}}\right)^{\frac{1}{\beta+2}}\left(\frac{\beta+1+W_{3i}-W_{4i}}{1-W_{3i}+W_{4i}}\right)^{\frac{1}{\beta+2}}\frac{1}{W_{4i}}=1 \tag{12}$$

Multiplying relation (11) by relation (12), we have:

$$W_{3i}W_{4i}=\frac{S_iD_iC_{oi}}{K_1K_2e^2}$$

Substituting W_{4i} and W_{3i} into relations (11) and (12), respectively, we get:

$$f(W_{3i}) = \left(W_{3i}^2 + (\beta+1)W_{3i} - A_i\right)W_{3i}^{\beta+2} + B_{1i}\left(W_{3i}^2 - W_{3i} - A_i\right) = 0 \quad (13)$$

And

$$f(W_{4i}) = \left(W_{4i}^2 + W_{4i} - A_i\right)W_{4i}^{\beta+2} + B_{2i}\left(W_{4i}^2 - (\beta+1)W_{4i} - A_i\right) = 0 \quad (14)$$

where:

$$A_i = \frac{S_i D_i C_{oi}}{K_1 K_2 e^2}, B_{1i} = \left(\frac{S_i}{K_1 e}\right)^{\beta+2}\left(\frac{2D_i C_{oi}}{\alpha D_i'}\right) \text{ and } B_{2i} = \left(\frac{D_i C_{oi}}{K_2 e}\right)^{\beta+2}\left(\frac{\alpha D_i'}{2D_i C_{oi}}\right)^2$$

Referring to the left hand side of relations (13) and (14) as $f(W_{ji})$, $j=3,4$, respectively. It could be easily proved that $f(0) < 0$ and $f(1) > 0$, this means that there exists a root $W_{ji} \in (0,1)$, j = 3 , 4. The trial and error approach can be used to calculate these roots. However, we shall first verify any root W_{ji}^*, j = 3, 4 calculated from equations (13) and (14) maximize $g(W_{ji}^*)$, $j = 3,4$, respectively. This is confirmed by the second derivative to $\ln g(W_{3i}, W_{4i})$ with respect to W_{3i} and W_{4i}, respectively, which are always negative.

Thus, the roots W_{3i}^* and W_{4i}^* calculated from equations (13) and (14) maximize the dual function $g(W_{3i}, W_{4i})$. Hence the optimal solution is W_{ji}^*, $j = 1,2,3,4$, where W_{3i}^*, W_{4i}^* are the solutions of (13), (14) and W_{1i}^*, W_{2i}^* are calculated by substituting the values of W_{3i}^* and W_{4i}^* in (9).

To find the optimal economic production run size Q_i^*, we apply Duffin and Peterson's theorem [3] of geometric programming as:

$$\frac{D_i C_{oi}}{Q_i^*} = W_{1i}^* \, g(W_{3i}^*, W_{4i}^*)$$

and

$$\frac{\alpha D_i' Q_i^{*(\beta+1)}}{2} = W_{2i}^* \, g(W_{3i}^*, W_{4i}^*)$$

Solving these relations, the optimal production run size is:

$$Q_i^* = \left(\frac{2D_i C_{oi}}{\alpha D_i'} \frac{W_{2i}^*}{W_{1i}^*}\right)^{\frac{1}{\beta+2}}, \quad i = 1,2,3, \dots ,n \quad (15)$$

Substituting the values of Q_i^* in (4) after adding the ignored terms, we get

$$\min \ TC = \sum_{i=1}^{n}\left[C_{pi} D_i + \left(\frac{1}{2}\alpha D_i^{\beta+1} D_i' C_{oi}^{\beta+1}\right)^{\frac{1}{\beta+2}}\left\{\left(\frac{W_{1i}^*}{W_{2i}^*}\right)^{\frac{1}{\beta+2}} + \left(\frac{W_{2i}^*}{W_{1i}^*}\right)^{\frac{\beta+1}{\beta+2}}\right\}\right] \quad (16)$$

As a special case, we assume $K_{1,2} \to \infty$, $\beta = 0 \Rightarrow (W_{3i,4i}^* \to 0,\ W_{1i,2i}^* = \frac{1}{2})$, and $C_{hi}(Q_i) = \text{constant}$. This is the classical economic production run size model.

4. STATISTICAL QUALITY CONTROL:

We shall compute the decision variable Q_i^* whose values are to be determined to minimize the annual relevant total cost and used to confirm that the production process is in control for three items (n = 3) and different values of β. The parameters of the model are shown in TABLE 1:

i	D_i	d_i	C_{oi}	C_{pi}	S_i	α_i
1	100 Units	300 Units	$ 200	$ 10	2.0	$ 1
2	070 Units	200 Units	$ 140	$ 08	1.5	$ 1
3	040 Units	100 Units	$ 100	$ 05	1.0	$ 1

TABLE 1

Assume the total available storage area and the total setup costs are given by $K_1 = 1200\ ft^2$ and $K_2 = \$\ 1000$, respectively.

It follows that the optimal values of production batch quantity Q_i^*, minimum total cost and subgroup ranges are given in TABLE 2 for each values of β:

β	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Q_1^*	170	134	107	087	072	060	051	044	039
Q_2^*	126	099	079	064	054	045	039	034	030
Q_3^*	086	069	055	046	038	033	028	025	022
Min TC	2203	2314	2439	2577	2727	2890	3064	3249	3445
R_r	084	065	052	041	034	027	023	019	017

TABLE 2

Applying control limits (CL) method when σ is unknown as:

The average of the subgroup ranges (CL) is:

$$\overline{R} = \frac{\sum_{r=1}^{9} R_r}{9} = \frac{362}{9} = 40.22$$

And the standard deviation of the subgroup ranges is:

$$S_R = \sqrt{\frac{\sum_{r=1}^{9} \left(R_r - \overline{R}\right)^2}{9}} = \sqrt{\frac{4117.36}{9}} = 21.39$$

The lower control limit is $LCL = \overline{R} - 3S_R = 0$, the average of the subgroup ranges is $\overline{R} = 40.22$ and the upper control limit is $UCL = \overline{R} + 3S_R = 104.39$. It is clear that $LCL < \overline{R} < UCL$. Therefore the production process is in control.

CONCLUSION:

This work presents statistical quality control for multi-item inventory model that considers order quantity as decision variable. An analytical solution of the production lot-size model with varying holding cost and two restrictions is derived using geometric programming approach. Finally, we used the optimal order quantity of each of the 3 items and subgroup ranges method to investigate quality control of the production process.

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