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ON TEXTURE α-SEPARATION AXIOMS IN DITOPOLOGICAL TEXTURE SPACES

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ABSTRACT

The content of this paper is to study the basic separation axioms in α open sets under the Ditopological texture setting. Here we also analyse the relationships between them and discuss their characterizations of these separation axioms.

Keywords: *Texture spaces, Ditopology, Ditopological Texture spaces, Texture $\alpha -T_0$ space, Texture $\alpha -T_1$ space, Texture $\alpha -T_2$ space.*

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1. INTRODUCTION

L. M. Brown [2] laid foundation to the notion of Textures which was initially called as fuzzy structures as a point-set for the study of fuzzy sets in 1998. Here in textures, it offers a convenient setting for the investigation of complement-free concepts in general. Extensive research has been done on texture using generalized form of sets by many authors. [4, 5, 15].

In this paper we present some classes of new spaces namely the T_α-T_0, T_α-T_1, T_α-T_2 spaces in dichotomous topologies or ditopology. Let S be a set, a texturing T [2] of S is a subset of P(S). If

(1) (T, ⊆) is a complete lattice containing S and φ, and the meet and join operations in (T, ⊆) are related with the intersection and union operations in (P(S), ⊆) by the equalities $\wedge_{\{i\in I\}} A_i = \cap_{\{i\in I\}} A_i$, $A_i \in T$, $I \in \mathbf{I}$, for all index sets I, while $\vee_{\{i\in I\}} A_i = \cup_{\{i\in I\}} A_i$, $A_i \in T$, $i \in \mathbf{I}$, for all finite index sets I.

(2) T is completely distributive.

(3) T separates the points of S. That is, given $s_1 \neq s_2$ in S we have $A \in T$ with $s_1 \in A, s_2 \notin A$, or $A \in T$ with $s_2 \in A, s_1 \notin A$.

If S is textured by T we call (S, T) a texture space or simply a texture.

For a texture (S; T), most properties are conveniently defined in terms of the p-sets $P_S = \cap\{A \in T \backslash s \in A\}$ and the q-sets, $Q_S = \vee\{A \in T/s \notin A\}$ The following are some basic examples of textures.

Examples 1.1: Some examples of texture spaces,

(1) If X is a set and P(X) the power set of X, then (X; P(X)) is the discrete texture on X. For $x \in X$, $P_X = \{x\}$ and $Q_X = X\backslash\{x\}$.

(2) Setting $I = [0; 1]$, $T = \{[0; r); [0; r]/r \in I\}$ gives the unit interval texture (I; T). For $r \in I$, $P_r = [0; r]$ and $Q_r = [0; r)$.

(3) $T = \varphi, \{a, b\}, \{b\}, \{b, c\}, S\}$ is a simple texturing of $S = \{a, b, c\}$ $P_a = \{a, b\}$, $P_b = \{b\}$ and $P_c = b, c\}$.

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Definition1.2. [4] The texture (S, T) is called coseparated if $Q_s \subset Q_t \Rightarrow P_s \subseteq P_t$ for all s, t $\in$ S.

Since a texturing T need not be closed under the operation of taking the set complement, the notion of topology is replaced by that of dichotomous topology or ditopology, namely a pair (τ, κ) of subsets of T, where the set of open sets τ satisfies
1. S, $\varphi \in \tau$,
2. $G_1; G_2 \in \tau$ then $G_1 \cap G_2 \in \tau$ and
3. $G_i \in \tau$, $i \in I$ then $\vee_i G_i \in \tau$,

and the set of closed sets κ satisfies
1. S, $\varphi \in \kappa$
2. $K_1; K_2 \in \kappa$ then $K_1 \cup K_2 \in \kappa$ and
3. $K_i \in \kappa$, $i \in I$ then $\cap K_i \in \kappa$. Hence a ditopology is essentially a 'topology" for which there is no a priori relation between the open and closed sets.

For A $\in$ T we define the closure [A] or cl(A) and the interior [A] or int(A) under (τ, κ) by the equalities
$[A] = \cap\{K \in \kappa / A \subseteq K\}$ and $]A[= \vee\{G \in \tau / G \subseteq A\}$:

An mapping σ: T → T is said to be complementation on (S, T) if κ = σ(τ), then (S, T, σ, τ, κ) is said to be a complemented ditopological texture space. The ditopology (τ, τ^c) is clearly complemented for the complementation $\pi_X : P(X) \rightarrow P(X)$ given by $\pi_X(Y) = X \backslash Y$.

We denote by O(S; T; τ, κ), or when there can be no confusion by O(S), the set of open sets in S. Likewise, C(S; T; τ, κ) or C(S) will denote the set of closed sets.

Definition1.3. For a ditopological texture space (S; T; τ, κ):
1. A $\in$ T is called α-open (b-open) if A $\subseteq$ intclintA (A$\subseteq$clint(A) $\cup$ intcl(A)). B$\in$T is called α-closed (resp. b-closed) if clintclB $\subseteq$ B (intclB $\cup$clintB $\subseteq$ B)

We denote by αO(S; T; τ, κ) (bO(S; T; τ, κ)), or simply by αO(S) (bO(S)), the set of α-open sets (b-open sets) in S. Likewise, αC(S; T; τ, κ) (bC(S; T; τ, κ)), or αC(S) (bC(S)) will denote the set of α-closed (b-closed sets) sets.

Now using αO(S) and αCO(S) we construct a new α-topology and α-closed topology or α- ditopology, namely a pair (τα, κα) of subsets of T, where the set of α-open sets τα satisfies
1. S, $\varphi \in \tau\alpha$,
2. $G1; G2 \in \tau\alpha$ then $G1 \cap G2 \in \tau\alpha$ and
3. $G_i \in \tau\alpha$, $i \in I$ then $\vee_i G_i \in \tau\alpha$,

and the set of α closed sets κα satisfies
1. S, $\varphi \in \kappa\alpha$
2. $K_1; K_2 \in \kappa\alpha$ then $K_1 \cup K_2 \in \kappa\alpha$ and
3. $K_i \in \kappa\alpha$, $i \in I$ then $\cap K_i \in \kappa\alpha$. Hence a αditopology is essentially a 'topology" for which there is no a priori relation between the α open and α closed sets.

Definition1.4. Let A ditopological texture space (S, T, τ, κ) is said to be
1. $\alpha - R_0$ if $G \in \tau\alpha$, $G \not\subset Q_s \Rightarrow \alpha cl(P_s) \subseteq G$.

2. Co-$\alpha - R_0$ if $F \in \kappa\alpha$, $P_s \not\subset F \Rightarrow F \subseteq \alpha int(Q_s)$.

3. $\alpha - R_1$ if $G \in \tau\alpha$, $G \not\subset Q_s$, $P_t \not\subset G \Rightarrow$ there exists $H \in \tau\alpha$ with $H \not\subset Q_s$, $P_t \not\subset cl(H)$.

4. Co-$\alpha - R_1$, if $F \in \alpha C(S)$, $P_s \not\subset F$, $F \not\subset Q_t \Rightarrow$ there exists $F \in \kappa\alpha$ with $P_s \not\subset K$, $int(K) \not\subset Q_t$.

5. $\alpha -$Regular, if $G \in \tau\alpha, G \not\subset Q_s \Rightarrow$ there exists $H \in \tau\alpha$ with $H \not\subset Q_s$, $cl(H) \subseteq G$.

6. Co-α Regular if $F \in \kappa\alpha$, $P_s \not\subset F \Rightarrow$ there exists $K \in \kappa\alpha$ with $P_s \not\subset K$, $F \subseteq int(K)$.

2. Texture α-separation axioms

Definition 2.1: A ditopological space (S, T, τ, κ) is said to be Texture α-$T_0(T_\alpha$-$T_0)$ if it satisfies the equivalent conditions obtained by setting $\mathcal{A} = (\tau\alpha \cup \kappa\alpha)^{\vee}$ in Theorem.2.2 and $\mathcal{B} = (\tau\alpha \cup \kappa\alpha)^{\cap}$ in Theorem.2.3.

Theorem.2.2. [4] Let $\mathcal{A} \subseteq S$ contains S,φ and be closed under arbitrary joins. Then the following are equivalent.

1. For every $A \in T$, there exists $A_j \in \mathcal{A}$, $j \in J$, with $A = \cap_{j\in J} A_j$.
2. For $s, t \in S$, $P_s \not\subset P_t \Rightarrow$ there exists $A \in \mathcal{A}$ with $P_t \subseteq A$ and $P_s \not\subset A$.
3. For $s,t \in S$, $P_s \not\subset P_t \Rightarrow$ there exists $A \in \mathcal{A}$ with $P_t \subseteq A \subseteq Q_s$.
4. There exists a complete family of dipods $(L_k, M_k)_{k\in K}$ satisfying
   $L'_k \not\subset L_k \Rightarrow$ there exists $A \in \mathcal{A}$ with $L_k \subset A \subset M_k$
5. for every $A \in T$ there exists $A_i^j \in \mathcal{A}$, $j \in J, i \in I_j$ with $A = \vee_{j\in J} \cap_{i\in I_j} A_i^j$
6. For $s,t \in S$, $Q_s \not\subset Q_t \Rightarrow$ there exists $A \in \mathcal{A}$ with $P_s \not\subset A \not\subset Q_t$.
7. For $s, t \in S$, $Q_s \not\subset Q_t \Rightarrow$ there exists $A \in \mathcal{A}$ with $P_t \subseteq A \subseteq Q_s$.
8. $Q_t \in \mathcal{A}$ for every $t \in S$.

Theorem 2.3: [4] Let $B \subseteq S$ contains S, φ and be closed under arbitrary joins. Then the following are equivalent.

1. For every $B \in T$, there exists $B_j \in \mathcal{B}$, $j \in J$, with $B = \vee_{j\in J} B_j$.
2. For $s, t \in S$, $Q_s \not\subset Q_t \Rightarrow$ there exists $B \in \mathcal{B}$ with $P_s \not\subset B \not\subset Q_t$
3. For $s, t \in S$, $Q_s \not\subset Q_t \Rightarrow$ there exists $B \in \mathcal{B}$ with $P_t \subset B \subset Q_s$.
4. There exists a complete family of dipods $(L_k, M_k)_{k\in K}$ satisfying $M^{\emptyset} \not\subset M_K$
   $\Rightarrow$ there exists with $B \in \mathcal{B}$ with $L_k \subset B \subset M'k$
5. For every $B \in T$ there exists $Bi^j \in \mathcal{B}$, $j \in J, i \in I_j$ with $B = \cap_{j\in J} \vee_{i\in Ij} Bi^j$
   If (S,T) is coseparated, each of the following is also equivalent to the above:
6. For $s, t \in S$, $P_s \not\subset P_t \Rightarrow$ there exists $B \in \mathcal{B}$ with $P_s \not\subset B \not\subset Q_t$.
7. For $s, t \in S$, $P_s \not\subset P_t \Rightarrow$ there exists $B \in \mathcal{B}$ with $P_t \subseteq B \subseteq Q_s$.
8. $P_s \in \mathcal{B}$ for every $s \in S$

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Theorem 2.4: Characterizations of T_α-T_0. Let (S, T, τ, κ) be a ditopological texture space. Then following are equivalent. :

1. $P_s \not\subset P_t \Rightarrow$ there exists $C_j \in \tau\alpha \cup \kappa\alpha$, $j \in J$ with $P_t \subseteq \vee_{j\in J} C_j \subseteq Q_s$.
2. $Q_s \not\subset Q_t \Rightarrow$ there exists $C_j \in \tau\alpha \cup \kappa\alpha$, $j \in J$ with $P_t \subseteq \cap_{j\in J} C_j \subseteq Q_s$.
3. For $A \in T$ there exists $Ci^j \in \tau\alpha \cup \kappa\alpha$, $j \in J$, $i \in I$, with $A = \vee_{j\in J} \cap_{i\in I} Ci^j$
4. $Q_s \not\subset Q_t \Rightarrow$ there exists $C \in \tau\alpha \cup \kappa\alpha$ with $P_s \not\subset C \not\subset Q_t$.
5. $cl(P_s) \subseteq cl(P_t)$ and $int(Q_s) \subseteq int(Q_t) \Rightarrow Q_s \subseteq Q_t$.
6. For $s \in S$ we have $Q_s = \vee_{j\in J} C_j$ for $C_j \in \tau\alpha \cup \kappa\alpha$.
   If (S, T) is coseparated the following condition also characterizes the $T_\alpha T_0$ property,
7. For all $s \in S$ we have $P_s = \cap_{j\cap J} C_j$ for $C_j \in \tau\alpha \cup \kappa\alpha$

Proof: Here (1) and (7) are equivalent for any collection $\mathcal{B}$ by Theorem.2.3. Therefore, in particular these are also equivalent for $B = (\tau\alpha \cup \kappa\alpha)^{\cap}$.

Similarly (2), (3), (6) are equivalent for any collection $\mathcal{A}$ in Theorem.2.2. Therefore, it is also true for $A=(\tau\alpha \cup \kappa\alpha)^{\vee}$. Since Theorem 2.2 holds for $\mathcal{A} = \tau\alpha \cup \kappa\alpha)^{\vee}$,then any element of T can be written in the form of $\cap_{j\in J} (\vee_{i\in I} C_i)$ with $C_i \in \tau\alpha \cup \kappa\alpha$, by completely distributive property this set is equal to $\vee_{\alpha\in I_j} (\cap C_i{}^j)$. Thus any element of $\mathcal{A}$ can be written in the form of $\mathcal{B}$ Similarly the converse holds. Therefore the two theorems (2.2 and 2.3) are equivalent for this choice of $\mathcal{A}$ and $\mathcal{B}$.

(3)⇒(4): Let $Q_s \not\subset Q_t$ then by definition $Q_s = \vee\{P_t | P_s \not\subset P_t\}$ so there exists $t \in S$ with $P_s \not\subset P_t$ and $P_t \not\subset Q_t$, using (3) we can write $P_t = \vee_{j\in J} \cap_{i\in I_j} C_i{}^j, C_i{}^j \in \tau\alpha \cup \kappa\alpha$ so we have $j \in J$ with $\cap_{i\in I_j} C_i \not\subset Q_t$ then $P_s \not\subset Ci^j \not\subset Q_t$ for some $i \in I_j$.

(2) ⇒ (4): Since every $C = \bigcap_{j \in J} C_j$. Therefore (2) can be written as $P_t \subseteq C \subseteq Q_S$. Thus we obtained $P_S \not\subset C \not\subset Q_t$.

(4) ⇒ (2): Similarly the converse.

(4) ⟹ (5): if $Q_S \not\subset Q_t$ we have $C \in \tau\alpha \cup \alpha C(S)$ with $P_S \not\subset C \not\subset Q_t$ using(4). Then two cases arise,

Case (i): If $C \in \tau\alpha$ then $P_S \not\subset C \Rightarrow C \subseteq Q_S$ which implies $C \subseteq intQ_S$ and thus we have $intQ_S \not\subset intQ_t$

Case (ii): If $C \in \alpha C(S)$ then $C \not\subset Q_t \Rightarrow P_t \subseteq C \Rightarrow cl(P_t) \subseteq C$ which implies $cl(P_S) \not\subset cl(P_t)$.

(5) ⟹ (2): If $Q_S \not\subset Q_t$ then $cl(P_S) \not\subset cl(P_t)$ or $int(Q_S) \not\subset int(Q_t)$.If first occur then $P_t \subseteq cl(P_t) \subseteq Q_S$ and if the other happen then $P_t \subseteq int(Q_S) \subseteq Q_S$.

Definition.2.5. A ditopological texture space is said to be,
1. $T\alpha$-T_1 if it is $T\alpha$-T_0 and α-R_0.
2. co T_α-T_1 if it is T_α-T_0 and co-α R_0.
3. b_i-$T\alpha$-T_1 if it is T_α-T_0 and $b_i - \alpha R_0$.

Theorem.2.6. Let (S, T, τ, κ) be a ditopological texture space,
1. (S, T, τ, κ) is $T_\alpha - T_1$ if and only if it satisfies the conditions of Theorem 2.3 with $\mathcal{B} = \kappa\alpha$. In particular, the following are characteristic of a T_α–T_1 ditopological space.
(i) For any $A \in T$ we have $F_i \in \kappa\alpha$, $i \in I$ with $A = \bigvee_{i \in I} F_i$.
(ii) For $s, t \in S$, $Q_S \not\subset Q_t \Rightarrow$ there exists $F \in \kappa\alpha$ with $P_S \not\subset F \not\subset Q_t$.
(iii) If (S, T) is coseparated then $P_S \in \kappa\alpha$ for each $s \in S$.

2. (S, T, τ, κ) is co-T_α-T_1 if and only if it satisfies the conditions of Theorem.2.2 with $\mathcal{A} = \tau\alpha$, In particular, the following properties.
(i) For any $A \in T$ we have $G_i \in \tau\alpha$), $i \in I$ with $A = \bigcap_{i \in I} G_i$.
(ii) For $s, t \in S$, $Q_S \not\subset Q_t \Rightarrow$ there exists $G \in \tau\alpha$ with $P_S \not\subset G \not\subset Q_t$.
(iii) $Q_S \in \tau\alpha$ for all $s \in S$.

Proof: Here we prove (2). Let the space be co-T_α-T_1, (i.e) it is T_α–T_0 and co-α R_0. To prove it satisfies the conditions with $\mathcal{A} = \tau\alpha$. Let us consider for any $s, t \in S$ satisfying $Q_S \not\subset Q_t$, we have $B \in \tau\alpha \cup \kappa\alpha$ with $P_S \not\subset B \not\subset Q_t$, since the space is $T\alpha$-T_0. Then two cases arise,

Case (i): If $B \in \tau\alpha$ then $B = G \in \tau\alpha$ which satisfies $P_t \subseteq G \subseteq Q_S$.

Case (ii): If $B \in \kappa\alpha$ then $P_S \not\subset B$ which implies $B \subseteq int(Q_S)$ by co α-R_0 then $G = intQ_S$ then we have $P_t \subseteq G \subseteq Q_S$, Thus the (ii)of (2) is proved. Since all the above conditions are equivalent it is enough to prove anyone of them, hence proved for $\mathcal{A} = \tau\alpha$.

Conversely, Let (S, T, τ, κ) satisfies Theorem 2.2 with $\mathcal{A} = \tau\alpha$. Then we have to prove it is T_α-T_0 and co-αR_0. It is obliviously true for $A = (\tau\alpha \cap \kappa\alpha)^V$.(i.e) it is T_α–T_0. Take $F \in \kappa\alpha$ such that $P_S \not\subset F$ in Theorem 2.2, we have $F = \bigcap_{j \in J} G_j$, $G_j \in \alpha O(S)$. From the equivalent conditions, $Q_S \in \tau\alpha$. Hence $F \subseteq Q_S$, (i.e.) $F \subseteq int(Q_S)$.Hence the proof..

Similarly we can prove (1).

Definition.2.7: A ditopological texture space is called,

1. T_α-T_2 if it is T_α-T_0 and α-R_1.
2. co- T_α-T_2 if it is T_α-T_0 and co−αR.
3. b_i-T_α-T_2 if it is T_α-T_0 and b_i-αR_1.

Theorem 2.8: The following are equivalent for a ditopology (S, T, τ, κ)

1. (S, T, τ, κ) is bi-T_α-T_2.
2. For $s, t \in S$, $Q_s \not\subset Q_t \Rightarrow$ there exists $H \in \tau\alpha$, $K \in \kappa\alpha$ with $H \subseteq K$, $P_s \not\subset K$ and $H \not\subset Q_t$.
3. For $A \in T$ there exist $H_i^j \in \tau\alpha$, $K_i^j \in \kappa\alpha$, $i \in I$ and $j \in J$ with $H_i^j \subseteq K_i^j$, for all i, j and $A = \vee_{i \in I} \cap_{j \in J_i} H_i^j = \vee_{i \in I} \cap_{j \in J_i} K_i^j$.

Proof: (1) ⟹ (2): Let $Q_s \not\subset Q_t$. Since the given space is bi-$T_\alpha T_2$ it is $T\alpha - T_0$ so we have $B \in \tau\alpha \cup \kappa\alpha$ with $P_s \not\subset B \not\subset Q_t$ by theorem 2.2.

Case(i): If $B \in \tau\alpha$ then we have $H \in \tau\alpha$ with $P_s \not\subset cl(H)$, $H \not\subset Q_t$ by α-R_0.Here take K=cl(H) then we get the required result.

Case (ii): If $B \in \kappa\alpha$ then we have $K \in \kappa\alpha$ with $P_s \not\subset K$, $int(K) \not\subset Q_t$ by co-α-R_1.Here take H=int(K)then we get the required result.

(2) ⟹ (3): For $A \in T$ we can write $A = \vee\{P_t \mid A \not\subset Q_t\} = \cap\{Q_s \mid P_s \not\subset A\}$. For s, t such that $A \not\subset Q_t$ and $P_s \not\subset A$ we have $Q_s \not\subset Q_t$ and so there exist $H_s^t \in \alpha O(S)$, $K_s^t \in \alpha C(S)$ with $H_s^t \subseteq K_s^t$, and $P_s \not\subseteq K_s^t$, $H_s^t \not\subseteq Q_t$. Hence we get $A = \vee_{\{A \not\subseteq Q_t\}} \cap_{\{P_s \not\subseteq A\}} H_s^t = \vee_{\{A \not\subseteq Q_t\}} \cap_{\{P_s \not\subseteq A\}} K_s^t$

(3) ⟹ (1): Similarly we can prove this result.

Definition 2.9: Let (S, T, τ, κ), be ditopological texture spaces then it is said to be T-α normal if $G \in \tau\alpha$ and $F \in \kappa\alpha$ with $F \subseteq G$ there exists $H \in \tau\alpha$ with $F \subseteq H \subseteq cl(H) \subseteq G$.

Remark 2.10: From the Definitions it is clear that
(i) Every Tα-T_2 space is $T\alpha T_1$space.
(ii) Every Tα-T_1 space is Tα-T_0space.

The converse need not be true always.

Theorem 2.11: Let $(S, P(X), \tau, \kappa, \sigma)$ be a complemented ditopological texture space then we have the following result
1. S be T α normal space.
2. for each $A \in \tau\alpha$ and each $U \in \tau\alpha$ containing A, there exists $G \in \tau\alpha \cap \kappa\alpha$ such that $A \subseteq G \subseteq U$.
3. for each pair of disjoint $A, B \in \kappa\alpha$ there exists disjoint τα U and V such that $A \subseteq U$ and $B \subseteq V$.

Proof:
(1) ⇒ (2): It is clear from the definition.

(2) ⇒ (3): Let A and B be any pair of disjoint α closed sets. Then we have $A \subseteq S - B \in \tau\alpha$ and there exists $U \in \tau\alpha \cap \kappa\alpha$ such that $A \subseteq U \subseteq S - B$. Now put V = S-U, then we obtain $A \subseteq U$, $B \subseteq V \in \tau\alpha$ and $U \cap V = \varphi$.

The converse need not be true always.

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