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On supra N-closed and supra sN-closed set in supra Topological spaces

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ABSTRACT

In this paper, we introduce new class of sets called supra N-closed sets and supra sN-closed sets. We obtain the basic properties and their relation- ships with other classes of sets in supra topological spaces.

Keywords: *supra N-closed, supra sN-closed, supra N-continuity, supra sN-continuity.*

1. INTRODUCTION

In 1983, A. S. Mashhour *et al.* [6] introduced the supra topological spaces and studied, continuous functions and s^* continuous functions. T. Noiri and O. R. Syed[5] introduced Ω closed sets. $\pi\Omega$ and $\pi\Omega$s closed sets in supra topological space was introduced by Arokiarani and M. Trinita Pricilla[1].

In this paper, we introduce the concept of supra N-closed sets and supra sN- closed sets and studied its basic properties. Also we introduce the concept of supra N and supra sN continuous functions and investigated their relationship with other classes of functions in supra topological spaces.

2. PRELIMINARIES

Definition: 2.1[6] A subfamily μ of X is said to be supra topology on X, if
i) $X, \varphi \in \mu$
ii) If $A_i \in \mu\ \forall i \in J$ then $\cup A_i \in \mu$

(X, μ) is called supra topological space.

The element of μ are called supra open sets in (X, μ) and the complement of supra open set is called supra closed sets and it is denoted by μ^c

Definition: 2.2[6] The supra closure of a set A is denoted by $cl^\mu(A)$, and is defined as supra $cl(A) = \cap\{B: B$ is supra closed and $A \subseteq B\}$.

The supra interior of a set A is denoted by $int^\mu(A)$, and is defined as supra $int(A) = \cup\{B: B$ is supra open and $A \supseteq B\}$.

Definition: 2.3[6] Let (X, τ) be a topological space and μ be a supra topology on X. We call μ a supra topology associated with τ, if $\tau \subseteq \mu$.

Definition: 2.4[4] Let (X, μ) be a supra topological space. A set A of X is called supra semi- open set, if $A \subseteq cl^\mu(int^\mu(A))$. The complement of supra semi-open set is supra semi-closed set.

Definition: 2.5[7] Let (X, μ) be a supra topological space. A set A of X is called supra α -open set, if $A \subseteq int^\mu(cl^\mu(int^\mu(A)))$. The complement of supra α -open set is supra α-closed set.

Definition: 2.6 [5] Let (X, μ) be a supra topological space. A set A of X is called supra Ω closed set, if $scl^\mu(A) \subseteq int^\mu(U)$,whenever $A \subseteq U$, U is supra open set. The complement of the supra Ω closed set is supra Ω open set.

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Definition: 2.7 [5] Let (X, μ) be a supra topological space. A set A of X is called supra Ωs closed set, if $scl^{\mu}(A) \subseteq int^{\mu}cl^{\mu}(U)$, whenever A ⊆ U , U is supra open set. The complement of the supra Ωs closed set is supra Ωs open set.

Definition: 2.8 [5] The supra Ω closure of a set A is denoted by $\Omega cl^{\mu}(A)$, and defined as $\Omega cl^{\mu}(A) = \cap$ {B: B is supra Ω closed and A⊆ B}.

The supra Ω semi closure of a set A is denoted by $\Omega scl^{\mu}(A)$, and defined as $\Omega scl^{\mu}(A) = \cap$ {B: B is supra Ω semi closed and A⊆B}.

3. Supra N-closed set and Supra sN-closed set

Definition: 3.1 Let (X, μ) be a supra topological space. A set A of X is called supra N-closed set if $\Omega cl^{\mu}(A) \subseteq U$, whenever A ⊆ U, U is supra α open set. The complement of supra N-closed set is supra N-open set.

Definition: 3.2 Let (X, μ) be a supra topological space. A set A of X is called supra sN-closed set if $\Omega scl^{\mu}(A) \subseteq U$, whenever A ⊆ U, U is supra α open set. The complement of supra sN-closed set is supra sN-open set.

Theorem: 3.3 Every supra closed set is supra N-closed.

Proof: Let A ⊆ U, U is α^{μ} open set, since A is supra closed set then $cl^{\mu}(A) = A \subseteq U$. We know that $\Omega cl^{\mu}(A) \subseteq cl^{\mu}(A) \subseteq U$, implies $\Omega cl^{\mu}(A) \subseteq U$. Therefore A is supra N-closed.

The converse of the above theorem need not be true as shown by the following example.

Example: 3.4 Let (X, μ) be a supra topological space, where X= {a, b, c} and μ = {X, φ, {a}}. Here{a, b} is supra N-closed set but it is not supra closed set.

Theorem: 3.5 Every supra semi closed set is supra N-closed set.

Proof: Let A ⊆ U, U is α^{μ} open set, since A is supra semi closed set then $scl^{\mu}(A) = A \subseteq U$. We know that $\Omega cl^{\mu}(A) \subseteq scl^{\mu}(A) \subseteq U$, implies $\Omega cl^{\mu}(A) \subseteq U$.Therefore A is supra N-closed.

The converse of the above theorem need not be true as shown by the following example.

Example: 3.6 Let (X, μ) be a supra topological space, where X= {a, b, c} and μ = {X, φ, {a}}. Here {a, b} is supra N-closed set but it is not supra semi closed set.

Theorem: 3.7 Every supra Ω closed set is supra N-closed set.

Proof: Let A ⊆ U, U is α^{μ} open set, since A is supra Ω closed set then $\Omega cl^{\mu}(A) = A \subseteq U$. Therefore A is supra N-closed.

The converse of the above theorem need not be true as shown by the following example.

Example: 3.8 Let (X, μ) be a supra topological space, where X= {a, b, c} and μ = {X, φ, {a}}. Here {a} is supra N-closed set but it is not supra Ω closed set.

Theorem: 3.9 Every supra N-closed set is supra sN-closed set.

Proof: Let A ⊆ U, U is α^{μ} open set, since A is supra N closed set we have $\Omega cl^{\mu}(A) \subseteq U$. We know that every Ω closed set is Ωsemi closed, implies $\Omega scl^{\mu}(A) \subseteq \Omega cl^{\mu}(A) \subseteq U$, Therefore A is supra sN-closed.

The converse of the above theorem need not be true as shown by the following example.

Example: 3.10 Let (X, μ) be a supra topological space, where X= {a, b, c} and μ = {X, φ, {a, b}, {b, c}}. Here {b} is supra sN-closed set but it is not supra N-closed set.

From the above theorem and examples we have the following diagram:

$$\begin{array}{ccccc} & & \text{Supra closed} & & \\ & & \downarrow & & \\ \text{supra } \Omega \text{ closed} & \rightarrow & \textbf{supra N-closed} & \leftarrow & \text{supra semi closed} \\ & & \downarrow & & \\ \text{supra } \Omega\text{s-closed} & \rightarrow & \textbf{supra sN-closed} & & \end{array}$$

4. Supra N-continuous, supra N-irresolute, supra sN-continuous and supra sN-irresolute functions

Definition: 4.1 Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A function f:(X, τ) → (Y, σ) is called supra N-continuous function if f^{-1}(V) is supra N-closed in (X, τ) for every supra closed set V of (Y, σ).

Definition: 4.2 Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A function f:(X, τ) → (Y, σ) is called supra sN-continuous function if f^{-1}(V) is supra sN-closed in (X, τ) for every supra closed set V of (Y, σ).

Definition: 4.3 Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A function f:(X, τ) → (Y, σ) is called supra N-irresolute if f^{-1}(V) is supra N-closed in (X, τ) for every supra N-closed set V of (Y, σ).

Definition: 4.4 Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A function f:(X, τ) → (Y, σ) is called supra sN-irresolute if f^{-1}(V) is supra sN-closed in (X, τ) for every supra sN-closed set V of (Y, σ).

Theorem: 4.5 Every supra continuous function is supra N-continuous.

Proof: Let f:(X, τ) → (Y, σ) be a supra continuous function. Le V be a supra closed set in Y. Since f is supra continuous, f^{-1} (V) is supra closed set in X. We know that every supra closed set is supra N-closed, then f^{-1}(V) is supra N-closed in X. Therefore f is supra N-continuous.

The converse of the above theorem need not be true. It is shown by the example given below.

Example: 4.6 Let X=Y= {a, b, c} and τ = {X, φ, {a}, {b, c}}, σ = {Y, φ, {a}}. f:(X, τ) → (Y, σ) be the function defined by f(a)=b, f(b)=c, f(c)=a. Here f is supra N- continuous but it is not supra continuous, since V={b, c} is closed in Y but f^{-1}({b, c}) = {a, b} is not a closed set in X.

Theorem: 4.7 Every supra Ω-continuous function is supra N-continuous.

Proof: Let f:(X, τ) → (Y, σ) be a supra Ω-continuous function. Let V be a closed set in Y. Since f is supra Ω-continuous, f^{-1}(V) is supra Ω-closed set in X. We know that every supra Ω- closed set is supra N-closed, then f^{-1}(V) is supra N-closed in X. Therefore f is supra N-continuous.

The converse of the above theorem need not be true.

Example: 4.8 Let X=Y={a, b, c} and τ = {X, φ, {a}} , σ = {Y, φ, {a} , {b, c}}. f:(X, τ) → (Y, σ) be the function defined by f(a)=a, f(b)=c, f(c)=b. Here f is supra N- continuous but not supra Ω-continuous, since V={a} is closed in Y but f^{-1}({a}) = {a} is not a Ω-closed set in X.

Theorem: 4.9 Every supra N-continuous function is supra sN-continuous.

Proof: Let f :(X, τ) $\rightarrow$(Y, σ) be a supra N-continuous function. Let V be a closed set in Y. Since f is supra N-continuous, f^{-1}(V) is supra N-closed set in X. We know that every supra N-closed set is supra sN-closed, then f^{-1}(V) is supra sN-closed in X. Therefore f is supra sN-continuous.

The converse of the above theorem need not be true.

Example: 4.10 Let X=Y= $\{a, b, c\}$ and $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\} \{b, c\}\}$, $\sigma = \{Y, \varphi, \{a, b\}, \{b, c\}\}$.

f:(X, τ) $\rightarrow$ (Y, σ) be the function defined by f(a)=b, f(b)=c, f(c)=a. Here f is supra sN-continuous but not supra N-continuous, since V=$\{c\}$ is closed in Y but $f^{-1}(\{c\}) = \{b\}$ is not a N-closed set in X.

Theorem: 4.11 Every supra N-irresolute is supra N-continuous.

Proof: Let f:(X, τ) $\rightarrow$ (Y, σ) be a supra N-irresolute. Let V be any supra closed set in Y, then V is supra N-closed set. since f is supra N-irresolute, f^{-1} (V) is a supra N-closed set in X. Therefore f is supra N-continuous.

The converse of the above theorem need not be true.

Example: 4.12 Let X=Y= $\{a, b, c\}$ and $\tau = \{X, \varphi, \{a, b\} \{b, c\}\}$, $\sigma = \{Y, \varphi, \{a\}\}$. f:(X, τ) $\rightarrow$ (Y, σ) be the function defined by f(a)=b, f(b)=a, f(c)=c. Here f is supra N- continuous but not supra N-irresolute, since V= $\{a, b\}$ is N-closed in Y but $f^{-1}(\{a, b\}) = \{a, b\}$ is not an N-closed set in X.

Theorem: 4.13 Every supra N-irresolute is supra sN-continuous.

Proof: Let f:(X, τ) $\rightarrow$ (Y, σ) be a supra N-irresolute function. Let V be a closed set in Y. We know that every closed set is N-closed, therefore V is N-closed. since f is supra N-irresolute, f^{-1} (V) is a supra N-closed set in X. we know that every supra N-closed set is sN-closed. Therefore f is supra sN-continuous./

The converse of the above theorem need not be true.

Example: 4.14 Let X=Y= $\{a, b, c\}$ and $\tau = \{X, \varphi, \{a, b\} \{b, c\}\}$, $\sigma = \{Y, \varphi, \{a\}\}$. f:(X, τ) $\rightarrow$ (Y, σ) be the function defined by f(a)=b, f(b)=a, f(c)=c. Here f is supra sN- continuous but not supra N-irresolute, since V= $\{a, b\}$ is N-closed in Y but $f^{-1}(\{a, b\}) = \{a, b\}$ is not an N-closed set in X.

Theorem: 4.15 Every supra N-irresolute is supra sN-irresolute.

Proof: Let f :(X, τ) $\rightarrow$ (Y, σ) be a supra N-irresolute function. Let V be N-closed set in Y. since f is supra N-irresolute, f^{-1}(V) is a supra N-closed set in X. we know that every supra N-closed set is sN-closed. Therefore f is supra sN-irresolute.

The converse of the above theorem need not be true.

Example: 4.16 Let X=$\{a, b, c\}$ and $\tau = \{X, \varphi, \{a, b\} \{b, c\}\}$, $\sigma = \{X, \varphi, \{a\}\}$. f:(X, τ) $\rightarrow$ (Y, σ) be the function defined by f(a)=b, f(b)=c, f(c)=a. Here f is supra sN- irresolute but not supra N-irresolute, since V=$\{b, c\}$ is N-closed in Y but $f^{-1}(\{b, c\}) = \{a, b\}$ is not an N-closed set in X.

From the above theorem and examples we have the following diagram:

Supra continuous **Supra N-irresolute** $\rightarrow$ **supra sN-irresolute**
$\downarrow$ $\downarrow$
supra Ω continuous $\rightarrow$ **supra N-continuous** $\leftarrow$ supra semi continuous
$\downarrow$
supra Ωs-continuous $\rightarrow$ **supra sN-continuous**

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