International Journal of Mathematical Archive-4(2), 2013, 314-327
IJMA Available online through www.ijma.info ISSN 2229 – 5046

GRAPHS RELATED TO KNOTS AND LINKS GENERATING BY MOVES CHESS

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(Received on: 29-03-12; Revised & Accepted on: 16-05-12)

ABSTRACT

In this paper we will clarify the analysis and movement of different chess pieces to apply the chess game on knot theory.

INTRODUCTION

Chess is a game with deep historical origins dating back as for as the 6th century AD. Most evidence suggests the first version of the game emerged in India under the name of Chaturanga. From India the Game quickly spread to Persia and subsequently the Arab world. Chess eventually spread to Europe in the 10th century AD because of increasing trade ties between Europe and the Arab world. Around the same time chess reached Japan and other Eastern nations. Many scholars profess that another variation of chess many have excited in China as far back as the 2nd century B C, although the game is thought to have differed widely from the game you are about to play. Until the 15th century AD several of the pieces, most notably the Queen and Bishop, were significantly less mobile than their modern counter parts. Around the end of the 15th century many of the chessmen developed their modern powers, enhancing the speed and dynamics of the game and creating the game we play today. At the beginning of each game the chessboard is set up in the initial position. Each player begins with 8 pieces (1 King, 1 Queen, 2 Rooks, 2 Bishops and 2 Knights) and 8 pawns which are located in their respective positions.

Knot theory deals with *knots, links, braids,* and related objects. It is a branch of algebraic topology which started already in the late 1880's. Most Scientists believed that the universe was per. Lord Kelvin (William Thomson, 1824-1907) proposed that different elements consists of different configurations of knots. He described atoms to be knots in the fabric of this ether.

This theory led many Scientists to believe that could understand the chemical elements by simply studying different types of knots and thus this led to a completely new field of study in Math.

Around 1980 the Scientists outside mathematics became interested in knot theory again, as it proved quite helpful in the study of DNA. [4]

One of the most important applications of the knot theory arises in study of DNA. It was discovered in 1953 by James Watson and Francis Crick that the basic structure of DNA molecules consists of two stands which are twisted together in a right-hounded helix. The geometric of DNA can exist in linear form or in closed circular form. Science 1990 one knows that the circular form of DNA can be knotted and two or more circular DNA forms can be linked together.

Another application of Knot theory molecular chemistry, particle physics [7], [8], statistical mechanics [6], Islamic and Pharaohs Arts[1], and more. [9], [10], [11].

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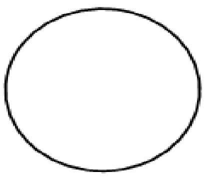
BASIC DEFINITIONS AND FACTS

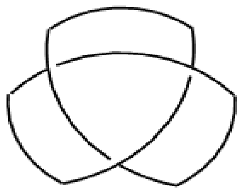
Definition 1: A knot K is a subspace of the three dimensional Euclidean space R^3, which is a topological image of the circleS^1 (Topological embedded circle in R^3) embedding K: $S^1 \rightarrow R^3$ of the circle S^1 into three dimensional Euclidean space R^3.

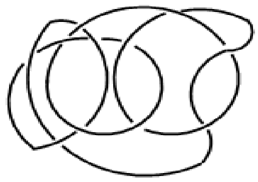
Some authors use a slightly more general definition:

A knot is an embedding k: $S^1 \rightarrow S^3$. Here S^3 is to be considered as the one- point compactification of R^3: $S^1 = R^3 \cup \infty$. Then a knot may close at infinity ∞.

Examples





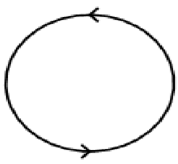


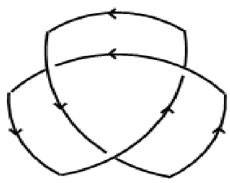
circle S^1 = unknotleft trefoil some more complicated knot

A natural generalization of knot theory is the study of embedding $S^k \rightarrow R^{k+n}$ or $S^k \rightarrow S^{n+k}$. We shall not consider here such higher- dimensional problems.

A knot can be taken with or without orientation. In the following, we mainly consider oriented knots.

Examples:

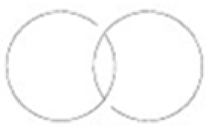


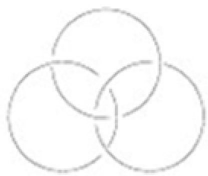


OrientedS^1oriented trefoil

Definition 2: A link L is an embedding of a topological sum of finitely many copies of a circle S^1 into three dimensional topological space R^3, L :$S^1 \sqcup S^1 \sqcup S^1 ... \sqcup S^1 \rightarrow R^3$.Therestriction of L to one of the copies of S^1 is called a *component* of L.

Examples:





n- unlink $= S^1 \sqcup \cdots \sqcup S^1$ 2- chain Borromean rings

Note that each component of a link is a knot.

Definition 3: A diagram of a knot or link is a projection of the latter into a plane with marking of each crossing (under-crossing or over crossing) in the image of the projection. That means a knot diagram is a picture of a projection of a knot onto a plane, a diagram in R^2 is made up of a number of arcs and crossings. At a crossing one arc is the over pass and the other to make up an under pass. and it not allowed the following :

Definition 4: Graph of a diagram of a knot or link is the graph consisting of the Crossings as the vertices of the graphs and the arcs between two crossings as The edges. With other words:

A graphia Γ is a link-graph (knot-graph) if there is a link L (a knot K)such that a suitable projection of L (of K) give Γ . We write $\Gamma(L)$ resp. $\Gamma(K)$.

Definition 5: Two graphs G and H are said to be isomorphic (written $G \sim H$) if there are bijections θ: V (G) $\rightarrow$ V (H) and ϕ : E(G) $\rightarrow$ E(H) such that ψG (e) = uv if and only if ψ_H (ϕ(e))= $\theta(u)\theta(v)$; such a pair (θ, ϕ)of mappings is called an isomorphism between G and H. [2]

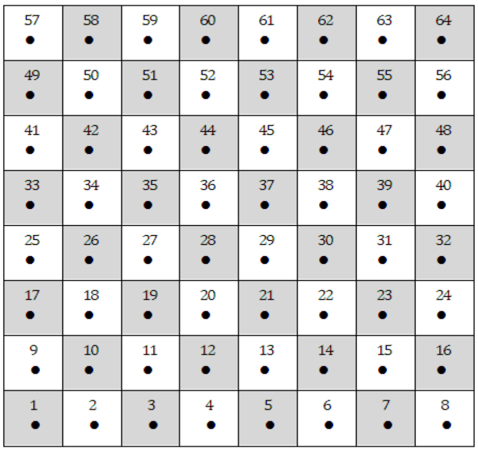
Definition 6: A Knot K_0 is equivalent to a knot K_n if there exists a sequence of knots $K_1,K_2,...,K_{n-1}$ such that K_i is an elementary deformation of K_{i-1} for $1 < i \leq n$.

Definition 7: Prime Knot [17] Primeknot is a knot is in a certain sense, indecomposable. Specifically, it is a non-trivial knot which cannot be written as the knot sum of two non-trivial knots. Knots that are not prime are said to be composite. It can be a nontrivial problem to determine whether a given knot is prime or not. A nice family of examples prime knots are the torus knots. These are formed by wrapping a circle around a tours p time in one direction and q times in the other, where p and q are co prime integers. The simplest prime knot is the trefoil with three crossing. Thetrefoil is actually a (3, 2) tours knot. The figure eight knot with four crossing is the simplest non-tours knot. Any knot with bridge number equal to 2 is a prime knot.

Definition 8: Prime Link [15], [16] A prime link is a link that cannot be represented as a knot sum of other links. Doll and Hoste (1991) list polynomials for oriented links of nine or fewer crossings, and Rolfsen (1976) gives a table of links with small numbers of components and crossings.

Definition 9: Alternating knot An alternating knot is a knot which possesses a knot diagram in which crossings alternating between under-and overpasses. Not all knot diagram of alternating knots need be alternating diagram. The trefoil knot and figure-eight knot are alternating knots, as are all prime knots with seven or fewer crossings.

Chessboard consists of 64 squares. Thirty two of them are black squares and 32 are white squares. On this chessboard, there are 16 white pieces and 16 black ones. These pieces are arranged on the chessboard with a specific way: the square from 1 to 8. Chess pieces are put twice from right to left as follows: Rook, Knight, Bishop, Queen, King, Bishop, Knight, and Rook, in front of all of them. Do the same in the other side.



Change of chess pieces leads to the change of the movement of each piece as known. We will clarify the analysis and movement of different chess pieces as follows:

1. The King

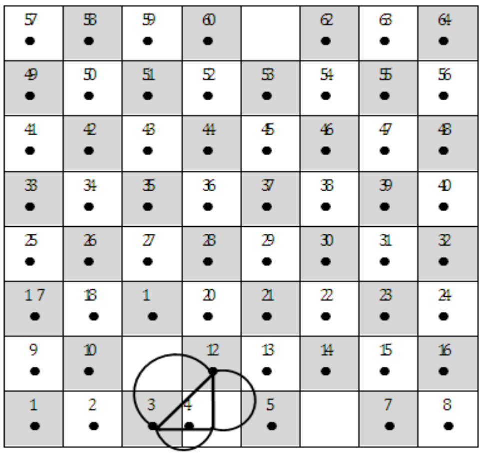
The King is the most valuable chess piece. The entire objective of the game is to capture the opposition's King. When a King is threatened by another chessmen it is said to be in check and when a King has no moves to escape check it is said to be in checkmate. If a player's King is Checkmated he has been defeated. For this reason it is important you always guard your King. In the early stages of the game (the opening) you should focus on finding safe shelter for your king which is usually along the side of the board. Mastering the King's movements is relatively easy. Your King can move one square in any direction (horizontally, vertically, or diagonally).

2. The Bishop

The Bishop moves in diagonal lines, forward and back, over any number of unoccupied squares. Each player starts with two Bishops, located on either side of the King and Queen. There are no restrictions on the distance for each move, but it is limited to diagonal movement in any direction. Since Bishops are restricted to diagonal movement, each Bishop will always remain on squares which are the same color as the one it started on, because of this many refer to them as light-squared and dark-squared Bishop.

3. The Rook

The Rook moves in a simple linear fashion. Each player starts with two Rooks, one in each corner of the first file closest to them. The Rook moves horizontally or vertically, forward or back, over any number of unoccupied squares as shown in the diagram. The Rook is the second most powerful piece and is referred to as a major piece. If you have the opportunity to take a Rook at the cost of your Bishop or Knight, you will usually do so since the Rook is a more powerful piece. If you do this you are said to have won the exchange.



4. The Knight

The Knight, colloquially known as the horse, has one of the most unusual movements in chess making it a challenging piece to master but a powerful piece once mastered. Each player starts with two Knights on their first rank beside their Rooks. The Knight moves in an L-shape, meaning two squares along a rank or file and then one square to the side. The manner in which pieces moves has changed drastically over the years, yet the Knights movement has remained unchanged since the seventh century AD.

5. The Queen

The Queen is the most powerful chess piece. Each player starts the game with one Queen, placed in the middle of their first rank next to their King. The Queen has the most flexibility in movement out of any piece which is precisely why it is the most powerful piece in the game.

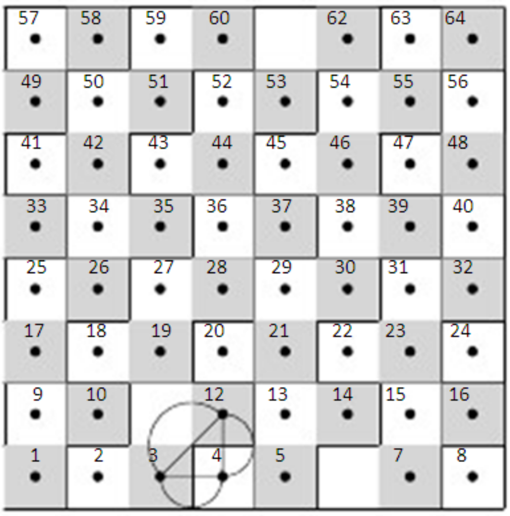
The Queen has various situations where it is most effective. Generally the Queen is most powerful during the middle game or end game. The Queen is most powerful at these stages because the board is open, the enemy King is less defended and your opponents will have more loose pieces. in a straight line vertically, or horizontally, or diagonally, covering as many unoccupied squares.

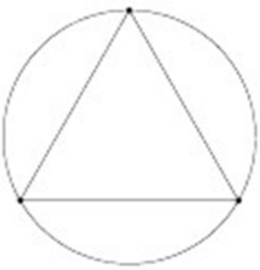
The Queen can move.

2. MAIN RESULTS

The movement of the King.

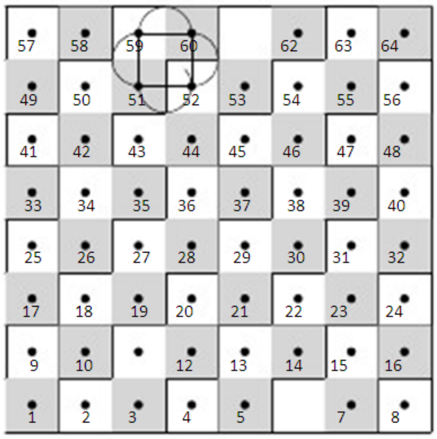
The King moves from Square 4 to 12, then 3 and back to 4. By repeating these movements we will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this

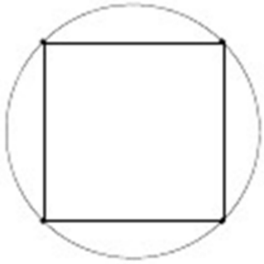




This graphs corresponds to knot 3_1

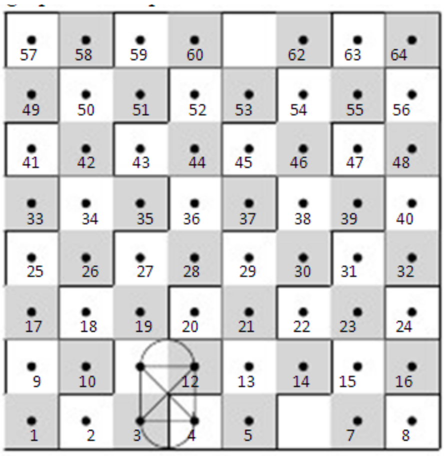
The King moves from square 60 to 52, then 51, then 59 and back to 60. By repeating these movements we will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this

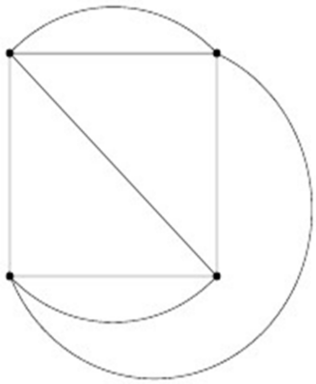




This graphs corresponds to knot 4_1^2

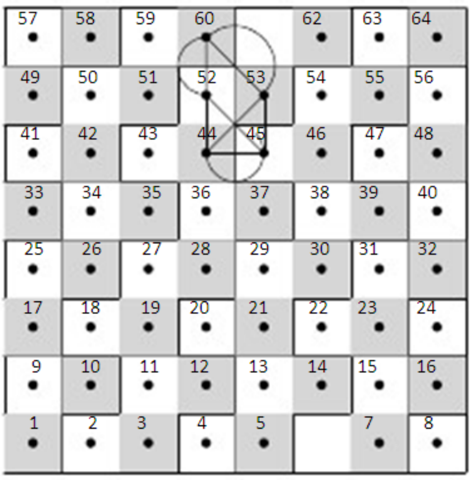
The King moves from square 4 to 12, then 11, and back to 12, then 4, then 11, then 3 and 4. By repeating these movements we will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this





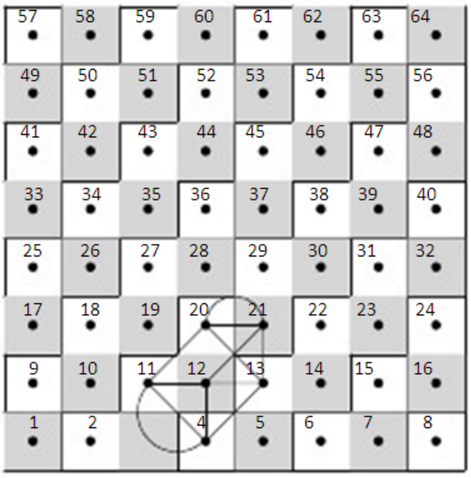
This graphs corresponds to knot 4_1

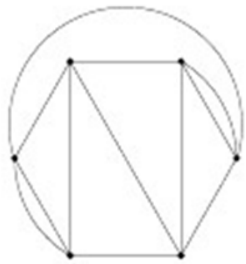
The King moves from square 60 to 53, then 44, then 45, then 53 and back to 60, then 52, then 45, then 44, then 52 and 60. By repeating these movements we will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this



This graphs corresponds to knot 5_2

The King moves from Square 4 to 12, then 13, then 21, then 20, then13 and back To 4, then 11, then 20, then 21, then 12, then 11, and 4. By repeating these movements we will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this





This graphs corresponds to knot 6_3

Theorem 1: Link graph of n g on, $n \geq 3$ can be obtain from the movements of the King.

Proof: From the movement of the king we can obtain the closed graph after n moves, which $\geq$ n3,and back to all moves we obtain Link graph of n g on, $n \geq 3$.

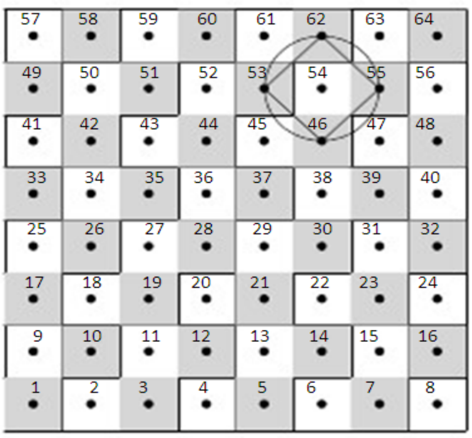
Theorem 2: Link graph without multiple $\left(6_1^3, 8_{18}, \ldots\right)$ can't be obtain from the movements of the King.

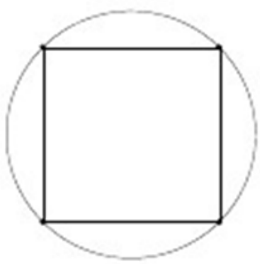
Proof: We find that the King moves in one square only whether horizontallor vertically or with the angle 45°. One of the characteristics of the link graph that is opposite to the kont is that deg(v) = 4 .Consequently, there is a number of (n - 4) of the heads that cannot be connected. In order to connected them , the King must move in two squares or more, and this contradicts the movement of the King.

2. Movement of Bishop and Rook:

Movement of the partner's Bishop

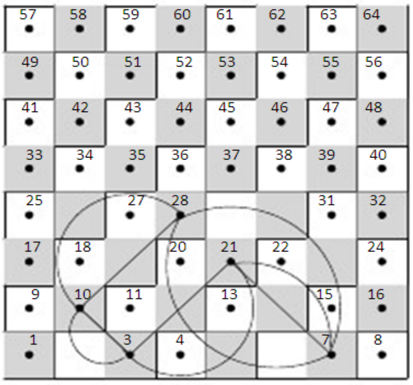
The Bishop moves from square 62 to 53, then 46, then 55 and back to 62. By repeating these movements we will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this

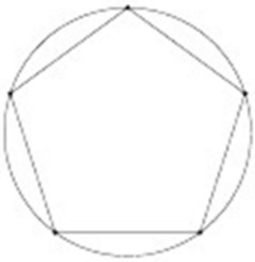




This graphs corresponds to knot 4_1^2

The Bishop moves from square 7 to 28, then 10, then 3, then 21 and back to7. By repeating these movements we will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this

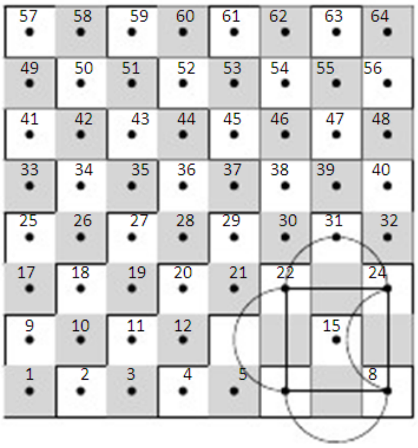


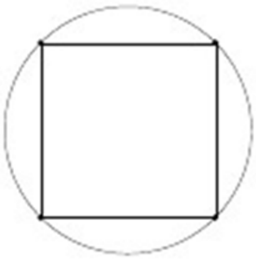


This graphs corresponds to knot 5_1

Movement of the partner's Rook:

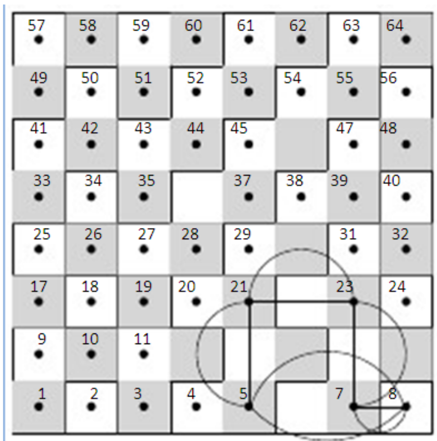
The Rook moves from square 8 to 6, then 22, then 24, and back to 8. By repeating these movements we will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this

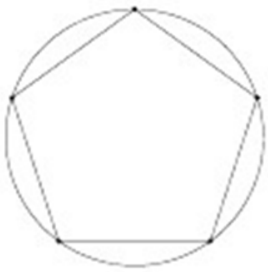




This graphs corresponds to knot 4_1^2

The Rook moves from square 8 to 7, then 23, then21, then 5, and back to 8. By repeating these movements we will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this





This graphs corresponding to knot 5_1

Theorem 3: We can obtain only *n-gon*, n ≥ 4 graphs corresponds to *n-gon* knot from the movement of Bishop and Rook.

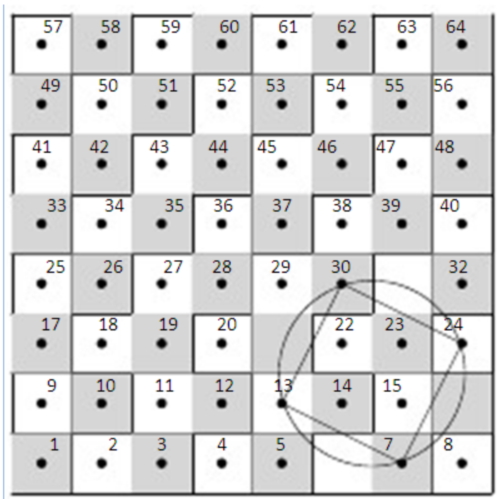
Proof: As for the movement of Bishop, it moves with the angle 45°. It moves from a black square to another. In order to get a closed curve, it is necessary to have the number of edge equal to 4 because it does not move horizontally or vertically.

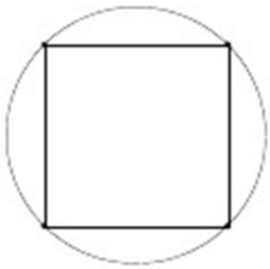
As for the movement of Rook, it moves horizontally or vertically because it goes from a square to another. We can only get a closed curve after four movements because it does not move with the angle 45°.

3- The Knight's movement:

It is known that the Knight moves on the form L. This means that if the Knight is in the white square to move on the form of letter L to become in the black square. This means that by the horse's movements we will get several black and white squares. Some of its movements were analyzed as follows:

The Knight moves from square 7 to 24, then 30, then 13, and back to 7. By repeating these movements we will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this





This graphs corresponds to knot 4^2_1

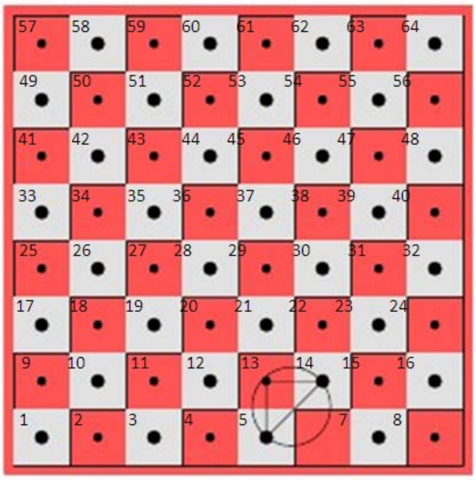
Theorem 4: We can obtain *2n-gon*, n ≥ 2 graphs corresponds to *2n-gon* knot from the movement of Knight.

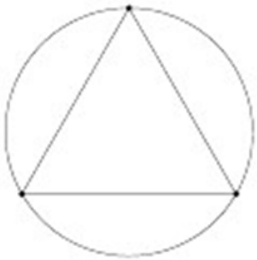
Proof: The Knight moves in an L- shape, when the Knight moves from black (white) to another black(white) square after 2 moves. In order to obtain a closed curve we must 2n moves. By this we can get a link graphs with 2n- gon, n ≥ 2.

4- The Queen's movement:

It is known that the Queen has the movements of all chess pieces except that of the Knight. Therefore if we analysis its movements we will get some graphs corresponding to knots as follows:

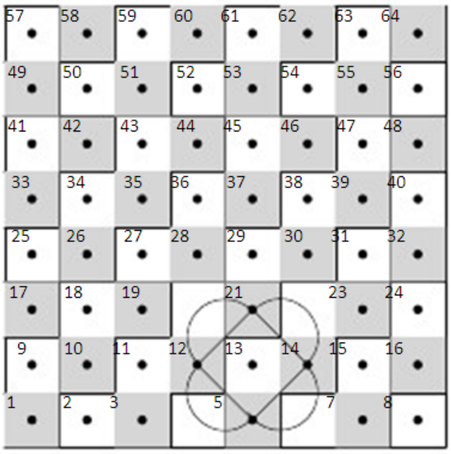
The Queen moves from Square 5 to14, then 13 and back to 5. By repeating these movements we will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this

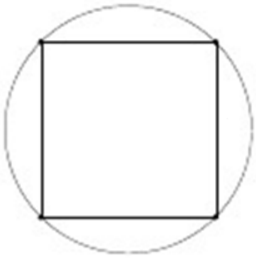




This graphs corresponds to knot 3_1

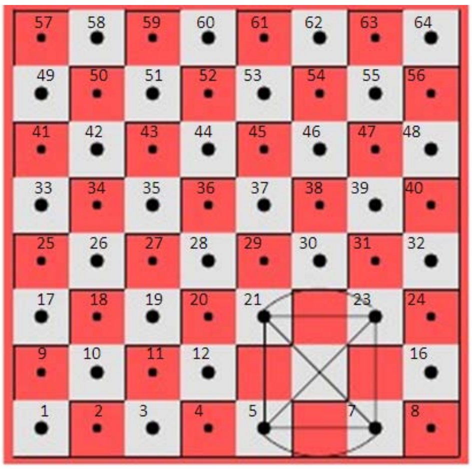
The Queen moves from square 5 to 14, then 21, then 12, and back to 5. By repeating these movements we will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this

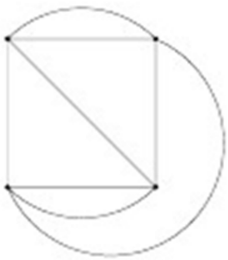




This graphs corresponds to knot 4_1^2

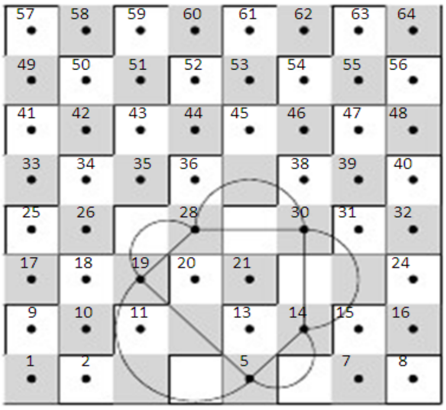
The Queen moves from square 5 to 7, then 21, then 23, then 7, and back to 5 then 23, then 21, and back to 5. We will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this





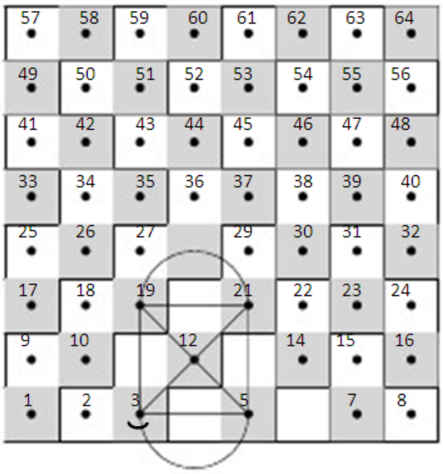
This graphs corresponds to knot 4_1

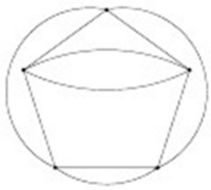
The Queen moves from square 5 to 14, then 30, then 28, then 19, and back to 5, By repeating these movements. We will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this



This graphs corresponds to knot 5_1

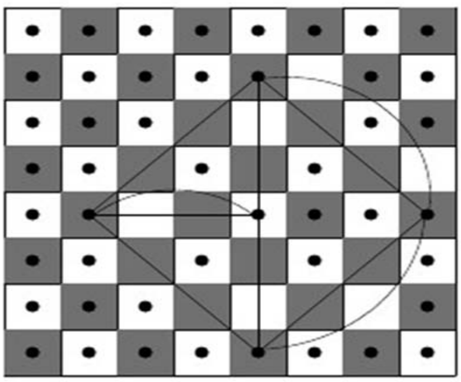
The Queen moves from square 5 to 12, then 21, then 19, then 21 and back to 5, then 3, and 5. We will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this





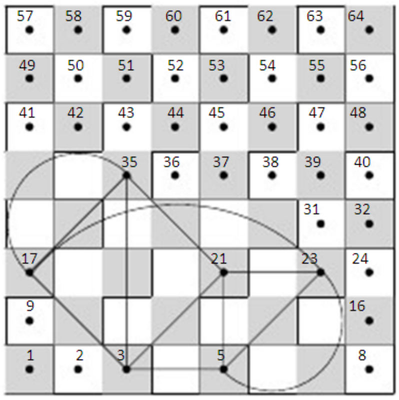
This graphs corresponds to knot 5_1^2

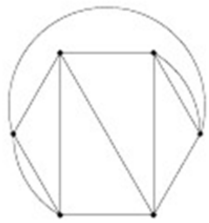
The Queen moves from Square 5 to 32, then 53, and back to 32, then 5, then 29, and to back 53, then 26, and back to 5. We will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this



This graphs corresponds to knot 5_2

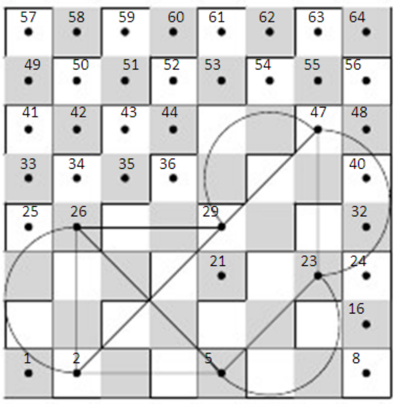
The Queen moves from square 5 to 21, then 23, and back to 5, then 3, then 35, and to back 21, then 3, and then 17. Then 35, and back to 17, then 35. We will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this

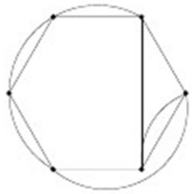




This graphs corresponds to knot 6_3

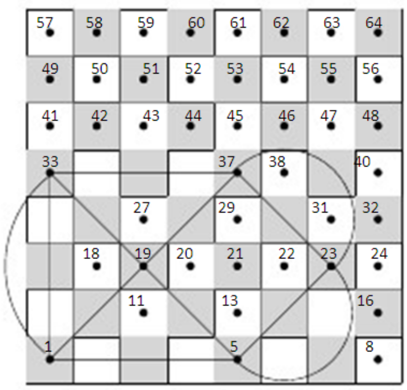
The Queen moves from square 5 to 23 ,then 47, then 29, and back To 47, then 23, then 26, then 29, and to back 26,then 2, and then 5. We will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this

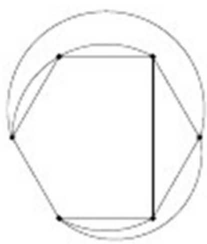




This graphs corresponds to knot 6_1

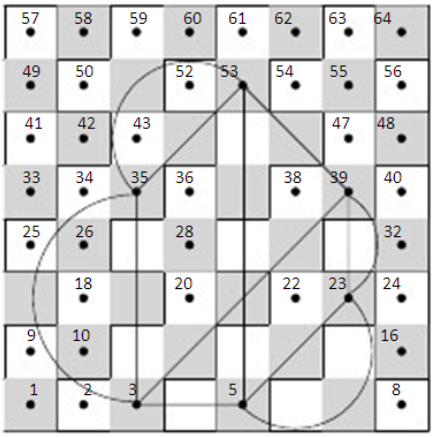
The Queen moves from square 5 to 23, then 37 and to back 23, then 5, then 19 and back to 37, then 33, then 1, and to back 33, then 1, and 5. We will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this

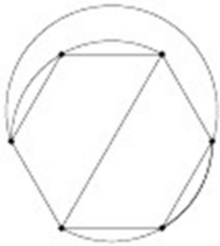




This graphs corresponds to knot 6_2

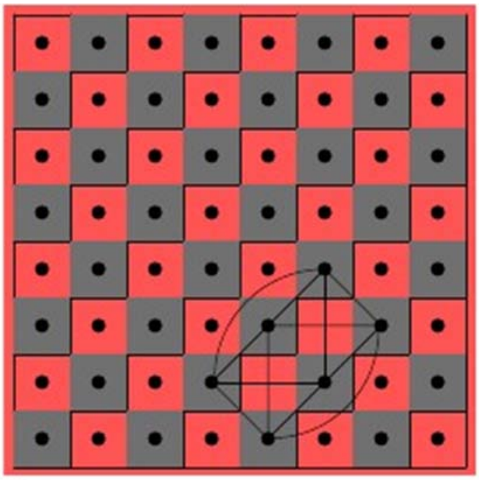
The Queen moves from square 5 to 23, then 39, and to back 23, then 5, then 3, then 35, then 53, and 5. We will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this

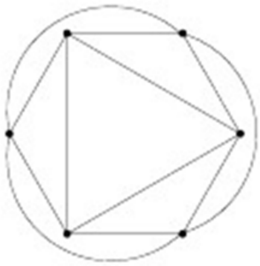




This graphs corresponds to knot 6_2^2

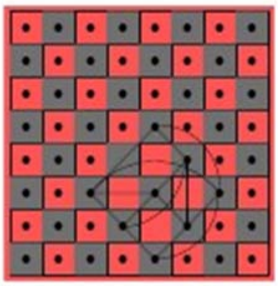
The Queen moves from square 5 to 14, then 30, then 23, then 21, then 12, and back then 30, then 21, then 5, then 12, then 14, then 23, and back to 5. We will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this

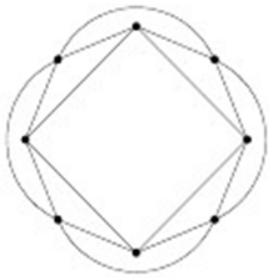




This graphs corresponds to knot 6_1^3

The Queen moves from square 5 to 23, then 37, then 19, then 12, then 30, then 21, then 14 and to back 5, then 37, then 6, then14, then 23, then 19, then 21, then 12, and back to 5.We will have this graph which has an isomorphic representation with a cyclic arrangement of the vertices like this





This graphs corresponds to knot 8_{18}

Theorem 5: The Queen's movement represents the all of graphs corresponds to knots.

Proof: According to the movement of Queen, we find that it makes all the movements of the chessboard. As for the movement of Knight, we can use the movement of Queen for it. This occurs by two movements, one horizontally and another vertically. By this we can get a Link graph for all knots.

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Source of support: Nil, Conflict of interest: None Declared