International Journal of Mathematical Archive-4(2), 2013, 298-310
IJMA Available online through www.ijma.info ISSN 2229 – 5046

# DOMINATING SETS OF SQUARE OF CENTIPEDES

**A. Vijayan[1] & K. Lal Gipson[*]**

*[1]Associate Professor, Department of Mathematics, Nesamony Memorial Christian College, Marthandam, Kanayakumari District, Tamil Nadu, South India*

*[*]Assistant Professor, Department of Mathematics, Mar Ephraem College of Engineering and Technology,Elavuvilai, Mathandam, Kanayakumari District, Tamil Nadu, South India*

***(Received on: 15-01-13; Revised & Accepted on: 15-02-13)***

***ABSTRACTS***

*Let G = (V, E) be a simple graph. A set S $\subseteq$ V is a dominating set of G, if every vertex in V-S is adjacent to atleast one vertex in S. Let $P_n^{*2}$ be the square of centipede corresponding to the path $P_n$ and let $D(P_n^{*2}, i)$ denote the family of all dominating sets of $P_n^{*2}$ with cardinality i. In this paper, some properties of the dominating sets of centipedes are exhibited. Also, we characterized the number of dominating sets of $P_n^{*2}$ and $P_n^{*2} - \{2n\}$ of cardinality i.*

***Keywords:*** *domination set, domination number.*

## 1. INTRODUCTION

Let G = (V, E) be a simple graph of order |V| = n. For any vertex v$\in$V, the open neighbourhood of v is the set N (v) = {u$\in$V/uv$\in$E} and the closed neighbourhood of v is the set N[v] = N(v) $\cup${v}. For a set S $\subseteq$ V, the open neighbourhood of S is N(S) = $\bigcup_{v\in S}$ N (v) and the closed neighbourhood of S is N[S] = N(S) $\cup$ S. A set S $\subseteq$ V is a dominating set of G, if N[S] = V, or equivalently, every vertex in V-S is adjacent to atleast one vertex in S. The domination number of a graph G is defined as the minimum size of a dominating set of vertices in G and it is denoted as $\gamma(G)$. A path is a connected graph in which end vertices have degree one and the remaining vertices have degree two, and is denoted by $P_n$.The centipede $P_n^*$ consists of a path $P_n$ in which each vertex imbedded with a pendant edge and a pendant vertex.

**Definition: 1.1:** The 2nd power of a graph with the same set of vertices as G and an edge between two vertices if and only if there is a path of length at most 2 between them.

As usual we use $\lfloor x \rfloor$ for the largest integer less than or equal to x and $\lceil x \rceil$ for the smallest integer greater than or equal to x. Also, we denote the set {1, 2… n} by [n], throughout this paper.

## 2. DOMINATING SETS OF SQUARE OF CENTIPEDES

For the construction of the dominating sets of the square of centipede, $P_n^{*2}$, we need to investigate the dominating sets of $P_n^{*2} - \{2n\}$.In this section we investigate dominating sets of $P_n^{*2}$. Let $D(P_n^{*2}, i)$ be the family of dominating sets of $P_n^{*2}$ with cardinality i. We shall find recursive formula for $|D(P_n^{*2}, i)|$ .We need the following lemmas to obtain the result of this section:

**Lemma 2.1:** $\gamma(P_n^2) = \left\lceil \frac{n}{5} \right\rceil$

By lemma 2.1 and the definition of domination number, one has the following lemma:

**Corresponding author:** *K. Lal Gipson[*]*
*[*]Assistant Professor, Department of Mathematics, Mar Ephraem College of Engineering and Technology,Elavuvilai, Mathandam, Kanayakumari District, Tamil Nadu, South India*

**Lemma 2.2:** For every $n \in N$,

i) $\gamma(P_n^{*2}) = \left\lceil \frac{n}{3} \right\rceil$

ii) $\gamma(P_n^{*2} - \{2n\}) = \left\lfloor \frac{n}{3} \right\rfloor$

iii) $D(P_n^{*2}, i) = \Phi \text{ if and only if } i \prec \left\lceil \frac{n}{3} \right\rceil \text{ or } i \succ 2n$

iv) $D(P_n^{*2} - \{2n\}, i) = \Phi \text{ if and only if } i \prec \left\lfloor \frac{n}{3} \right\rfloor \text{ or } i \succ 2n-1$

**Proof:**

i) Clearly $\{3, 7, 11, \ldots 2n-1\}$ is a minimum dominating set for $P_n^{*2}$ .If n is even or odd it contains $\left\lceil \frac{n}{3} \right\rceil$ elements.

Hence $\gamma(P_n^{*2}) = \left\lceil \frac{n}{3} \right\rceil$.

ii) Clearly $\{3, 7, 11, \ldots 2n-3\}$ is a minimum dominating set for $P_n^{*2} - \{2n\}$. If n is even or odd it contains $\left\lfloor \frac{n}{3} \right\rfloor$ elements.

Hence $\gamma(P_n^{*2} - \{2n\}) = \left\lfloor \frac{n}{3} \right\rfloor$

iii) It follows from (i) and the definition of dominating set.

iv) It follows from (ii) and the definition of dominating set.

For the construction of $D(P_n^{*2}, i)$ we consider

$D(P_n^{*2} - \{2n\}, i-1), D(P_{n-1}^{*2}, i-1), D(P_{n-1}^{*2} - \{2n-2\}, i-1), \text{ and } D(P_{n-3}^{*2}, i-1).$

The families of these dominating sets can be empty or otherwise. Thus, we have eight combinations, whether these five families are empty or not. Two of these combinations are not possible (Lemma 2.3 (i), (ii) & (iii)).

Also, the combinations $D(P_n^{*2} - \{2n\}, i-1) = D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-3}^{*2}, i-1) = \Phi$ do not need to be considered because it implies that $D(P_n^{*2}, i) = \Phi$ (See lemma 2.2 (iii)). Thus we only need to consider four combinations or cases. We consider those cases in Lemma **(2.3).**

**Lemma 2.3:**

i) If $D(P_n^{*2} - \{2n\}, i-1) = D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = \Phi \text{ then } D(P_{n-3}^{*2}, i-1) = \Phi$ .

ii) If $D(P_n^{*2} - \{2n\}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-3}^{*2}, i-1) = \Phi \text{ then } D(P_{n-1}^{*2}, i-1) = \Phi$ .

iii) If $D(P_n^{*2} - \{2n\}, i-1) = D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-3}^{*2}, i-1) = \Phi$ , $\text{then } D(P_n^{*2}, i) = \Phi$ .

iv) If $D(P_n^{*2} - \{2n\}, i-1) \neq \Phi \; D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-3}^{*2}, i-1) = \Phi$ , $\text{then } D(P_n^{*2}, i) \neq \Phi$

v) If $D(P_n^{*2} - \{2n\}, i-1) \neq \Phi, , D(P_{n-1}^{*2} - \{2n-2\}, i-1) = \Phi$ ,then $D(P_{n-1}^{*2}, i-1) \neq \Phi$ .

**Proof:**

i) If $D(P_n^{*2} - \{2n\}, i-1) = D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = \Phi$ , by lemma 2.2(iii),(iv) $i-1 \prec \left\lfloor \frac{n}{3} \right\rfloor$ or $i-1 \succ 2n-1$, $i-1 \prec \left\lceil \frac{n-1}{3} \right\rceil$ or $i-1 \succ 2n-2$, $i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor$ or $i-1 \succ 2n-3$.

Therefore, $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ *or* $i-1 \succ 2n-1$

Suppose, $D(P_{n-3}^{*2}, i-1) \neq \Phi$ then $\left\lceil \frac{n-3}{3} \right\rceil \leq i-1 \leq 2(n-3)$.

We have $\left\lfloor \frac{n-1}{3} \right\rfloor \leq \left\lceil \frac{n-3}{3} \right\rceil \leq i-1$. But $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$, which is a contradiction.

Therefore, $D(P_{n-3}^{*2}, i-1) = \Phi$.

ii) If $D(P_n^{*2} - \{2n\}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-3}^{*2}, i-1) = \Phi$, by lemma 2.2(iii),(iv)

$i-1 \prec \left\lfloor \frac{n}{3} \right\rfloor$ or $i-1 \succ 2n-1$, $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ or $i-1 \succ 2n$-3, $i-1 \prec \left\lceil \frac{n-3}{3} \right\rceil$ or $i-1 \succ 2n$-6.

There fore

$i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ or $i-1 \succ 2n-1$. Therefore $i-1 \prec \left\lceil \frac{n-3}{3} \right\rceil$ or $i-1 \succ 2n-2$ holds.

Hence, $D(P_{n-1}^{*2}, i-1) = \Phi$.

iii) If $D(P_n^{*2} - \{2n\}, i-1) = D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-3}^{*2}, i-1) = \Phi$. by lemma 2.2(iii),(iv),

$i-1 \prec \left\lfloor \frac{n}{3} \right\rfloor$ or $i-1 \succ 2n-1$, $i-1 \prec \left\lceil \frac{n-1}{3} \right\rceil$ or $i-1 \succ 2n-2$, $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ or

$i-1 \succ 2n$-3, $i-1 \prec \left\lceil \frac{n-3}{3} \right\rceil$ or $i-1 \succ 2n$-6.

Therefore, $i-1 \prec \left\lceil \frac{n-3}{3} \right\rceil$ or $i-1 \succ 2n$-1.

That is, $i \prec \left\lceil \frac{n-3}{3} \right\rceil + 1 \leq \left\lfloor \frac{n}{3} \right\rfloor$ or $i \succ 2n$.

Therefore, $i \prec \left\lfloor \frac{n}{3} \right\rfloor$ or $i \succ 2n$

Hence, $D(P_n^{*2}, i) = \Phi$.

iv) If $D(P_n^{*2} - \{2n\}, i-1) \neq \Phi$, $D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-3}^{*2}, i-1) = \Phi$,

then $\left\lfloor \frac{n}{3} \right\rfloor \leq i-1 \leq 2n-1$,

$i-1 \prec \left\lceil \frac{n-1}{3} \right\rceil$ or $i-1 \succ 2n-2$, $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ or $i-1 \succ 2n$-3 and

$i-1 \prec \left\lceil \frac{n-3}{3} \right\rceil$ or $i-1 \succ 2n$-6.

Therefore, we have $2n-2 \prec i-1 \leq 2n-1$.

Therefore, $2n-1 \leq i-1 \leq 2n-1$.

Therefore, i-1=2n-1

Therefore,  i=2n

Therefore $D(P_n^{*2}, i) \neq \Phi$.

v) If $D(P_n^{*2}-\{2n\},i-1)\neq\Phi, D(P_{n-1}^{*2}-\{2n-2\},i-1)=\Phi, then\ D(P_{n-1}^{*2},i-1)\neq\Phi$

We have $\left\lfloor\frac{n}{3}\right\rfloor\le i-1\le 2n-1$ and $i-1\prec\left\lfloor\frac{n-1}{3}\right\rfloor$ or $i-1\succ 2n$-3.

Therefore, $2n-2\le i-1\le 2n-1$.

Therefore, $2n-1\le i\le 2n$.

Therefore, i=2n-1 or i=2n.

Therefore, in either case, we have $D(P_{n-1}^{*2},i-1)\neq\Phi$.

**Theorem 2.4:** For every $n\ge 7$

i) $D(P_n^{*2}-\{2n\},i-1)\neq\Phi$ and $D(P_{n-1}^{*2},i-1)=D(P_{n-1}^{*2}-\{2n-2\},i-1)=D(P_{n-3}^{*2},i-1)=\Phi$, if and only if i=2n.

ii) $D(P_n^{*2}-\{2n\},i-1)\neq\Phi$, $D(P_{n-1}^{*2},i-1)\neq\Phi$, $D(P_{n-1}^{*2}-\{2n-2\},i-1)=\Phi$, $D(P_{n-3}^{*2},i-1)=\Phi$, if and only if i=2n-1

iii) $D(P_n^{*2}-\{2n\},i-1)\neq\Phi, D(P_{n-1}^{*2},i-1)\neq\Phi, D(P_{n-1}^{*2}-\{2n-2\},i-1)\neq\Phi, and\ D(P_{n-3}^{*2},i-1)=\Phi$ if and only if i=2n-2 or 2n-3 or 2n-4.

**Proof:**

i) (⇒) Since $D(P_{n-1}^{*2},i-1)=D(P_{n-1}^{*2}-\{2n-2\},i-1)=D(P_{n-3}^{*2},i-1)=\Phi$, we have

$i-1\prec\left\lceil\frac{n-1}{3}\right\rceil$ or $i-1\succ 2n$-2, $i-1\prec\left\lfloor\frac{n-1}{3}\right\rfloor$ or $i-1\succ 2n$-3, $i-1\prec\left\lceil\frac{n-3}{3}\right\rceil$ or $i-1\succ 2(n-3)$.

Also, $\left\lfloor\frac{n}{3}\right\rfloor\le i-1\le 2n-1.$

Therefore, $i-1\prec\left\lceil\frac{n-1}{3}\right\rceil$ does not hold.

Therefore, $i-1\succ 2n-2$.

Therefore $i-1\ge 2n-1$.

Therefore, $i\ge 2n$

Together, we have i=2n.

(⇐) It follows from Lemma 2.2(iii), (iv).

ii) (⇒)Since $D(P_n^{*2}-\{2n\},i-1)\neq\Phi$, $D(P_{n-1}^{*2},i-1)\neq\Phi$, $D(P_{n-1}^{*2}-\{2n-2\},i-1)=\Phi$, by Lemma 2.2(iii),(iv), we have $\left\lfloor\frac{n}{3}\right\rfloor\le i-1\le 2n-1$, $\left\lceil\frac{n-1}{3}\right\rceil\le i-1\le 2n-2$, and $i-1\prec\left\lfloor\frac{n-1}{3}\right\rfloor$ or $i-1\succ 2n$-3.Also $i-1\prec\left\lceil\frac{n-1}{3}\right\rceil$

Then $2n-3\prec i-1\le 2n-2$.

Therefore, i-1=2n-2

Thus, i=2n-1

(⇐) It follows from Lemma 2.2(iii), (iv).

iii) ($\Rightarrow$) Since

$D(P_n^{*2}-\{2n\},i-1)\neq\Phi, D(P_{n-1}^{*2},i-1)\neq\Phi, D(P_{n-1}^{*2}-\{2n-2\},i-1)\neq\Phi, and\ D(P_{n-3}^{*2},i-1)=\Phi$, by Lemma 2.2(iii),(iv), we have $\left\lfloor\frac{n}{3}\right\rfloor\leq i-1\leq 2n-1$, $\left\lceil\frac{n-1}{3}\right\rceil\leq i-1\leq 2n-2$, $\left\lfloor\frac{n-1}{3}\right\rfloor\leq i-1\leq 2n-3$.Also .

$i-1\prec\left\lceil\frac{n-3}{3}\right\rceil$ or $i-1\succ 2n-6$ .Also $i-1\geq 2n-5$.

Therefore, $2n-5\leq i-1\leq 2n-3$.Therefore, $2n-4\leq i\leq 2n-2$.

Hence i=2n-2,2n-3,2n-4

($\Leftarrow$) It follows from Lemma 2.2(iii), (iv).

Now, we construct following Theorem.

**Theorem 2.5:**

i) If $D(P_n^{*2}-\{2n\},i-1)\neq\Phi$ ,and $D(P_{n-1}^{*2},i-1)=D(P_{n-1}^{*2}-\{2n-2\},i-1)=D(P_{n-3}^{*2},i-1)=\Phi$ then

$$D(P_n^{*2},i)=\{X\cup\{2n\}/X\in D(P_n^{*2}-\{2n\},i-1)\ \}$$

ii) If $D(P_n^{*2}-\{2n\},i-1)\neq\Phi$ , $D(P_{n-1}^{*2},i-1)\neq\Phi$ , $D(P_{n-1}^{*2}-\{2n-2\},i-1)\neq\Phi$

and $D(P_{n-3}^{*2},i-1)=\Phi$ , then

$$D(P_n^{*2},i)=\{X_1\cup\{2n\}/X_1\in D(P_n^{*2}-\{2n\},i-1), X_2\cup\{\{2n-1\}/D(P_{n-1}^{*2},i-1)\}$$
$$\{X_3\cup\{2n-2,2n-3\}/X_3\in D(P_{n-1}^{*2}-\{2n-2\},i-1)\}$$

iii) If $D(P_n^{*2}-\{2n\},i-1)\neq\Phi$ $D(P_{n-1}^{*2},i-1)\neq\Phi$ $D(P_{n-1}^{*2}-\{2n-2\},i-1)\neq\Phi$ and $D(P_{n-3}^{*2},i-1)\neq\Phi$ ,then

$$\{X_1\cup\{2n\}/X_1\in D(P_n^{*2}-\{2n\},i-1),$$
$$X_2\cup\{2n-1\}/X_2\in D(P_{n-1}^{*2},i-1),$$
$$X_3\cup\{2n-3\}/X_3\in D(P_{n-1}^{*2}-\{2n-2\},i-1)\ \},$$
$$X_4\cup\begin{cases}\{2n-1\}/X_4\in D(P_{n-3}^{*2},i-1),1\notin D(P_{n-3}^{*2},i-1)\\ \{2n-3\}/X_4\in D(P_{n-3}^{*2},i-1),1\in D(P_{n-3}^{*2},i-1)\end{cases}$$

**Proof:**

i) By theorem 2.1(i),i=2n.Since in this case $D(P_n^{*2},i)=D(P_n^{*2},2n)=\{[2n]\}$ and

$D(P_n^{*2}-\{2n\},i-1)=D(P_n^{*2}-\{2n\},2n-1)=\{[2n-1]\}$,

then we have the result.

ii) Let

$Y_1=\{\ \ X_1\cup\{2n\}/X_1\in D(P_n^{*2}-\{2n\},i-1)\ \ \}$

$Y_2=\{\ \ X_2\cup\{2n-1\}/X_2\in D(P_{n-1}^{*2},i-1)\ \ \}$ and

$Y_3=\{\ \ X_3\cup\{2n-2,2n-3\ \ \}/X_3\in D(P_{n-1}^{*2}-\{2n-2\},i-1),\ \ \}$ .

Obviously, $Y_1\cup Y_2\cup Y_3\subseteq D(P_n^{*2},i)$ (2.1)

Now let $Y\in D(P_n^{*2},i)$ .If $2n\in Y$ , then at least one of the vertives labeled 2n-1 or 2n-2 is in Y.In either cases, $Y=X_1\cup\{2n\}\ forsome\ X_1\in D(P_n^{*2}-\{2n\},i-1)$ ,that is $Y\in Y_1$.

If 2n $\notin Y$ and 2n-1 $\in Y$ , then 2n-2 $\in Y$ , So $Y=X_2\cup\{2n-1\}\ forsome\ D(P_{n-1}^{*2},i-1)$ ,that is $Y\in Y_2$ .Now suppose that2n-2 $\in Y$ ,2n $\notin Y$ and 2n-1 $\notin Y$ ,then at least one of the vertices labeled 2n-3,2n-4 is in Y. If 2n-4 $\in Y$ , then

$Y=\{\ \ X_3\cup\{2n-2,2n-3\ \ \}\ forsome\ X_3\in D(P_{n-1}^{*2}-\{2n-2\},i-1)\ \ \}$; that is Y $\in Y_3$.

 

Thus, we have proved that $D(P_n^{*2}, i) \subseteq Y_1 \cup Y_2 \cup Y_3$. (2.2)

From (2.1) and (2.2), We have

$$D(P_n^{*2}, i) = \{X_1 \cup \{2n\} / X_1 \in D(P_n^{*2} - \{2n\}, i-1), X_2 \cup \{2n-1\} / D(P_{n-1}^{*2}, i-1)\}$$
$$\{X_3 \cup \{2n-2, 2n-3\} / X_3 \in D(P_{n-1}^{*2} - \{2n-2\}, i-1),\}$$

iii) Let

$Y_1 = X_1 \cup \{2n\} / X_1 \in D(P_n^{*2} - \{2n\}, i-1),$

$Y_2 = X_2 \cup \{2n-1\} / X_2 \in D(P_{n-1}^{*2}, i-1),$

$Y_3 = X_3 \cup \{2n-3\} / X_3 \in D(P_{n-1}^{*2} - \{2n-2\}, i-1)\}$, and

$$Y_4 = X_4 \cup \begin{cases} \{2n-1\} / X_4 \in D(P_{n-3}^{*2}, i-1), 1 \notin D(P_{n-3}^{*2}, i-1) \\ \{2n-3\} / X_4 \in D(P_{n-3}^{*2}, i-1), 1 \in D(P_{n-3}^{*2}, i-1) \end{cases}.$$

Obviously, $Y_1 \cup Y_2 \cup Y_3 \cup Y_4 \subseteq D(P_n^{*2}, i)$ (2.3)

Now, let $Y \in D(P_n^{*2}, i)$. If $2n \in Y$, then at least one of the vertices labeled 2n-1 or 2n-2 is in Y. In either cases, $Y = X_1 \cup \{2n\}$ *forsome* $X_1 \in D(P_n^{*2} - \{2n\}, i-1)$ ; that is $Y \in Y_1$.

If 2n $\notin Y$ and 2n-1 $\in Y$ ,then 2n-2 $\in Y$ ,so $Y = X_2 \cup \{2n-1\}$ *forsome* $D(P_{n-1}^{*2}, i-1)$ that is $Y \in Y_2$ .Now suppose that 2n-2 $\in Y$ ,2n $\notin Y$ and 2n-1 $\notin Y$ ,then at least one of the vertices labeled 2n-3,2n-4 is in Y.If 2n-4 $\in Y$, then $Y = \{X_3 \cup \{2n-3\}$ *forsome* $X_3 \in D(P_{n-1}^{*2} - \{2n-2\}, i-1)\}$, that is Y $\in Y_3$.Now suppose that 2n-3 $\in Y$ ,2n-2 $\notin Y$ ,2n-1 $\notin Y$ ,then at leat one of the vertices labeled 2n-4,2n-5 is in Y.If 2n-5 $\in Y$, then $Y = X_4 \cup \begin{cases} \{2n-1\} \text{ forsome} X_4 \in D(P_{n-3}^{*2}, i-1), 1 \notin D(P_{n-3}^{*2}, i-1) \\ \{2n-3\} \text{ forsome} X_4 \in D(P_{n-3}^{*2}, i-1), 1 \in D(P_{n-3}^{*2}, i-1) \end{cases}$.

Thus we have proved that $D(P_n^{*2}, i) \subseteq Y_1 \cup Y_2 \cup Y_3 \cup Y_4$. (2.4)

From (2.3) and (2.4)

$$\{X_1 \cup \{2n\} / X_1 \in D(P_n^{*2} - \{2n\}, i-1),$$
$$X_2 \cup \{2n-1\} / X_2 \in D(P_{n-1}^{*2}, i-1),$$
$$X_3 \cup \{2n-3\} / X_3 \in D(P_{n-1}^{*2} - \{2n-2\}, i-1)\},$$
$$X_4 \cup \begin{cases} \{2n-1\} / X_4 \in D(P_{n-3}^{*2}, i-1), 1 \notin D(P_{n-3}^{*2}, i-1) \\ \{2n-3\} / X_4 \in D(P_{n-3}^{*2}, i-1), 1 \in D(P_{n-3}^{*2}, i-1)\} \end{cases}$$

**Theorem 2.6:** For every n $\geq$ 7,

$$|D(P_n^{*2}, i)| = |D(P_n^{*2} - \{2n\}, i-1)| + |D(P_{n-1}^{*2}, i-1)| + |D(P_{n-1}^{*2} - \{2n-2\}, i-1)| + |D(P_{n-3}^{*2}, i-1)|$$

**Proof:** We consider the three cases in Theorem 2.2

i) By Theorem 2.2( i), $D(P_n^{*2}, i) = \{X \cup \{2n\} / X \in D(P_n^{*2} - \{2n\}, i-1)\}$.Therefore we have the result in this case.

ii) ByTheorem2.2(ii), we have

$D(P_n^{*2}, i) = A_1 \cup A_2 \cup A_3$ Where,

$|A_1| = D\left(P_n^{*2} - \{2n\}, i-1\right), |A_2| = D(P_{n-1}^{*2}, i-1), |A_3| = D\left(P_{n-1}^{*2} - \{2n-2\}, i-1\right)$.Since for every

$X_1 \in A_1, X_2 \in X_1$ ,and for every $X_2 \in A_2$ , $2n \notin X_2$ ,so $A_1 \cap A_2 = \Phi$ .Also since for every $X_2 \in A_2$ and $X_3 \in A_3$ , $2n-1 \in X_2, and\, 2n-1 \notin X_3$ , we have $A_2 \cap A_3 = \Phi$ .For every $X_3 \in A_3$ , $2n-1 \notin X_3$ , and 2n-3 $\notin X_3$ ,but for every $X_1 \in A_1$ ,atleast one of 2n-3 or 2n-1 is in $X_3$ ;because $X_3 = Y_3 \cup \{2n\}$ for some $Y_3 \in D\left(P_n^{*2} - \{2n\}, i-1\right)$ ,so atleast one of 2n-3 of 2n-1 must be in $Y_3$. Hence we have the result.

iii) By Theorem 2.2(ii) $D(P_n^{*2}, i) = A_1 \cup A_2 \cup A_3 \cup A_4$ ,Where,
$\left|A_1\right| = D(P_n^{*2} - \{2n\}, i-1), \left|A_2\right| = D(P_{n-1}^{*2}, i-1), \left|A_3\right| = D(P_{n-1}^{*2} - \{2n-2\}, i-1)$ and $\left|A_4\right| = D(P_{n-3}^{*2}, i-1)$
Since for every $X_1 \in A_1, X_2 \in X_1$ ,and for every $X_2 \in A_2$ , $2n \notin X_2$ ,so $A_1 \cap A_2 = \Phi$ .Also since for every $X_2 \in A_2$ and $X_3 \in A_3$ , $2n-1 \in X_2, and\, 2n-1 \notin X_3$ ,we have $A_2 \cap A_3 = \Phi$ .For every $X_3 \in A_3$ , $2n-1 \notin X_3$ , and 2n-3 $\notin X_3$ ,but for every $X_1 \in A_1$ , atleast one of 2n-3 or 2n-1 is in $X_3$ ,because $X_3 = Y_3 \cup \{2n\}$ for some $Y_3 \in D\left(P_n^{*2} - \{2n\}, i-1\right)$ so atleast one of 2n-3 of 2n-1 must be in $Y_3$. For every $X_4 \in A_4$ ,2n-1 $\notin X_4$ and2n-3 $\notin X_4$ but for every $X_1 \in A_1$ , atleast one of 2n-3 or 2n-1 is in, $X_4$ because $X_4 = Y_4 \cup \{2n\}$ for some $Y_4 \in D\left(P_n^{*2} - \{2n\}, i-1\right)$ , atleast one of 2n-3 of 2n-1 must be in $Y_4$ .Hence we have the result.

**3. Domination sets of** $\left(P_n^{*2} - \{2n\}\right)$

For the construction of $\left(P_n^{*2} - \{2n\}\right)$ , we consider

$D(P_{n-1}^{*2}, i-1), D(P_{n-1}^{*2} - \{2n-2\}, i-1), D(P_{n-2}^{*2}, i-1), D(P_{n-2}^{*2} - \{2n-4\}, i-1)\; and\; D(P_{n-3}^{*2} - \{2n-6\}, i-1)$

The families of these dominating sets can be empty or otherwise. Thus, we have eight combinations, whether these five families are empty or not. Two of these combinations are not possible (Lemma 2.3 (i), (ii) & (iii)).

Also, the combinations
$D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-2}^{*2}, i-1) = D(P_{n-2}^{*2} - \{2n-4\}, i-1) = D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$ does not need to be considered because it implies that $D(P_n^{*2} - \{2n\}, i-1) = \Phi$
(See lemma 2.5 (iii)). Thus we only need to consider four combinations or cases. We consider those cases in theorem **(2.7).**

**Lemma 3.1:**

i) If $D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-2}^{*2}, i-1) = D(P_{n-2}^{*2} - \{2n-4\}, i-1) = \Phi$ ,then $D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$ .

ii) If $D(P_{n-1}^{*2}, i-1) = D(P_{n-2}^{*2}, i-1) = D(P_{n-2}^{*2} - \{2n-4\}, i-1) = D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$ ,then $D(P_{n-1}^{*2} - \{2n-2\}, i-1) = \Phi$ .

iii) If $D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-2}^{*2}, i-1) = \Phi,$
$D(P_{n-2}^{*2} - \{2n-4\}, i-1) = D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$ , then $D(P_n^{*2} - \{2n\}, i-1) = \Phi$ .

iv) $D(P_{n-1}^{*2}, i-1) \neq \Phi$ and
$D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-2}^{*2}, i-1) = D(P_{n-2}^{*2} - \{2n-4\}, i-1) = D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$ then
$D(P_n^{*2} - \{2n\}, i-1) \neq \Phi$ .

v) If $D(P_{n-1}^{*2}, i-1) \neq \Phi, D(P_{n-1}^{*2} - \{2n-2\}, i-1) \neq \Phi\; and$
$D(P_{n-2}^{*2} - \{2n-4\}, i-1) = \Phi\; then\; D(P_{n-2}^{*2}, i-1) = \Phi$

**Proof:**

i) Since $D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-2}^{*2}, i-1) = D(P_{n-2}^{*2} - \{2n-4\}, i-1) = \Phi$ , by lemma 2.2(iii),(iv) we have,

$i-1 \prec \left\lceil \frac{n-1}{3} \right\rceil$ or $i-1 \succ 2n$ -2, $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ or $i-1 \succ 2n-3$, $i-1 \prec \left\lceil \frac{n-2}{3} \right\rceil$ or $i-1 \succ 2n-4$

$i-1 \prec \left\lfloor \frac{n-2}{3} \right\rfloor$ or $i-1 \succ 2n-5$.

From the above relations, it follows that $i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor$ or $i-1 \succ 2n-7$.

Therefore, $D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$ .

ii) Since $D(P_{n-1}^{*2}, i-1) = D(P_{n-2}^{*2}, i-1) = D(P_{n-2}^{*2} - \{2n-4\}, i-1) = D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$ so by Lemma 2.2(iii),(iv), we have

$i-1 \prec \left\lceil \frac{n-1}{3} \right\rceil$ or $i-1 \succ 2n$ -2, $i-1 \prec \left\lceil \frac{n-2}{3} \right\rceil$ or $i-1 \succ 2n-4$, $i-1 \prec \left\lfloor \frac{n-2}{3} \right\rfloor$ or $i-1 \succ 2n-5$,

$i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor$ or $i-1 \succ 2n-7$.

Therefore, $i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor$ or $i-1 \succ 2n$ -2.

Hence, $i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor$ or i-1 $i-1 \succ 2n$ -3 hold.

Hence, $D(P_{n-1}^{*2} - \{2n-2\}, i-1) = \Phi$ .\`

iv) If $D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-2}^{*2}, i-1) = \Phi$,

$D(P_{n-2}^{*2} - \{2n-4\}, i-1) = D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$ ,so by Lemma 2.2(iii),(iv), we have,

$i-1 \prec \left\lceil \frac{n-1}{3} \right\rceil$ or $i-1 \succ 2n$ -2 , $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ or $i-1 \succ 2n$ -3, $i-1 \prec \left\lceil \frac{n-2}{3} \right\rceil$ or $i-1 \succ 2n-4$,

$i-1 \prec \left\lfloor \frac{n-2}{3} \right\rfloor$ or $i-1 \succ 2n-5$, $i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor$ or $i-1 \succ 2n-7$.

Hence, $i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor$ or $i-1 \succ 2n$ -2 . $i \succ 2n-1$

Therefore, $i \prec \left\lfloor \frac{n}{3} \right\rfloor$ or $i \succ 2n-1$.

Therefore, $D(P_{n}^{*2} - \{2n\}, i-1) = \Phi$ .

v) Since $D(P_{n-1}^{*2}, i-1) \neq \Phi$ and

$D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-2}^{*2}, i-1) = D(P_{n-2}^{*2} - \{2n-4\}, i-1) = D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$ So, by Lemma 2.2(iii),(iv), we have,

$\left\lceil \frac{n-1}{3} \right\rceil \leq i-1 \leq 2n-2$, $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ or $i-1 \succ 2n$ -3, $i-1 \prec \left\lceil \frac{n-2}{3} \right\rceil$ or $i-1 \succ 2n-4$,

$i-1 \prec \left\lfloor \frac{n-2}{3} \right\rfloor$ or $i-1 \succ 2n-5$, $i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor$ or $i-1 \succ 2n-7$.

Since $i-1 \geq \left\lceil \frac{n-1}{3} \right\rceil$, the possibilities of $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$, $i-1 \prec \left\lceil \frac{n-2}{3} \right\rceil$.

$i-1 \prec \left\lfloor \frac{n-2}{3} \right\rfloor$ or $i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor$ do not occur. Therefore, from $\left\lfloor \frac{n-1}{3} \right\rfloor \prec i-1 \leq 2n-2$,

We have $i-1 \leq 2n-2$.

The other possibilities, we obtain that $i-1 \succ 2n-3$.

Therefore, $i-1 \geq 2n-2$. But $i-1 \leq 2n-2$.

Therefore, i-1 = 2n-2.

Therefore, i= 2n-1.

Hence, $D(P_n^{*2} - \{2n\}, i-1) \neq \Phi$.

vi) Since $D(P_{n-1}^{*2}, i-1) \neq \Phi, D(P_{n-1}^{*2} - \{2n-2\}, i-1) \neq \Phi$ *and* $D(P_{n-2}^{*2} - \{2n-4\}, i-1) = \Phi$, by Lemma 2.2(iii),(iv), we have,

$\left\lceil \frac{n-1}{3} \right\rceil \leq i-1 \leq 2n-2$, $\left\lfloor \frac{n-1}{3} \right\rfloor \leq i-1 \leq 2n-3$ and $i-1 \prec \left\lfloor \frac{n-2}{3} \right\rfloor$ or $i-1 \succ 2n-5$.

Then $\left\lfloor \frac{n-1}{3} \right\rfloor \leq i-1 \leq 2n-3$ and $i-1 \succ 2n-5$.

Therefore, $i-1 \leq 2n-3 \, and \, i-1 \geq 2n-4$.

Therefore, $i-1 = 2n-4 \;\; o \;\; i-1 =r 2n-3$.

Therefore, $i = 2n-3 \;\; or \, i-1 = 2n-2$.

Therefore, $D(P_n^{*2} - \{2n\}, i-1) \neq \Phi$.

Now we state when these cases for the families $D(P_{n-1}^{*2}, i-1), D(P_{n-1}^{*2} - \{2n-2\}, i-1), D(P_{n-2}^{*2}, i-1)$, $D(P_{n-2}^{*2} - \{2n-4\}, i-1)$, $D(P_{n-3}^{*2} - \{2n-6\}, i-1)$ can occur.

**Theorem 3.2:**

For every $n \geq 7$

i) $D(P_{n-1}^{*2}, i-1) \neq \Phi$ and $D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-2}^{*2}, i-1) = D(P_{n-2}^{*2} - \{2n-4\}, i-1) = D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$ if and only if i=2n-1.

ii) $D(P_{n-1}^{*2}, i-1) \neq \Phi$, $D(P_{n-1}^{*2} - \{2n-2\}, i-1) \neq \Phi$, $D(P_{n-2}^{*2}, i-1) = \Phi$ *and* $D(P_{n-2}^{*2} - \{2n-4\}, i-1) = \Phi$ if and only if $i = 2n-2$ .

iii) $D(P_{n-1}^{*2}, i-1) \neq \Phi$, $D(P_{n-1}^{*2} - \{2n-2\}, i-1) \neq \Phi, D(P_{n-2}^{*2}, i-1) \neq \Phi$, $D(P_{n-1}^{*2} - \{2n-4\}, i-1) \neq \Phi$ and $D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$ if and only if i=2n-4, i=2n-5.

**Proof:**

i) $(\Rightarrow)$ Since $D(P_{n-1}^{*2}, i-1) \neq \Phi$ and , $D(P_{n-1}^{*2} - \{2n-2\}, i-1) = D(P_{n-2}^{*2}, i-1) = D(P_{n-2}^{*2} - \{2n-4\}, i-1) = D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$ we have,

$i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ or $i-1 \succ 2n-3, i-1 \prec \left\lceil \frac{n-2}{3} \right\rceil$ or $i-1 \succ 2n-4, i-1 \prec \left\lfloor \frac{n-2}{3} \right\rfloor$ or $i-1 \succ 2n-5$,

$i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor$ or $i-1 \succ 2n-7$.

Also, $\left\lceil \frac{n-1}{3} \right\rceil \le i-1 \le 2n-2$.

Therefore $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ does not hold.

Therefore $i-1 \succ 2n-3$. Hence $i-1 \ge 2n-2$. Therefore $i \ge 2n-1$.

But $i-1 \le 2n-2$, $i \le 2n-1$.

Therefore i= 2n-1.

($\Leftarrow$) It follows from Lemma 2.2(iii), (iv).

ii) Since $D(P_{n-1}^{*2}, i-1) \ne \Phi$, $D(P_{n-1}^{*2} - \{2n-2\}, i-1) \ne \Phi$, $D(P_{n-2}^{*2}, i-1) = \Phi \; and \; D(P_{n-2}^{*2} - \{2n-4\}, i-1) = \Phi$, so by Lemma 2.2(iii),(iv), we have, $\left\lceil \frac{n-1}{3} \right\rceil \le i-1 \le 2n-2$, $\left\lfloor \frac{n-1}{3} \right\rfloor \le i-1 \le 2n-3$.

Also, $i-1 \prec \left\lceil \frac{n-2}{3} \right\rceil$ or $i-1 \succ 2n-4$, $i-1 \prec \left\lfloor \frac{n-2}{3} \right\rfloor$ or $i-1 \succ 2n-5$.

Therefore, $\left\lceil \frac{n-1}{3} \right\rceil \le i-1 \le 2n-3$.

Therefore, $i-1 \prec \left\lceil \frac{n-2}{3} \right\rceil$ and $i-1 \prec \left\lfloor \frac{n-1}{3} \right\rfloor$ do not hold.

Therefore, $i-1 \succ 2n-4$. Therefore, $i-1 \ge 2n-3$.

But $i-1 \le 2n-3$.

Therefore, i=2n-2.

($\Leftarrow$) It follows from Lemma 2.2(iii), (iv).

iii) $D(P_{n-1}^{*2}, i-1) \ne \Phi$, $D(P_{n-1}^{*2} - \{2n-2\}, i-1) \ne \Phi$, $D(P_{n-2}^{*2}, i-1) \ne \Phi$ $D(P_{n-1}^{*2} - \{2n-4\}, i-1) \ne \Phi$ and $D(P_{n-3}^{*2} - \{2n-6\}, i-1) = \Phi$, So by Lemma 2.2(iii),(iv), we have, $\left\lceil \frac{n-1}{3} \right\rceil \le i-1 \le 2n-2$, $\left\lfloor \frac{n-1}{3} \right\rfloor \le i-1 \le 2n-3$, $\left\lceil \frac{n-2}{3} \right\rceil \le i-1 \le 2n-4$, $\left\lfloor \frac{n-2}{3} \right\rfloor \le i-1 \le 2n-5$ .Therefore $\left\lceil \frac{n-1}{3} \right\rceil \le i-1 \le 2n-5$.

Also, $i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor$ or $i-1 \succ 2n-7$ .Therefore, $i-1 \prec \left\lfloor \frac{n-3}{3} \right\rfloor$ does not hold.

Therefore, $i-1 \succ 2n-7$ .Therefore, $i-1 \ge 2n-6$ .But $i-1 \le 2n-5$.

Therefore, i-1= 2n-6 or 2n-5.

Therefore, i= 2n-5 or 2n-4.

**Theorem 3.3:**

i) $D(P_{n-1}^{*2}, i-1) = \Phi$, $D(P_{n-1}^{*2} - \{2n-2\}, i-1) \ne \Phi$, $D(P_{n-2}^{*2}, i-1) \ne \Phi$ and $D(P_{n-3}^{*2} - \{2n-6\}, i-1) \ne \Phi$ then

 

$$D(P_n^{*2} - \{2n\}, i) = \{ X_1 \cup \{2n-1\} / X_1 \in D(P_{n-1}^{*2} - \{2n-2\}, i-1),$$
$$X_2 \cup \{2n-2\} / X_2 \in D(P_{n-1}^{*2}, i-1),$$
$$X_3 \cup \{2n-3\} / X_3 \in D(P_{n-2}^{*2} - \{2n-4\}, i-1),$$
$$X_4 \cup \{2n-5\} / X_4 \in D(P_{n-3}^{*2} - \{2n-6\}, i-1), \}$$

ii) $D(P_{n-1}^{*2}, i-1) \neq \Phi$ and $D(P_{n-1}^{*2} - \{2n-2\}, i-1) = \Phi, D(P_{n-2}^{*2}, i-1) = \Phi$ , $D(P_{n-3}^{*2} - \{2n-6\}, i-1) \neq \Phi$ then $D(P_n^{*2} - \{2n\}, i) = \{ X \cup \{2n-1\} / X \in D(P_{n-1}^{*2}, i-1) \}$

**Proof:**

i) Let

$$Y_1 = \{ X_1 \cup \{2n-1\} / X_1 \in D(P_{n-1}^{*2} - \{2n-2\}, i-1)$$
$$Y_2 = \{ X_2 \cup \{2n-2\} / X_2 \in D(P_{n-1}^{*2}, i-1)\}$$
$$Y_3 = \{ X_3 \cup \{2n-3\} / X_3 \in D(P_{n-2}^{*2} - \{2n-4\}, i-1)\}$$
$$Y_4 = \{ X_4 \cup \{2n-5\} / X_4 \in D(P_{n-3}^{*2} - \{2n-6\}, i-1)\}$$

Obviously, $Y_1 \cup Y_2 \cup Y_3 \cup Y_4 \subseteq D(P_n^{*2} - \{2n\}, i)$ (2.1)

Now let $Y \in D(P_n^{*2} - \{2n\}, i)$ .If $2n \in Y$ ,then at least one of vertices labeled 2n-1 or 2n-2 is in Y.In either cases, $Y = X_1 \cup \{2n\}$ *forsome* $X_1 \in D(P_{n-1}^{*2} - \{2n-2\}, i-1)$;that is $Y \in Y_1$.

If 2n $\notin Y$ and 2n-1 $\in Y$ ,then 2n-2 $\in Y$ ,so $Y = X_2 \cup \{2n-1\}$ *forsome* $D(P_{n-1}^{*2}, i-1)$ that is $Y \in Y_2$ .Now suppose that2n-2 $\in Y$, 2n $\notin Y$ and 2n-1 $\notin Y$ , then at least one of vertices labeled 2n-3,2n-4 is in Y. If 2n-4 $\in Y$, then $Y = X_3 \cup \{2n-3\}$ *forsome* $X_3 \in D(P_{n-2}^{*2} - \{2n-4\}, i-1)$ , that is Y $\in Y_3$. Now suppose that 2n-5 $\in Y$, 2n-3 $\notin Y$ and 2n-4 $\notin Y$, then at least one of vertices labeled 2n-3, 2n-5 is in Y. If 2n-4 $\in Y$, then $Y = X_4 \cup \{2n-5\}$ *forsome* $X_4 \in D(P_{n-3}^{*2} - \{2n-6\}, i-1)$;
that is Y $\in Y_4$.

Thus we have proved

$D(P_n^{*2} - \{2n\}, i) \subseteq Y_1 \cup Y_2 \cup Y_3 \cup Y_4$ $D(P_n^{*2}, i) \subseteq Y_1 \cup Y_2 \cup Y_3$ (2.2)

From (2.1) and (2.2)

We have

$$D(P_n^{*2} - \{2n\}, i) = \{ X_1 \cup \{2n-1\} / X_1 \in D(P_{n-1}^{*2} - \{2n-2\}, i-1),$$
$$X_2 \cup \{2n-2\} / X_2 \in D(P_{n-1}^{*2}, i-1),$$
$$X_3 \cup \{2n-3\} / X_3 \in D(P_{n-2}^{*2} - \{2n-4\}, i-1),$$
$$X_4 \cup \{2n-5\} / X_4 \in D(P_{n-3}^{*2} - \{2n-6\}, i-1), \}$$

ii) By lemma 2.1,i=2n-1.Therefore $D(P_n^{*2} - \{2n\}, i) = D(P_n^{*2} - \{2n\}, 2n-1) = \{[2n-1]\}$ and $D(P_{n-1}^{*2}, i-1) = D(P_{n-1}^{*2}, 2n-2) = \{[2n-2]\}$. So we have the result.

**Theorem3.4:** For every n $\geq$ 7

$$|D(P_n^{*2} - \{2n\}, i)| = |D(P_{n-1}^{*2}, i-1)| + |D(P_{n-1}^{*2} - \{2n-2\}, i-1)| + |D(P_{n-2}^{*2}, i-1)| + |D(P_{n-2}^{*2} - \{2n-4\}, i-1)| + |D(P_{n-3}^{*2} - \{2n-6\}, i-1)|$$

 

**Proof:** We consider the three cases in Theorem 2.2

i) By Theorem 2.2 $D(P_n^{*2} - \{2n\}, i) = A_1 \cup A_2 \cup A_3 \cup A_4$, where. $|A_1| = D(P_{n-1}^{*2} - \{2n-2\}, i-1)$, $|A_2| = D(P_{n-2}^{*2}, i-1)$, $|A_3| = D(P_{n-2}^{*2} - \{2n-4\}, i-1)$, $|A_4| = D(P_{n-3}^{*2} - \{2n-6\}, i-1)$.
Since for every $X_1 \in A_1, X_2 \in X_1$ , and for every $X_2 \in A_2$, $2n \notin X_2$ ,so $A_1 \cap A_2 = \Phi$ .Also since for every $X_2 \in A_2$ and $X_3 \in A_3$, $2n-1 \in X_2, and\, 2n-1 \notin X_3$ ,we have $A_2 \cap A_3 = \Phi$ .For every $X_3 \in A_3$, $2n-1 \notin X_3$ ,and 2n-3 $\notin X_3$,but for every $X_1 \in A_1$, atleast one of 2n-3 or 2n-1 is in $X_3$ ,because $A_3 \cap A_4 = \Phi$ . For every $X_4 \in A_4$ for some so atleast one of 2n-3 of 2n-1 must be in $Y_4$ . $2n-1 \notin X_3$ ,and2n-3 $\notin X_3$,but for every $X_1 \in A_1$, atleast one of the 2n-3 or 2n-1 is in $X_4$ ,because $X_4 = Y_4 \cup \{2n\}$ for some $Y_4 \in D(P_n^{*2} - \{2n\}, i)$ so atleast one of the 2n-3 of 2n-1 must be $Y_4$ . Hence we have the result.

ii) By Theorem 2.2( ii), $|D(P_n^{*2} - \{2n\}, i-1)| = |D(P_{n-1}^{*2}, i-1)|$.Since in this case $|D(P_{n-1}^{*2} - \{2n-2\}, i-1)| = |D(P_{n-2}^{*2}, i-1)|$ =0 and $|D(P_{n-2}^{*2} - \{2n-4\}, i-1)| = |D(P_{n-3}^{*2} - \{2n-6\}, i-1)|$

= 0, we have the result in this case.

The following table illustrates the number of dominating sets are of $D(P_n^{*2}, i)$ and $D(P_n^{*2} - \{2n\}, i)$, for n = 1,2,…,10 and i = 1,2,…,20.

| i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| n | | | | | | | | | | | | | | | | | | | | |
| $P_1^{*2}$ | 2 | 1 | | | | | | | | | | | | | | | | | | |
| $P_2^{*2} - \{4\}$ | 3 | 3 | 1 | | | | | | | | | | | | | | | | | |
| $P_2^{*2}$ | 2 | 6 | 4 | 1 | | | | | | | | | | | | | | | | |
| $P_3^{*2} - \{6\}$ | 2 | 9 | 10 | 5 | 1 | | | | | | | | | | | | | | | |
| $P_3^{*2}$ | 1 | 8 | 18 | 15 | 6 | 1 | | | | | | | | | | | | | | |
| $P_4^{*2} - \{8\}$ | 1 | 10 | 29 | 33 | 21 | 7 | 1 | | | | | | | | | | | | | |
| $P_4^{*2}$ | 0 | 6 | 30 | 58 | 53 | 28 | 8 | 1 | | | | | | | | | | | | |
| $P_5^{*2} - \{10\}$ | 0 | 7 | 41 | 97 | 115 | 83 | 36 | 9 | 1 | | | | | | | | | | | |
| $P_5^{*2}$ | 0 | 3 | 32 | 110 | 193 | 190 | 118 | 45 | 10 | 1 | | | | | | | | | | |
| $P_6^{*2} - \{12\}$ | 0 | 3 | 39 | 158 | 331 | 402 | 316 | 164 | 55 | 11 | 1 | | | | | | | | | |
| $P_6^{*2}$ | 0 | 1 | 22 | 141 | 398 | 660 | 682 | 471 | 218 | 66 | 12 | 1 | | | | | | | | |
| $P_7^{*2} - \{14\}$ | 0 | 1 | 25 | 183 | 595 | 1149 | 1415 | 1189 | 698 | 285 | 78 | 13 | 1 | | | | | | | |
| $P_7^{*2}$ | 0 | 0 | 11 | 127 | 540 | 1377 | 2239 | 2421 | 1825 | 971 | 362 | 91 | 14 | 1 | | | | | | |
| $P_8^{*2} - \{16\}$ | 0 | 0 | 12 | 138 | 695 | 1979 | 3671 | 4688 | 4254 | 2797 | 1333 | 453 | 105 | 15 | 1 | | | | | |
| $P_8^{*2}$ | 0 | 0 | 4 | 97 | 606 | 2161 | 4907 | 7641 | 8462 | 6832 | 4064 | 1774 | 557 | 120 | 16 | 1 | | | | |
| $P_9^{*2} - \{18\}$ | 0 | 0 | 4 | 92 | 595 | 2767 | 7068 | 12548 | 16103 | 15294 | 10896 | 5838 | 2331 | 677 | 136 | 17 | 1 | | | |
| $P_9^{*2}$ | 0 | 0 | 1 | 43 | 510 | 2251 | 8056 | 17061 | 26066 | 29517 | 25208 | 16371 | 8078 | 2994 | 812 | 153 | 18 | 1 | | |
| $P_{10}^{*2} - \{20\}$ | 0 | 0 | 1 | 47 | 553 | 3001 | 10307 | 25117 | 43127 | 55583 | 54725 | 41579 | 39749 | 11080 | 3806 | 965 | 171 | 19 | 1 | |
| $P_{10}^{*2}$ | 0 | 0 | 0 | 17 | 320 | 2353 | 9998 | 29102 | 59414 | 89550 | 103191 | 92162 | 64241 | 50263 | 14766 | 31755 | 1135 | 190 | 20 | 1 |

 

## REFERENCES

[1] S. Alikhani and Y. H. Peng, Introduction to domination polynomial of a graph.arXiv:0905.2251v1 [math.co] 14 May 2009.

[2] S. Alikhani and Y. H. Peng, 2009, Domination sets and Domination Polynomials of paths, International journal of Mathematics and Mathematical Sciences. Article ID 542040.

[3] S. Alikhani and Y. H.P eng, Dominating sets of centipedes, Journal of Discrete Mathematical Sciences & Cryptography, Vol. 12 (2009), No. 4, pp. 411 – 428.

[4] G. Chartand and P. Zhang, Introduction to Graph Theory, McGraw-Hill, Boston, Mass, USA, 2005.

[5] T. W. Haynes, S. T. hedetniemi, and P. J. Slater, Fundamental of Domination in graphs,vol.208 of Monographs and Textbooks in Pure and Applied Mathematics, Marcel Dekker, New York,NY,USA,1998.

**Source of support: Nil, Conflict of interest: None Declared**