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On supra N-closed and supra sN-closed set in supra Topological spaces

L. Vidyarani* & M. Vigneshwaran

Department of Mathematics Kongunadu Arts and Science College, Coimbatore, (T.N.) India

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ABSTRACT

In this paper, we introduce new class of sets called supra N-closed sets and supra sN-closed sets. We obtain the basic properties and their relation- ships with other classes of sets in supra topological spaces.

Keywords: *supra N-closed, supra sN-closed, supra N-continuity, supra sN-continuity.*

1. INTRODUCTION

In 1983, A. S. Mashhour *et al.* [6] introduced the supra topological spaces and studied, continuous functions and s^* continuous functions. T. Noiri and O. R. Syed[5] introduced Ω closed sets. πΩ and πΩs closed sets in supra topological space was introduced by Arokiarani and M. Trinita Pricilla[1].

In this paper, we introduce the concept of supra N-closed sets and supra sN- closed sets and studied its basic properties. Also we introduce the concept of supra N and supra sN continuous functions and investigated their relationship with other classes of functions in supra topological spaces.

2. PRELIMINARIES

Definition: 2.1[6] A subfamily μ of X is said to be supra topology on X, if
i) X, φ ∈ μ
ii) If $A_i \in \mu$ $\forall i \in J$ then $\cup A_i \in \mu$

(X, μ) is called supra topological space.

The element of μ are called supra open sets in (X, μ) and the complement of supra open set is called supra closed sets and it is denoted by μ^c

Definition: 2.2[6] The supra closure of a set A is denoted by cl^μ (A), and is defined as supra cl(A) = ∩ {B: B is supra closed and A ⊆ B}.

The supra interior of a set A is denoted by int^μ (A), and is defined as supra int(A) =∪{B: B is supra open and A ⊇ B}.

Definition: 2.3[6] Let (X, τ) be a topological space and μ be a supra topology on X. We call μ a supra topology associated with τ, if τ ⊆ μ.

Definition: 2.4[4] Let (X, μ) be a supra topological space. A set A of X is called supra semi- open set, if A ⊆ $cl^\mu(int^\mu(A))$. The complement of supra semi-open set is supra semi-closed set.

Definition: 2.5[7] Let (X, μ) be a supra topological space. A set A of X is called supra α -open set, if A ⊆ $int^\mu(cl^\mu(int^\mu(A)))$. The complement of supra α -open set is supra α-closed set.

Definition: 2.6 [5] Let (X, μ) be a supra topological space. A set A of X is called supra Ω closed set, if $scl^\mu(A) \subseteq int^\mu(U)$,whenever A ⊆ U, U is supra open set. The complement of the supra Ω closed set is supra Ω open set.

Corresponding author: *L. Vidyarani**
Department of Mathematics Kongunadu Arts and Science College Coimbatore, India

Definition: 2.7 [5] Let (X, μ) be a supra topological space. A set A of X is called supra Ωs closed set, if scl^{μ} (A) ⊆ $int^{\mu}cl^{\mu}$(U), whenever A ⊆ U , U is supra open set. The complement of the supra Ωs closed set is supra Ωs open set.

Definition: 2.8 [5] The supra Ω closure of a set A is denoted by Ωcl^{μ} (A), and defined as Ωcl^{μ} (A) =∩ {B: B is supra Ω closed and A⊆ B}.

The supra Ω semi closure of a set A is denoted by Ωscl^{μ} (A), and defined as Ωscl^{μ} (A) = ∩ {B: B is supra Ω semi closed and A⊆B}.

3. Supra N-closed set and Supra sN-closed set

Definition: 3.1 Let (X, μ) be a supra topological space. A set A of X is called supra N-closed set if Ωcl^{μ} (A) ⊆ U, whenever A ⊆ U, U is supra α open set. The complement of supra N-closed set is supra N-open set.

Definition: 3.2 Let (X, μ) be a supra topological space. A set A of X is called supra sN-closed set if Ωscl^{μ} (A) ⊆ U, whenever A ⊆ U, U is supra α open set. The complement of supra sN-closed set is supra sN-open set.

Theorem: 3.3 Every supra closed set is supra N-closed.

Proof: Let A ⊆ U, U is α^{μ} open set, since A is supra closed set then cl^{μ} (A) = A ⊆ U. We know that Ωcl^{μ}(A) ⊆ cl^{μ} (A) ⊆ U, implies Ωcl^{μ}(A) ⊆ U. Therefore A is supra N-closed.

The converse of the above theorem need not be true as shown by the following example.

Example: 3.4 Let (X, μ) be a supra topological space, where X= {a, b, c} and μ = {X, φ, {a}}. Here{a, b} is supra N-closed set but it is not supra closed set.

Theorem: 3.5 Every supra semi closed set is supra N-closed set.

Proof: Let A ⊆ U, U is α^{μ} open set, since A is supra semi closed set then scl^{μ} (A) = A ⊆ U. We know that Ωcl^{μ}(A) ⊆ scl^{μ}(A) ⊆ U, implies Ωcl^{μ}(A) ⊆ U.Therefore A is supra N-closed.

The converse of the above theorem need not be true as shown by the following example.

Example: 3.6 Let (X, μ) be a supra topological space, where X= {a, b, c} and μ = {X, φ, {a}}. Here {a, b} is supra N-closed set but it is not supra semi closed set.

Theorem: 3.7 Every supra Ω closed set is supra N-closed set.

Proof: Let A ⊆ U, U is α^{μ} open set, since A is supra Ω closed set then Ωcl^{μ}(A) = A ⊆ U. Therefore A is supra N-closed.

The converse of the above theorem need not be true as shown by the following example.

Example: 3.8 Let (X, μ) be a supra topological space, where X= {a, b, c} and μ = {X, φ, {a}}. Here {a} is supra N-closed set but it is not supra Ω closed set.

Theorem: 3.9 Every supra N-closed set is supra sN-closed set.

Proof: Let A ⊆ U, U is α^{μ} open set, since A is supra N closed set we have Ωcl^{μ} (A) ⊆ U . We know that every Ω closed set is Ωsemi closed, implies Ωscl^{μ}(A) ⊆ Ωcl^{μ}(A) ⊆ U, Therefore A is supra sN-closed.

The converse of the above theorem need not be true as shown by the following example.

Example: 3.10 Let (X, μ) be a supra topological space, where X= {a, b, c} and μ = {X, φ, {a, b}, {b, c}}. Here {b} is supra sN-closed set but it is not supra N-closed set.

From the above theorem and examples we have the following diagram:

Supra closed
↓
supra Ω closed → **supra N-closed** ← supra semi closed
↓
supra Ωs-closed → **supra sN-closed**

4. Supra N-continuous, supra N-irresolute, supra sN-continuous and supra sN-irresolute functions

Definition: 4.1 Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A function f:(X, τ) → (Y, σ) is called supra N-continuous function if f^{-1}(V) is supra N-closed in (X, τ) for every supra closed set V of (Y, σ).

Definition: 4.2 Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A function f:(X, τ) → (Y, σ) is called supra sN-continuous function if f^{-1}(V) is supra sN-closed in (X, τ) for every supra closed set V of (Y, σ).

Definition: 4.3 Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A function f:(X, τ) → (Y, σ) is called supra N-irresolute if f^{-1}(V) is supra N-closed in (X, τ) for every supra N-closed set V of (Y, σ).

Definition: 4.4 Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A function f:(X, τ) → (Y, σ) is called supra sN-irresolute if f^{-1}(V) is supra sN-closed in (X, τ) for every supra sN-closed set V of (Y, σ).

Theorem: 4.5 Every supra continuous function is supra N-continuous.

Proof: Let f:(X, τ) → (Y, σ) be a supra continuous function. Le V be a supra closed set in Y. Since f is supra continuous, f^{-1} (V) is supra closed set in X. We know that every supra closed set is supra N-closed, then f^{-1}(V) is supra N-closed in X. Therefore f is supra N-continuous.

The converse of the above theorem need not be true. It is shown by the example given below.

Example: 4.6 Let X=Y= {a, b, c} and τ = {X, φ, {a}, {b, c}}, σ = {Y, φ, {a}}. f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=c, f(c)=a. Here f is supra N- continuous but it is not supra continuous, since V={b, c} is closed in Y but f^{-1}({b, c}) = {a, b} is not a closed set in X.

Theorem: 4.7 Every supra Ω-continuous function is supra N-continuous.

Proof: Let f:(X, τ) → (Y, σ) be a supra Ω-continuous function. Let V be a closed set in Y. Since f is supra Ω-continuous, f^{-1}(V) is supra Ω-closed set in X. We know that every supra Ω- closed set is supra N-closed, then f^{-1}(V) is supra N-closed in X. Therefore f is supra N-continuous.

The converse of the above theorem need not be true.

Example: 4.8 Let X=Y={a, b, c} and τ = {X, φ, {a}} , σ = {Y, φ, {a} , {b, c}}. f:(X, τ) → (Y, σ) be the function defined by f(a)=a, f(b)=c, f(c)=b. Here f is supra N- continuous but not supra Ω-continuous, since V={a} is closed in Y but f^{-1}({a}) = {a} is not a Ω-closed set in X.

Theorem: 4.9 Every supra N-continuous function is supra sN-continuous.

Proof: Let f :(X, τ) →(Y, σ) be a supra N-continuous function. Let V be a closed set in Y. Since f is supra N-continuous, $f^{-1}(V)$ is supra N-closed set in X. We know that every supra N-closed set is supra sN-closed, then $f^{-1}(V)$ is supra sN-closed in X. Therefore f is supra sN-continuous.

The converse of the above theorem need not be true.

Example: 4.10 Let X=Y= {a, b, c} and τ = {X, φ, {a}, {b}, {a, b} {b, c}}, σ = {Y, φ, {a, b}, {b, c}}.

f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=c, f(c)=a. Here f is supra sN-continuous but not supra N-continuous, since V={c} is closed in Y but $f^{-1}(\{c\}) = \{b\}$ is not a N-closed set in X.

Theorem: 4.11 Every supra N-irresolute is supra N-continuous.

Proof: Let f:(X, τ) → (Y, σ) be a supra N-irresolute. Let V be any supra closed set in Y, then V is supra N-closed set. since f is supra N-irresolute, $f^{-1}(V)$ is a supra N-closed set in X. Therefore f is supra N-continuous.

The converse of the above theorem need not be true.

Example: 4.12 Let X=Y= {a, b, c} and τ = {X, φ, {a, b} {b, c}}, σ = {Y, φ, {a}}. f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=a, f(c)=c. Here f is supra N- continuous but not supra N-irresolute, since V= {a, b} is N-closed in Y but $f^{-1}(\{a, b\}) = \{a, b\}$ is not an N-closed set in X.

Theorem: 4.13 Every supra N-irresolute is supra sN-continuous.

Proof: Let f:(X, τ) → (Y, σ) be a supra N-irresolute function. Let V be a closed set in Y. We know that every closed set is N-closed, therefore V is N-closed. since f is supra N-irresolute, $f^{-1}(V)$ is a supra N-closed set in X. we know that every supra N-closed set is sN-closed. Therefore f is supra sN-continuous./

The converse of the above theorem need not be true.

Example: 4.14 Let X=Y= {a, b, c} and τ = {X, φ, {a, b} {b, c}}, σ = {Y, φ, {a}}. f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=a, f(c)=c. Here f is supra sN- continuous but not supra N-irresolute, since V= {a, b} is N-closed in Y but $f^{-1}(\{a, b\}) = \{a, b\}$ is not an N-closed set in X.

Theorem: 4.15 Every supra N-irresolute is supra sN-irresolute.

Proof: Let f :(X, τ) → (Y, σ) be a supra N-irresolute function. Let V be N-closed set in Y. since f is supra N-irresolute, $f^{-1}(V)$ is a supra N-closed set in X. we know that every supra N-closed set is sN-closed. Therefore f is supra sN-irresolute.

The converse of the above theorem need not be true.

Example: 4.16 Let X={a, b, c} and τ ={X, φ, {a, b} {b, c}}, σ={X, φ, {a}}. f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=c, f(c)=a. Here f is supra sN- irresolute but not supra N-irresolute, since V={b, c} is N-closed in Y but $f^{-1}(\{b, c\}) = \{a, b\}$ is not an N-closed set in X.

From the above theorem and examples we have the following diagram:

Supra continuous **Supra N-irresolute** → **supra sN-irresolute**
↓ ↓
supra Ω continuous → **supra N-continuous** ← supra semi continuous
↓
supra Ωs-continuous → **supra sN-continuous**

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