

## ON $bT$ -CLOSED SETS IN SUPRA TOPOLOGICAL SPACES

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### ABSTRACT

In this paper, we introduce a new class of set namely  $bT^\mu$ -closed sets in supra topological space. We further discuss the concept of  $bT^\mu$ -continuity and obtained their applications.

**Keywords:** supra  $bT$ -closed and supra  $bT$ -continuity.

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### 1. INTRODUCTION

In 1983 Mashhour et al [2] introduced Supra topological spaces and studied  $S$ - continuous maps and  $S^\mu$ -continuous maps. In 2010, Sayed *et al* [3] introduced and investigated several properties of supra  $b$ -open set and supra  $b$ -continuity. In 2011, Arockiarani and Trintia Pricilla [5] introduced and investigated several properties of a new type of sets called supra  $T$ -closed set and supra  $T$ -continuity maps. In this paper, we introduced the concept of  $bT^\mu$ -closed sets and study its basic properties. Also, we introduce the concept of  $bT^\mu$ -continuous functions and investigated several properties for these classes of functions in supra topological spaces.

### 2. PRELIMINARIES

**Definition: 2.1[2, 3]** A subfamily of  $\mu$  of  $X$  is said to be a supra topology on  $X$ , if

- (i)  $X, \phi \in \mu$
- (ii) if  $A_i \in \mu$  for all  $i \in J$  then  $\cup A_i \in \mu$ .

The pair  $(X, \mu)$  is called supra topological space. The elements of  $\mu$  are called supra open sets in  $(X, \mu)$  and complement of a supra open set is called a supra closed set.

**Definition: 2.2[3]** (i) The supra closure of a set  $A$  is denoted by  $cl^\mu(A)$  and is defined as  $cl^\mu(A) = \cap \{B : B \text{ is a supra closed set and } A \subseteq B\}$ .

(ii) The supra interior of a set  $A$  is denoted by  $int^\mu(A)$  and defined as  $int^\mu(A) = \cup \{B : B \text{ is a supra open set and } A \supseteq B\}$ .

**Definition: 2.3[2]** Let  $(X, \tau)$  be a topological spaces and  $\mu$  be a supra topology on  $X$ . We call  $\mu$  a supra topology associated with  $\tau$  if  $\tau \subset \mu$ .

**Definition: 2.4[3]** Let  $(X, \mu)$  be a supra topological space. A set  $A$  is called a supra  $b$ -open set if  $A \subseteq cl^\mu(int^\mu(A) \cup int^\mu(cl^\mu(A)))$ . The complement of a supra  $b$ -open set is called a supra  $b$ -closed set.

**Definition: 2.5[6]** Let  $(X, \mu)$  be a supra topological space. A set  $A$  of  $X$  is called supra generalized  $b$ -closed set (simply  $g^\mu b$ -closed) if  $bcl^\mu(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is supra open. The complement of supra generalized  $b$ -closed set is supra generalized  $b$ -open set.

**Definition: 2.6[5]** A subset  $A$  of  $(X, \mu)$  is called  $T^\mu$ -closed set if  $bcl^\mu(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g^\mu b$ -open in  $(X, \mu)$ . The complement of  $T^\mu$ -closed set is called  $T^\mu$ -open set.

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**Definition: 2.7[4]** A subset A of a supra topological space  $(X, \mu)$  is called supra regular open if  $A = \text{cl}^\mu(\text{int}^\mu(A))$ . The complement of supra regular open set is called supra regular closed set.

**Definition: 2.8[4]** A subset A of a supra topological space  $(X, \mu)$  is called supra generalized b- regular closed set if  $\text{bcl}^\mu(A) \subseteq U$  and whenever  $A \subseteq U$  and U is supra regular open of  $(X, \mu)$ . The complement of supra generalized b- regular closed set is called supra generalized b- regular open set.

### 3. BASIC PROPERTIES OF bT<sup>μ</sup>-CLOSED SETS

**Definition: 3.1** A subset A of a supra topological space  $(X, \mu)$  is called bT<sup>μ</sup>-closed set if  $\text{bcl}^\mu(A) \subseteq U$  whenever  $A \subseteq U$  and U is T<sup>μ</sup>-open in  $(X, \mu)$ .

The complement of supra bT<sup>μ</sup>-closed set is called supra bT<sup>μ</sup>-open set. We denote the family of all bT<sup>μ</sup>-closed set by bT<sup>μ</sup>(X, μ).

**Theorem: 3.2** Every supra closed set is bT<sup>μ</sup>-closed.

**Proof:** Let  $A \subseteq U$  and U is T<sup>μ</sup>-open set. Since A is supra closed then  $\text{cl}^\mu(A) = A \subseteq U$ . We know that  $\text{bcl}^\mu(A) \subseteq \text{cl}^\mu(A) \subseteq U$ , implies  $\text{bcl}^\mu(A) \subseteq U$ . Therefore A is bT<sup>μ</sup>-closed.

The converse of the above theorem need not be true as seen from the following example.

**Example: 3.3** Let  $X = \{a, b, c\}$  and  $\mu = \{X, \phi, \{a\}\}$ . Then the set  $\{a, c\}$  is bT<sup>μ</sup>-closed set in  $(X, \mu)$  but not supra closed.

**Theorem: 3.4** Every bT<sup>μ</sup>-closed set is g<sup>μ</sup>b-closed set.

**Proof:** Let  $A \subseteq U$  and U is supra open set. We know that every supra open set is T<sup>μ</sup>-open set, then U is T<sup>μ</sup>-open set. Since A is bT<sup>μ</sup>-closed set, we have  $\text{bcl}^\mu(A) \subseteq U$ . Therefore A is g<sup>μ</sup>b-closed set.

**Example: 3.5** Let  $X = \{a, b, c\}$ . and  $\mu = \{X, \phi, \{a\}\}$ . Then the set  $\{a, b\}$  is g<sup>μ</sup>b-closed but not bT<sup>μ</sup>-closed.

**Theorem: 3.6** Every bT<sup>μ</sup>-closed set is g<sup>μ</sup>br-closed set.

**Proof:** Let  $A \subseteq U$  and U is supra regular open set. We know that every supra regular open set is T<sup>μ</sup>-open set, then U is T<sup>μ</sup>-open set. Since A is bT<sup>μ</sup>-closed set, we have  $\text{bcl}^\mu(A) \subseteq U$ . Therefore A is g<sup>μ</sup>br-closed set.

**Example: 3.7** Let  $X = \{a, b, c\}$ . and  $\mu = \{X, \phi, \{a\}\}$ . Then the set  $\{a, b\}$  is g<sup>μ</sup>br-closed set but not bT<sup>μ</sup>-closed set.

**Theorem: 3.8** The union of two bT<sup>μ</sup>-closed set is bT<sup>μ</sup>-closed set.

**Proof:** Let A and B two bT<sup>μ</sup>-closed set. Let  $A \cup B \subseteq G$ , where G is T<sup>μ</sup>-open.

Since A and B are bT<sup>μ</sup>-closed sets. Therefore  $\text{bcl}^\mu(A) \cup \text{bcl}^\mu(B) \subseteq G$ . Thus  $\text{bcl}^\mu(A \cup B) \subseteq G$ . Hence  $A \cup B$  is bT<sup>μ</sup>-closed set.

**Theorem 3.9** Let A be bT<sup>μ</sup>-closed set of  $(X, \mu)$ . Then  $\text{bcl}^\mu(A) - A$  does not contain any non empty T<sup>μ</sup>-closed set.

**Proof: Necessity** Let A be bT<sup>μ</sup>-closed set. suppose  $F \neq \phi$  is a T<sup>μ</sup>-closed set of  $\text{bcl}^\mu(A) - A$ . Then  $F \subseteq \text{bcl}^\mu(A) - A$  implies  $F \subseteq \text{bcl}^\mu(A)$  and  $A^c$ . This implies  $A \subseteq F^c$ . Since A is bT<sup>μ</sup>-closed set,  $\text{bcl}^\mu(A) \subseteq U^c$ . Consequently,  $F \subseteq [\text{bcl}^\mu(A)]^c$ . Hence  $F \subseteq \text{bcl}^\mu(A) \cap [\text{bcl}^\mu(A)]^c = \phi$ . Therefore F is empty, a contradiction.

**Sufficiency:** Suppose  $A \subseteq U$  and that  $U$  is  $T^\mu$ -open. If  $\text{bcl}^\mu(A) \not\subseteq U$ . Then  $\text{bcl}^\mu(A) \cap U^c$  is a non empty  $T^\mu$ -closed subset of  $\text{bcl}^\mu(A) - A$ .

Hence  $\text{bcl}^\mu(A) \cap U^c = \emptyset$  and  $\text{bcl}^\mu(A) \subseteq U$ . Therefore  $A$  is  $bT^\mu$ -closed.

**Theorem: 3.10** If  $A$  is  $bT^\mu$ -closed set in a supra topological space  $(X, \mu)$  and  $A \subseteq B \subseteq \text{bcl}^\mu(A)$  then  $B$  is also  $bT^\mu$ -closed set.

**Proof:** Let  $U$  be  $T^\mu$ -open in set  $(X, \mu)$  such that  $B \subseteq U$ . Since  $A \subseteq B \Rightarrow A \subseteq U$  and since  $A$  is  $bT^\mu$ -closed set in  $(X, \mu)$   $\text{bcl}^\mu(A) \subseteq U$ , since  $B \subseteq \text{bcl}^\mu(A)$ . Then  $\text{bcl}^\mu(B) \subseteq U$ . Therefore  $B$  is also  $bT^\mu$ -closed set in  $(X, \mu)$

**Theorem: 3.11** Let  $A$  be  $bT^\mu$ -closed set then  $A$  is  $b^\mu$ -closed iff  $\text{bcl}^\mu(A) - A$  is  $T^\mu$ -closed.

**Proof:** Let  $A$  be  $bT^\mu$ -closed set. If  $A$  is  $b^\mu$ -closed, we have  $\text{bcl}^\mu(A) - A = \emptyset$ , which

is  $T^\mu$ -closed. Conversely, let  $\text{bcl}^\mu(A) - A$  is  $bT^\mu$ -closed. Then by the theorem 3.13,  $\text{bcl}^\mu(A) - A$  does not contain any non empty  $T^\mu$ -closed and  $\text{bcl}^\mu(A) - A = \emptyset$ . Hence  $A$  is  $b^\mu$ -closed.

**Theorem: 3.12** A subset  $A \subseteq X$  is  $bT^\mu$ -open iff  $F \subseteq \text{bint}^\mu(A)$  whenever  $F$  is  $T^\mu$ -closed and  $F \subseteq A$ .

**Proof:** Let  $A$  be  $bT^\mu$ -open set and suppose  $F \subseteq A$ , where  $F$  is  $T^\mu$ -closed. Then  $X - A$  is  $bT^\mu$ -closed set contained in the  $T^\mu$ -open set  $X - F$ . Hence  $\text{bcl}^\mu(X - A) \subseteq X - F$ . Thus  $F \subseteq \text{bint}^\mu(A)$ . Conversely, if  $F$  is  $T^\mu$ -closed set with  $F \subseteq \text{bint}^\mu(A)$  and  $F \subseteq A$ , then  $X - \text{bint}^\mu(A) \subseteq X - F$ . This implies that  $\text{bcl}^\mu(X - A) \subseteq X - F$ . Hence  $X - A$  is  $bT^\mu$ -closed. Therefore  $A$  is  $bT^\mu$ -open set.

**Theorem: 3.13** If  $B$  is  $T^\mu$ -open and  $bT^\mu$ -closed set in  $X$ , then  $B$  is  $b^\mu$ -closed.

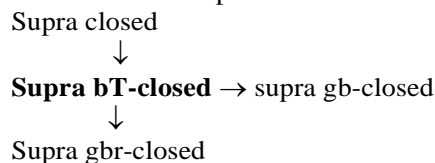
**Proof:** Since  $B$  is  $T^\mu$ -open and  $bT^\mu$ -closed then  $\text{bcl}^\mu(B) \subseteq B$ , but  $B \subseteq \text{bcl}^\mu(B)$ . Therefore  $B = \text{bcl}^\mu(B)$ . Hence  $B$  is  $b^\mu$ -closed.

**Corollary: 3.14** If  $B$  is supra open and  $bT^\mu$ -closed set in  $X$ . Then  $B$  is  $b^\mu$ -closed.

**Theorem: 3.15** Let  $A$  be supra  $g^\mu$ -b-open and  $bT^\mu$ -closed set. Then  $A \cap F$  is  $T^\mu$ -closed whenever  $F$  is supra b-closed.

**Proof:** Let  $A$  be supra  $g^\mu$ -b-open and  $bT^\mu$ -closed set then  $\text{bcl}^\mu(A) \subseteq A$  and also  $A \subseteq \text{bcl}^\mu(A)$ . Therefore  $\text{bcl}^\mu(A) = A$ . Hence  $A$  is supra b-closed. Since  $F$  is supra b-closed. Therefore  $A \cap F$  is supra b-closed in  $X$ . Hence  $A \cap F$  is  $T^\mu$ -closed in  $X$ .

From the above theorem and example we have the following diagram



#### 4. $bT^\mu$ -CONTINUOUS FUNCTIONS.

**Definition: 4.1** Let  $(X, \tau)$  and  $(Y, \sigma)$  be two topological spaces and  $\mu$  be an associated supra topology with  $\tau$ . A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called  $bT^\mu$ -Continuous if  $f^{-1}(V)$  is  $bT^\mu$ -closed in  $(X, \mu)$  for every closed set  $V$  of  $(Y, \sigma)$ .

**Definition: 4.2** Let  $(X, \tau)$  and  $(Y, \sigma)$  be two topological spaces and  $\mu$  be an associated supra topology with  $\tau$ . A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called  $bT^\mu$ -irresolute if  $f^{-1}(V)$  is  $bT^\mu$ -closed in  $(X, \mu)$  for every  $bT^\mu$ -closed set  $V$  of  $(Y, \sigma)$ .

**Theorem: 4.3** Every continuous function is  $bT^\mu$  - continuous.

**Proof:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a continuous function and  $A$  is closed in  $Y$ . Since  $f$  is continuous, then  $f^{-1}(A)$  is a closed set in  $X$ . Since  $\mu$  is associated with  $\tau$ , then  $\tau \subseteq \mu$ . Therefore  $f^{-1}(A)$  is supra closed in  $X$  and it is  $bT^\mu$ -closed in  $(X, \mu)$ . Hence  $f$  is  $bT^\mu$  - continuous.

**Remark: 4.4** The converse of the above theorem need not be true as seen from the following example.

**Example: 4.5** Let  $X=Y= \{a, b, c\}$ ,  $\tau = \{X, \phi\{a\}\}$  and  $\sigma = \{X, \phi, \{a\}\{b\}\{a, b\}\}$ .

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a function defined by  $f(a) = c$ ,  $f(b) = a$ ,  $f(c)=b$ . Let  $f^{-1}(\{b, c\}) = \{a, c\}$  is  $bT^\mu$ -closed but not closed. Then  $f$  is  $bT^\mu$ -continuous but not continuous.

**Theorem: 4.6** Every supra continuous function is  $bT^\mu$  - continuous.

**Proof:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a supra continuous and  $A$  is supra closed in  $Y$ . Since  $f$  is supra continuous, then  $f^{-1}(A)$  is supra closed in  $X$ . Since  $\mu$  is associated with  $\tau$ , then  $\tau \subseteq \mu$ . Therefore  $f^{-1}(A)$  is supra closed and it is  $bT^\mu$  - closed in  $(X, \mu)$ . Hence  $f$  is  $bT^\mu$  - continuous.

**Remark: 4.7** The converse of the above theorem need not be true as seen from the following example.

**Example: 4.8** Let  $X=\{a, b, c\}$ ,  $\tau = \{X, \phi, \{a\}\}$  and  $\sigma = \{Y, \phi, \{a\}\{b\}\{a, b\}\}$ . Let  $f:(X, \tau) \rightarrow (Y, \sigma)$  be a function defined by  $f(a) = c$ ,  $f(b) = a$ ,  $f(c) = b$ . Let  $f^{-1}(\{b, c\})=\{a, c\}$  is is  $bT^\mu$  -closed but not supra closed. Then  $f$  is  $bT^\mu$ -continuous but not supra continuous.

**Theorem: 4.9**

- (i) Every  $bT^\mu$  - continuous is  $g^\mu$ br - continuous.
- (ii) Every  $bT^\mu$ - irresolute is  $bT^\mu$  - continuous.

**Proof:** (i) Let  $f:(X, \tau) \rightarrow (Y, \sigma)$  be a  $bT^\mu$  - continuous function. Let  $V$  be a supra closed set in  $Y$ . Since  $f$  is  $bT^\mu$  -continuous,  $f^{-1}(V)$  is  $bT^\mu$  -closed in  $X$ . We know that every  $bT^\mu$  -closed is  $g^\mu$ br - closed set, then  $f^{-1}(V)$  is  $g^\mu$ br - closed set in  $X$ . Therefore  $f$  is  $g^\mu$ br -continuous.

(ii) Suppose  $f:(X, \tau) \rightarrow (Y, \sigma)$  be a  $bT^\mu$ - irresolute .Let  $V$  be any supra closed set in  $Y$ , then  $V$  is  $bT^\mu$ -closed. Since  $f$  is  $bT^\mu$ -irresolute,  $f^{-1}(V)$  is  $bT^\mu$  -closed in  $X$ . Hence  $f$  is  $bT^\mu$ -continuous.

**Remark: 4.10** The converse of the above theorem need not be true as seen from the following examples.

**Example: 4.11** (i) Let  $X=Y=\{a, b, c\}$ ,  $\tau = \{X, \phi\{a\}\{a, b\}\{b, c\}\}$  and  $\sigma = \{Y, \phi, \{a\}\}$ .

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a function defined by  $f(a) = c$ ,  $f(b) = b$ ,  $f(c) = a$ .  $f^{-1}(\{b, c\}) = \{a, b\}$  which is  $g^\mu$  br-continuous but not  $bT^\mu$  - continuous

**Example: 4.12** (ii) Let  $X=Y=\{a, b, c\}$ ,  $\tau = \{X, \phi, \{a\}\{a, b\}\{b, c\}\}$  and  $\sigma = \{Y, \phi, \{a\}\{a, b\}\}$ .

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a function defined by  $f(a) = c$ ,  $f(b) = b$ ,  $f(c) = a$ . Then  $f$  is  $bT^\mu$ -continuous. Since  $f^{-1}\{b, c\} = \{a, b\}$  is not  $bT^\mu$  closed in  $(X, \tau)$ .Therefore  $f$  is not  $bT^\mu$ -irresolute.

**Theorem: 4.13** Let  $f:(X, \tau) \rightarrow (Y, \sigma)$  and  $g:(Y, \sigma) \rightarrow (Z, \gamma)$  be any two function then

- (i)  $g \circ f$  is  $bT^\mu$  -continuous if  $g$  is supra continuous and  $f$  is  $bT^\mu$  -continuous.
- (ii)  $g \circ f$  is  $bT^\mu$ - irresolute if  $g$  is  $bT^\mu$  - irresolute and  $f$  is  $bT^\mu$  - irresolute.
- (iii)  $g \circ f$  is  $bT^\mu$  -continuous if  $g$  is  $bT^\mu$  -continuous and  $f$  is  $bT^\mu$  - irresolute.

**Proof:** (i) Let  $V$  be supra closed in  $(Z, \gamma)$ . Then  $g^{-1}(V)$  is supra closed in  $(Y, \sigma)$ . Since  $g$  is supra continuous, then  $f^{-1}(g^{-1}(V))=(gof)^{-1}(V)$  is  $bT^\mu$ -closed in  $(X, \tau)$ . Hence  $gof$  is  $bT^\mu$ -continuous.

(ii) Let  $V$  be  $bT^\mu$ -closed in  $(Z, \gamma)$ . Then  $g^{-1}(V)$  is  $bT^\mu$ -closed in  $(Y, \sigma)$ . Since  $g$  is  $bT^\mu$ -irresolute, then  $f^{-1}(g^{-1}(V))=(gof)^{-1}(V)$  is  $bT^\mu$ -closed in  $(X, \tau)$ . Hence  $gof$  is  $bT^\mu$ -irresolute.

(iii) Let  $V$  be supra closed in  $(Z, \gamma)$ . Then  $g^{-1}(V)$  is  $bT^\mu$ -closed in  $(Y, \sigma)$ . Since  $g$  is  $bT^\mu$ -continuous, then  $f^{-1}(g^{-1}(V))=(gof)^{-1}(V)$  is  $bT^\mu$ -closed in  $(X, \tau)$ . Hence  $gof$  is  $bT^\mu$ -continuous.

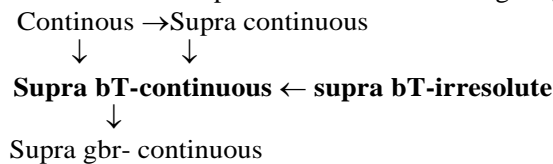
**Remark: 4.14** The composition of two  $bT^\mu$ -continuous function need not  $bT^\mu$ - continuous and it is shown by the following example.

**Example: 4.15** Let  $X = \{a, b, c, d\}$ ,  $\tau = \{X, \phi, \{a\} \{b\} \{a, b\}\}$  and  $\sigma = \{X, \phi, \{a\} \{c\} \{a, c\}\}$

Let  $f: (X, \tau) \rightarrow (X, \tau)$  be a function defined by  $f(a) = b, f(b) = c, f(c) = d$  and  $f(d)=a$ .

Let  $g: (X, \tau) \rightarrow (X, \sigma)$  be a function defined by  $g(a) = b, g(b) = c, g(c) = d$  and  $g(d)=a$ . Then  $f$  and  $g$  are  $bT^\mu$ -continuous, since  $\{b, c, d\}$  is supra closed in  $(X, \sigma)$ ,  $(gof)^{-1} \{b, c, d\} = \{a, b, d\}$  which is not  $bT^\mu$ -closed in  $(X, \tau)$ . Therefore  $gof$  is not  $bT^\mu$ - continuous.

From the above theorem and example we have the following diagram



### 5. APPLICATIONS

**Definition: 5.1** A supra topological space  $(X, \mu)$  is called  ${}_{bT}T_c^\mu$ -space. If every  $bT^\mu$ -closed set is supra closed set.

**Theorem: 5.2** Let  $(X, \tau)$  be a supra topological space then

- (i)  $O^\mu(\tau) \subset BT^\mu O(\tau)$
- (ii) A space  $(X, \tau)$  is  ${}_{bT}T_c^\mu$ -space iff  $O^\mu(\tau) = BT^\mu O(\tau)$ .

**Proof:**

(i) Let  $A$  be supra open. Then  $X-A$  is supra closed and so  $bT^\mu$ -closed. This implies that  $A$  is  $bT^\mu$ -open. Hence  $O^\mu(\tau) \subset BT^\mu O(\tau)$ .

(ii) Let  $(X, \tau)$  be  ${}_{bT}T_c^\mu$ -space. Let  $A \in BT^\mu O(\tau)$ , then  $X-A$  is  $bT^\mu$ -closed. By hypothesis  $X-A$  is supra closed and thus  $A \in O^\mu(\tau)$ . Hence  $O^\mu(\tau) = BT^\mu O(\tau)$ . Conversely, let  $O^\mu(\tau) = BT^\mu O(\tau)$ . Let  $A$  be  $bT^\mu$ -closed, then  $X-A$  is  $bT^\mu$ -open.

Hence  $X-A$  is supra open. Thus  $X$  is supra closed. This implies  $(X, \tau)$  is  ${}_{bT}T_c^\mu$ -space.

**Theorem: 5.3** If  $(X, \tau)$  is  ${}_{bT}T_c^\mu$ -space then for each  $x \in X$ ,  $\{x\}$  is either  $bT^\mu$ -closed or supra open.

**Proof:** Suppose  $(X, \tau)$  is  ${}_{bT}T_c^\mu$ -space. Let  $x \in X$  and assume that  $\{x\}$  is not supra open, then  $X-\{x\}$  is not supra closed. Then  $X-\{x\}$  is  $bT^\mu$ -closed. Since  $(X, \tau)$  is  ${}_{bT}T_c^\mu$ -space, then  $X-\{x\}$  is supra closed or equivalently  $\{x\}$  is supra open.

**Definition: 5.4** A supra topological space  $(X, \mu)$  is called  ${}_{gb}T_{bT}^\mu$ -space. If every  $g^\mu b$ -closed set is  $bT^\mu$ - closed set.

**Theorem: 5.5** Let  $(X, \tau)$  be a supra topological space then

- (i)  $BT^\mu O(\tau) \subset G^\mu BO(\tau)$
- (ii) A space  $(X, \tau)$  is  ${}_{gb}T_{bT}^\mu$ -space iff  $BT^\mu O(\tau) = G^\mu BO(\tau)$ .

**Proof:** (i) Let  $A$  be  $bT^\mu$ - open. Then  $X-A$  is  $bT^\mu$ - closed and so  $g^\mu b$ -closed. This implies that  $A$  is  $g^\mu b$ -open. Hence  $BT^\mu O(\tau) \subset G^\mu BO(\tau)$ .

(ii) Let  $(X, \tau)$  be  ${}_{gb}T_{bT}^\mu$ -space. Let  $A \in G^\mu BO(\tau)$ , then  $X-A$  is  $g^\mu b$ -closed. By hypothesis  $X-A$  is  $bT^\mu$ - closed and thus  $A \in BT^\mu O(\tau)$ . Hence  $BT^\mu O(\tau) = G^\mu BO(\tau)$ . Conversely, let  $BT^\mu O(\tau) = G^\mu BO(\tau)$ . Let  $A$  be  $g^\mu b$ -closed, then  $X-A$  is  $g^\mu b$ -open. Hence  $X-A$  is  $bT^\mu$ - open. Thus  $X$  is  $bT^\mu$ - closed. This implies  $(X, \tau)$  is  ${}_{gb}T_{bT}^\mu$ -space.

**Theorem: 5.6** If  $(X, \tau)$  is  ${}_{gb}T_{bT}^\mu$ -space then for each  $x \in X$ ,  $\{x\}$  is either  $g^\mu b$ -closed or  $bT^\mu$ - open.

**Proof:** Suppose  $(X, \tau)$  is  ${}_{gb}T_{bT}^\mu$ -space. Let  $x \in X$  and assume that  $\{x\}$  is not  $bT^\mu$ - open, then  $X-\{x\}$  is not  $bT^\mu$ -closed. Then  $X-\{x\}$  is  $g^\mu b$ -closed. Since  $(X, \tau)$  is  ${}_{gb}T_{bT}^\mu$ -space, then  $X-\{x\}$  is  $bT^\mu$ -closed or equivalently  $\{x\}$  is  $bT^\mu$ -open.

**Definition: 5.7** A supra topological space  $(X, \mu)$  is called  ${}_{gbr}T_{bT}^\mu$ - space. If every  $g^\mu br$ -closed set is  $bT^\mu$ - closed set.

**Theorem: 5.8** Let  $(X, \tau)$  be a supra topological space then

(i)  $BT^\mu O(\tau) \subset G^\mu BRO(\tau)$

(ii) A space  $(X, \tau)$  is  ${}_{bT}T_c^\mu$ -space iff  $BT^\mu O(\tau) = G^\mu BRO(\tau)$ .

**Proof:** (i) Let  $A$  be  $bT^\mu$ - open. Then  $X-A$  is  $bT^\mu$ - closed and so  $g^\mu br$ -closed. This implies that  $A$  is  $g^\mu br$ -open. Hence  $BT^\mu O(\tau) \subset G^\mu BRO(\tau)$ .

(ii) Let  $(X, \tau)$  be  ${}_{gbr}T_{bT}^\mu$ -space. Let  $A \in G^\mu BRO(\tau)$ , then  $X-A$  is  $g^\mu br$ -closed. By hypothesis  $X-A$  is  $bT^\mu$ - closed and thus  $A \in BT^\mu O(\tau)$ . Hence  $BT^\mu O(\tau) = G^\mu BRO(\tau)$ . Conversely, let  $BT^\mu O(\tau) = G^\mu BRO(\tau)$ . Let  $A$  be  $g^\mu br$ -closed, then  $X-A$  is  $g^\mu br$ -open. Hence  $X-A$  is  $bT^\mu$ - open. Thus  $X$  is  $bT^\mu$ - closed. This implies  $(X, \tau)$  is  ${}_{gbr}T_{bT}^\mu$ -space.

**Theorem: 5.9** If  $(X, \tau)$  is  ${}_{gbr}T_{bT}^\mu$ -space then for each  $x \in X$ ,  $\{x\}$  is either  $g^\mu br$ -closed or  $bT^\mu$ - open.

**Proof:** Suppose  $(X, \tau)$  is  ${}_{gbr}T_{bT}^\mu$ -space. Let  $x \in X$  and assume that  $\{x\}$  is not  $bT^\mu$ - open, then  $X-\{x\}$  is not  $bT^\mu$ -closed. Then  $X-\{x\}$  is  $g^\mu br$ -closed. Since  $(X, \tau)$  is  ${}_{gbr}T_{bT}^\mu$ -space, then  $X-\{x\}$  is  $bT^\mu$ -closed or equivalently  $\{x\}$  is  $bT^\mu$ -open.

**Theorem: 5.10**

(a) Every  ${}_{gbr}T_{bT}^\mu$ -space is  ${}_{bT}T_c^\mu$ -space.

(b) Every  ${}_{gbr}T_{bT}^\mu$ -space is  ${}_{gb}T_{bT}^\mu$ -space.

(c) Every  ${}_{bT}T_c^\mu$ -space is  ${}_{gb}T_{bT}^\mu$ -space.

**Proof:** It is obvious.

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