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ON G/A - SZEGED INDEX OF STANDARD GRAPHS

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#### Abstract

A modern graph invariant is Szeged Index and it has considerable applications in molecular chemistry. A recently introduced graph invariant is G/A - Szeged Index and it has a numerable applications in chemistry. In this paper, the G/A - Szeged indices of standard graphs are calculated. A modified G/A - Szeged Index of a graph is also introduced in which all the vertices of the graph are taken into consideration, thereby the variations in these indices of standard graphs are identified.


Keywords: Szeged Index, G/A - Szeged Index, modified G/A - Szeged Index.

## 1. INTRODUCTION

The useful concepts related to a molecular graph associated with alkanes are mainly Wiener index (See[5]) and Szeged index (see[3]). A recently introduced concept (see [2]) is $\mathrm{GA}_{2}$ Index and we coined it as G/A- Szeged Index (Geometric mean by Arthimetic mean - Szeged Index). As usual, no standard formula is available to find out this index as well, for any connected graph. In §2, we calculate the G/A - Szeged indices of standard graphs and in §3, we introduce modified G/A - Szeged index and observe the variation of these indices for the standard graphs.

Throughout this paper, we consider only non-empty, simple, finite and connected graph to avoid trivialities.
For the standard notation and results we refer Bondy \& Murthy [1].
For ready reference, we give the following:
Definition 1.1 [3]: $G$ is a (non-empty, simple, finite and connected) graph with vertex set $V(G)$ and edge set $E(G)$. Then the Szeged index of $G$, denoted by $\operatorname{Sz}(G)$, is defined to be $\sum_{e \in E(G)} n_{1}(e) \cdot n_{2}(e)$, where e $=u v, N_{1}(e \mid G)=\{w \in$ $\mathrm{V}(\mathrm{G}): \mathrm{d}(\mathrm{w}, \mathrm{u})<\mathrm{d}(\mathrm{w}, \mathrm{v})\}, \mathrm{N}_{2}(\mathrm{e} \mid \mathrm{G})=\{\mathrm{w} \in \mathrm{V}(\mathrm{G}): \mathrm{d}(\mathrm{w}, \mathrm{v})<\mathrm{d}(\mathrm{w}, \mathrm{u})\}$ and $\mathrm{n}_{1}(\mathrm{e} \mid \mathrm{G})=\mid \mathrm{N}_{1}\left(\mathrm{e}|\mathrm{G}|, \mathrm{n}_{2}(\mathrm{e} \mid \mathrm{G})=\left|\mathrm{N}_{2}(\mathrm{e} \mid \mathrm{G})\right|\right.$. (' $d$ ' denotes the distance function and ' $\|$ ', the cardinality function ).

When there is only one graph G, under consideration, we write 'e' only instead of 'e $\mid G$ '.

## Observations 1.2 [4]:

a) For the graph $K_{n}(n \geq 2), S z\left(K_{n}\right)=n(n-1) / 2$.
b) For the complete graph $K_{m, n}(m, n \geq 1), \operatorname{Sz}\left(K_{m, n}\right)=(m n)^{2}$.
c) For the cycle $C_{k}(k \geq 3), \operatorname{Sz}\left(\mathrm{C}_{\mathrm{k}}\right)=\mathrm{k}\left[\frac{k}{2}\right]^{2}$ ( [ ] denotes integral part).
d) For the path $\mathrm{P}_{\mathrm{n}}(\mathrm{n} \geq 2), \mathrm{Sz}\left(\mathrm{P}_{\mathrm{n}}\right)=\mathrm{n}\left(\mathrm{n}^{2}-1\right) / 6$.
e) For the wheel $\left(K_{1} \vee C_{n}\right)(n \geq 3), S z\left(K_{1} v C_{n}\right)=n\left\{(n-2)+\left[\frac{n}{2}\right]^{2}\right\}$.

## 2. G/A - SZEGED INDEX OF STANDARD GRAPHS

For convenience, we recollect the following:

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Definition 2.1 [2]: G is a graph (i.e non- empty, simple, finite and connected graph). Let 'e' be any edge of G. Then the
$\mathrm{G} / \mathrm{A}$ - Szeged index of G is defined to be $\sum_{e \in E(G)} \frac{\sqrt{n_{1}(e) n_{2}(e)}}{\left[n_{1}(e)+n_{2}(e)\right] / 2}$.

Theorem 2.2: For the complete graph $K_{n}(n \geq 2), G / A-S z\left(K_{n}\right)=n(n-1) / 2$.
Proof: For any e $\epsilon E\left(K_{n}\right)$, by Th.2.3 in [ 4 ], $n_{1}(e)=n_{2}(e)=1$.

$$
\begin{aligned}
\text { So } \mathrm{G} / \mathrm{A}-\mathrm{Sz}\left(\mathrm{~K}_{\mathrm{n}}\right) & =\sum_{e \in E\left(K_{n}\right)} 2 \frac{\sqrt{(1)(1)}}{(1+1)} \\
& =\left|E\left(\mathrm{~K}_{\mathrm{n}}\right)\right| \\
& =\frac{n(n-1)}{2} .
\end{aligned}
$$

Theorem 2.3: For the complete bipartite graph $K_{m, n}(m, n \geq 1)$,

$$
\mathrm{G} / \mathrm{A}-\mathrm{Sz}\left(\mathrm{~K}_{\mathrm{m}, \mathrm{n}}\right)=\frac{2(m n)^{3 / 2}}{m+n}
$$

Proof: For e $\epsilon E\left(K_{m, n}\right)$, by Th. (2.5) in [ 4$], n_{1}(e)=n$ and $n_{2}(e)=m$, further $\left|E\left(K_{m, n}\right)\right|=m n$.

$$
\begin{aligned}
\text { So } \mathrm{G} / \mathrm{A}-\mathrm{Sz}\left(\mathrm{~K}_{\mathrm{m}, \mathrm{n}}\right) & =\sum_{e \in E\left(K_{m, n}\right)} 2 \frac{\sqrt{(n)(m)}}{(n+m)} \\
& =2 \frac{\sqrt{(m)(n)}}{(m+n)}\left|E\left(K_{m, n}\right)\right| \\
& =\frac{2(m n)^{3 / 2}}{m+n}
\end{aligned}
$$

Theorem 2.4: For the cycle $C_{k}(k \geq 3), G / A-S z\left(C_{k}\right)=k$.
Proof: Clearly $\left|\mathrm{E}\left(\mathrm{C}_{\mathrm{k}}\right)\right|=\mathrm{k}$.
We divide this into two cases.
Case (i): k is even and say $\mathrm{k}=2 \mathrm{n}(\mathrm{n} \geq 2)$.
For any e $\epsilon E\left(C_{2 n}\right)$, by Th. (2.6) in [4], $n_{1}(e)=n_{2}(e)=n$.

$$
\text { So } \mathrm{G} / \mathrm{A}-\mathrm{Sz}\left(\mathrm{C}_{2 \mathrm{n}}\right)=\sum_{e \in E\left(C_{2 n}\right)} \frac{2 \sqrt{(n)(n)}}{n+n}
$$

$$
=\left|\mathrm{E}\left(\mathrm{C}_{2 \mathrm{n}}\right)\right|=2 \mathrm{n}
$$

Case (ii): $k$ is odd and any say $k=2 n-1(n \geq 2)$. For any e $\epsilon E\left(C_{2 n-1}\right)$, by Th. 2.6 in [ 4 ],

$$
\begin{aligned}
& \begin{aligned}
\mathrm{n}_{1}(\mathrm{e})=\mathrm{n}_{2}(\mathrm{e})=\mathrm{n}-1 & \\
\text { So G/A-Sz(C2n-1)} & =\sum_{e \in E\left(C_{2 n}\right)} \frac{2 \sqrt{(n-1)(n-1)}}{(n-1)+(n-1)} \\
& =\left|\mathrm{E}\left(\mathrm{C}_{2 \mathrm{n}-1}\right)\right| \\
& =2 \mathrm{n}-1 . \text { Hence follows the result. }
\end{aligned}
\end{aligned}
$$

Theorem 2.5: For the path $\mathrm{P}_{\mathrm{n}}(\mathrm{n} \geq 2), \mathrm{G} / \mathrm{A}-\mathrm{Sz}\left(\mathrm{P}_{\mathrm{n}}\right)=\frac{2}{n} \sum_{i=1}^{n-1} \sqrt{i(n-i)}$.
Proof: Let $V\left(P_{n}\right)=\left\{v_{1}, v_{2}, \ldots, v_{n}\right\}$. Any edge of $P_{n}$ is of the form $e_{i}=v_{i} v_{i+1}$ for $i=1 \ldots(n-1)$. For any $e_{i} \in E\left(P_{n}\right)$, by Th.2.2 in [4], $n_{1}\left(e_{i}\right)=i$ and $n_{2}\left(e_{i}\right)=(n-i)$.

So, $\mathrm{G} / \mathrm{A}-\mathrm{Sz}\left(\mathrm{P}_{\mathrm{n}}\right)=\sum_{i=1}^{n-1} 2 \frac{\sqrt{i(n-i)}}{(i+n-i)}=\frac{2}{n} \sum_{i=1}^{n-1} \sqrt{i(n-i)}$.

## Corollary 2.6:

a) $\mathrm{G} / \mathrm{A}-\mathrm{Sz}\left(\mathrm{P}_{2 \mathrm{~m}}\right)=\left\{\begin{array}{l}1 \text { if } \mathrm{m}=1 \\ \frac{4}{2 m}\left(\sum_{i=1}^{m-1} \sqrt{i(2 m-i)}+\frac{m}{2}\right) \text { for } \mathrm{m} \geq 2 .\end{array}\right.$

Observe that, with the convention $\sum_{i=r}^{s} \ldots=0$ if $\mathrm{s}<\mathrm{r}$, we get the index of $\mathrm{P}_{2}$ from the later formula by taking $\mathrm{m}=1$ ).
b) $\quad \mathrm{G} / \mathrm{A}-\mathrm{Sz}\left(\mathrm{P}_{2 \mathrm{~m}+1}\right)=\frac{4}{2 m+1}\left\{\left(\sum_{i=1}^{m-1} \sqrt{i(2 m-i)}\right)\right.$ for $\mathrm{m} \geq 1$.

Proof: By taking $n=2$ in Theorem (2.5), we get that $\mathrm{G} / \mathrm{A}-\mathrm{Sz}\left(\mathrm{P}_{2}\right)=\frac{2}{2} \sqrt{(1)(1)}=1$.
Let $\mathrm{n} \geq 4$ and be even. We can write $\mathrm{n}=2 \mathrm{~m}(\mathrm{~m} \geq 2)$.
So, $\mathrm{G} / \mathrm{A}-\mathrm{Sz}\left(\mathrm{P}_{2 \mathrm{~m}}\right)=\frac{2}{2 m}\left(\sum_{i=1}^{2 m-1} \sqrt{i(2 m-i)}\right)$

$$
\begin{equation*}
=\frac{2}{2 m}\left(\sum_{i=1}^{m-1} \sqrt{i(2 m-i)}+\sqrt{m(2 m-m)}+\sum_{i=m+1}^{2 m-1} i(2 m-i)\right) \tag{2.6.1}
\end{equation*}
$$

Replacing i by ( $2 \mathrm{~m}-\mathrm{i}$ ) in the third sum of (2.6.1), we observe that
$\sum_{i=m+1}^{2 m-1} i(2 m-i)=\sum_{i^{1}=1}^{m-1}\left(2 m-i^{1}\right)\left(i^{1}\right)=\sum_{i=1}^{m-1} \sqrt{i(2 m-i)}$.
Hence, from (2.6.1) we have

$$
\begin{aligned}
\mathrm{G} / \mathrm{A}-\mathrm{Sz}\left(\mathrm{P}_{2 \mathrm{~m}}\right) & =\frac{2}{2 m}\left[2 \sum_{i=1}^{m-1} \sqrt{i(2 m-i)}+m\right] \\
& =\frac{4}{2 m}\left[\sum_{i=1}^{m-1} \sqrt{i(2 m-i)}+\frac{m}{2}\right]
\end{aligned}
$$

This proves (a).
Let $\mathrm{n}=2 \mathrm{~m}+1(\mathrm{~m} \geq 1)$
$\begin{aligned} \text { Now G/A }-\operatorname{Sz}\left(\mathrm{P}_{2 \mathrm{~m}+1}\right) & =\frac{2}{2 m+1}\left[\sum_{i=1}^{2 m} \sqrt{i(2 m+1-i)}\right] \\ & =\frac{2}{2 m+1}\left[\sum_{i=1}^{m} \sqrt{i(2 m+1-i)}+\sum_{i=m+1}^{2 m} \sqrt{i(2 m+1-i)}\right]\end{aligned}$
Replacing ' i ' by ( $2 \mathrm{~m}+1-\mathrm{i}$ ') in the second sum, as in the previous case, we observe that this sum is same as the first one.
So, $\mathrm{G} / \mathrm{A}-\mathrm{Sz}\left(\mathrm{P}_{2 \mathrm{~m}+1}\right)=\frac{4}{2 m+1}\left[\sum_{i=1}^{m} \sqrt{i(2 m+1-i)}\right]$.

This proves (b) and thus the proof of the theorem is complete.

Theorem 2.7: For the wheel

$$
K_{1} \vee C_{n}(n \geq 3), G / A-S z\left(K_{1} \vee C_{n}\right)=n\left(1+\frac{2 \sqrt{n-2}}{n-1}\right)
$$

Proof: Let $V\left(K_{1} \vee C_{n}\right)=\left\{u_{0}, v_{1}, v_{2}, \ldots, v_{n}\right\}$ where $u_{0}$ is the centre (hub) of the wheel.

Now, $E\left(K_{1} v C_{n}\right)=\left\{u_{0} v_{i}: i=1,2, \ldots, n\right\} U\left\{v_{i} v_{i+1}: i=1,2, \ldots, n\right\}$ (with the convention $\left.v_{n+1}=v_{1}\right)$.
Denote $e_{i}=u_{0} v_{i}$ and $f_{i}=v_{i} v_{i+1}(i=1,2, n\}$. Now, from Th. 2.7 in [4], for any e $\in E\left(K_{1} v C_{n}\right), n_{1}(e)=n-2, n_{2}(e)=1$ and observe that the $f_{i}$ 's constitute $C_{n}$.

$$
\text { So, } \begin{aligned}
\mathrm{G} / \mathrm{A}-\mathrm{Sz}\left(\mathrm{~K}_{1} \mathrm{v}_{\mathrm{n}}\right) & =\sum_{i=1}^{n} \frac{2 \sqrt{n_{1}\left(e_{i}\right) n_{2}\left(e_{i}\right)}}{n_{1}\left(e_{i}\right)+n_{2}\left(e_{i}\right)}+G / A-S z(C n) \\
& =2 \sum_{i=1}^{n} \frac{\sqrt{(n-2)(1)}}{(n-2)+1}+n \text { (by virtue of Th.(2.4)) } \\
& =\frac{2 n}{n-1} \sqrt{(n-2)}+(n) \\
& =n\left(1+\frac{2 \sqrt{n-2}}{n-1}\right)
\end{aligned}
$$

## 3. MODIFIED G/A - SZEGED INDEX OF STANDARD GRAPHS

In the calculations of G/A - Szeged indices of $K_{n}(n \geq 2), C_{2 n-1}(n \geq 2)$ and $K_{1} v C_{n}(n \geq 3)$ the contribution of all the vertices of the corresponding graphs are not there. To avoid this, we propose the following modified index that involves all the vertices.

Definition 3.1: Let $G$ be a graph and $e=u v \epsilon E(G)$.

Denote $\mathrm{N}_{1}{ }^{*}(\mathrm{e} / \mathrm{G})=\{\mathrm{w} \in \mathrm{V}(\mathrm{G}): \mathrm{d}(\mathrm{w}, \mathrm{u}) \leq \mathrm{d}(\mathrm{w}, \mathrm{v})\}$,

$$
\mathrm{N}_{2}^{*}(\mathrm{e} / \mathrm{G})=\{\mathrm{w} \in \mathrm{~V}(\mathrm{G}): \mathrm{d}(\mathrm{w}, \mathrm{v})<\mathrm{d}(\mathrm{w}, \mathrm{u})\}
$$

and
$\mathrm{n}_{1}{ }^{*}(\mathrm{e} / \mathrm{G})=\left|\mathrm{N}_{1}{ }^{*}(\mathrm{e} / \mathrm{G})\right| \& \mathrm{n}_{2}{ }^{*}(\mathrm{e} / \mathrm{G})=\left|\mathrm{N}_{2}{ }^{*}(\mathrm{e} / \mathrm{G})\right|$.
The refined G/A - Szeged Index of G, denoted by G/A - Sz* ${ }^{*}(G)$ is defined as,
$\sum_{e \in E(G}\left\{2 \frac{\sqrt{n_{1}{ }^{*}(e / G) \cdot n_{2}{ }^{*}(e / G)}}{n_{1}{ }^{*}(e / G)+n_{2}{ }^{*}(e / G)}\right\}$
(Another way of defining this modified index is to keep < as it is in $\mathrm{N}_{1}{ }^{*}(\mathrm{e} / \mathrm{G})$ and changing < into $\leq$ in $\mathrm{N}_{2}{ }^{*}(\mathrm{e} / \mathrm{G})$.

Observation 3.2: For the graphs, $P_{n}(n \geq 2), K_{m, n}(m, n \geq 1), C_{2 n}(n \geq 2)$, we observe that this modified index is same as the previous one, since there are no leftout vertices.

Theorem 3.3: For $n \geq 2, G / A-\mathrm{Sz}^{*}\left(\mathrm{~K}_{\mathrm{n}}\right)=(\mathrm{n}-1)^{3 / 2}$.

Proof: For any e $\epsilon E\left(K_{n}\right)$, by Th. 3.3 in [ 4$], n_{1}{ }^{*}(\mathrm{e})=\mathrm{n}-1$ and $\mathrm{n}_{2}{ }^{*}(\mathrm{e})=1$.

$$
\text { So, } \begin{aligned}
\mathrm{G} / \mathrm{A}-\mathrm{Sz}\left(\mathrm{~K}_{1} \vee \mathrm{C}_{\mathrm{n}}\right) & =\sum_{e \in E\left(K_{n}\right)} \frac{2 \sqrt{(n-1)(1)}}{n-1+1} \\
& =\frac{2}{n} \sqrt{n-1}\left(\frac{n(n-1)}{2}\right) \\
& =(\mathrm{n}-1)^{3 / 2}
\end{aligned}
$$

Theorem 3.4: For $\mathrm{n} \geq 2, \mathrm{G} / \mathrm{A}-\mathrm{Sz}^{*}\left(C_{2 n-1}\right)=2 \sqrt{n(n-1)}(\leq 2 n-1)$

Proof: For any e $\epsilon \mathrm{E}\left(\mathrm{C}_{2 \mathrm{n}-1}\right)$, by Th. (3.4) in [4], $\mathrm{n}_{1}{ }^{*}\left(\mathrm{e}_{\mathrm{i}}\right)=\mathrm{n}$ and $\mathrm{n}_{2}{ }^{*}\left(\mathrm{e}_{\mathrm{i}}\right)=\mathrm{n}-1$.
So, G/A - Sz ${ }^{*}\left(C_{2 n-1}\right)=\sum_{e \in E\left(C_{2 n-1}\right)} \frac{2 \sqrt{n(n-1)}}{(n+n-1)}$

$$
\begin{aligned}
& =\frac{2}{2 n-1} \sqrt{n(n-1)}\left|E\left(C_{2 n-1}\right)\right| \\
& =\frac{2}{2 n-1} \sqrt{n(n-1)}(2 n-1) \\
& =2 \sqrt{n(n-1)}(\leq 2 n-1) \text { (since G.M. } \leq \text { A.M. })
\end{aligned}
$$

Observation 3.5: In the other way of defining the modified index, we get the same indices for $\mathrm{K}_{\mathrm{n}}$ and $\mathrm{C}_{2 \mathrm{n}-1}$ since $\mathrm{n}_{1}{ }^{*}(\mathrm{e})$ and $\mathrm{n}_{2}{ }^{*}(\mathrm{e})$ are interchanged in the corresponding calculations.

Theorem 3.6: For the wheel $K_{1} v C_{n}(n \geq 3)$,
$\mathrm{G} / \mathrm{A}-S z^{*}\left(K_{1} \vee C_{n}\right)=\left\{\begin{array}{l}\frac{2 n}{n+1}\left[\sqrt{n}+\sqrt{\frac{n}{2}\left(\frac{n}{2}+1\right)}\right] \text { if } \mathrm{n} \text { is even, } \\ \frac{n}{n+1}\left[2 \sqrt{n}+\sqrt{n^{2}+2 n-3}\right] \text { if } \mathrm{n} \text { is odd. }\end{array}\right.$
Proof: With the same notation of Th $2.7, \mathrm{n}_{1}{ }^{*}\left(\mathrm{e}_{\mathrm{i}}\right)=\mathrm{n}-2+2=\mathrm{n}$ and $\mathrm{n}_{2}{ }^{*}\left(\mathrm{e}_{\mathrm{i}}\right)=1$.
So,

$$
\begin{equation*}
\sum_{i=1}^{n} \frac{2 \sqrt{n_{1}^{*}\left(e_{i}\right) n_{2}^{*}\left(e_{i}\right)}}{n_{1}^{*}\left(e_{i}\right)+n_{2}^{*}\left(e_{i}\right)}=\frac{2 n}{n+1} \sqrt{n} \tag{3.6.1}
\end{equation*}
$$

Case (i): Suppose $n$ is even.
By Th. 3.6.1 in [4], we have
$\mathrm{n}_{1}{ }^{*}\left(\mathrm{f}_{\mathrm{i}}\right)=\mathrm{n} / 2+1$ and $\mathrm{n}_{2}{ }^{*}\left(\mathrm{f}_{\mathrm{i}}\right)=\mathrm{n} / 2$
So

$$
\begin{equation*}
\sum_{i=1}^{n} \frac{2 \sqrt{n_{1}^{*}\left(f_{i}\right) n_{2}^{*}\left(f_{i}\right)}}{n_{1}^{*}\left(f_{i}\right)+n_{2}^{*}\left(f_{i}\right)}=\frac{2}{n+1} n \sqrt{\frac{n}{2}\left(\frac{n}{2}+1\right)} \tag{3.6.2}
\end{equation*}
$$

By (3.6.1) and (3.6.2)
$\mathrm{G} / \mathrm{A}-\mathrm{Sz}^{*}\left(\mathrm{~K}_{1} \vee \mathrm{C}_{\mathrm{n}}\right)=\frac{2 n}{n+1}\left[\sqrt{n}+\sqrt{\frac{n}{2}\left(\frac{n}{2}+1\right)}\right]$.
Case (ii): Suppose n is odd.
By Th.3.6.2 in [4],
$\mathrm{n}_{1}{ }^{*}\left(\mathrm{f}_{\mathrm{i}}\right)=\frac{n-1}{2}+2=\frac{n+3}{2}$ and $\mathrm{n}_{1}{ }^{*}\left(\mathrm{f}_{\mathrm{i}}\right)=\frac{n-1}{2}$.
So, $\sum_{i=1}^{n} 2 \frac{\sqrt{n_{1}^{*}\left(f_{i}\right) n_{2}^{*}\left(f_{i}\right)}}{n_{1}^{*}\left(f_{i}\right)+n_{2}^{*}\left(f_{i}\right)}=\frac{2 n}{n+1} \sqrt{\left(\frac{n+3}{2}\right)\left(\frac{n-1}{2}\right)}$

$$
\begin{equation*}
=\frac{n}{n+1} \sqrt{n^{2}+2 n-3} \tag{3.6.3}
\end{equation*}
$$

By (3.6.1) and (3.6.3)
$\mathrm{G} / \mathrm{A}-\mathrm{Sz}^{*}\left(\mathrm{~K}_{1} \vee \mathrm{C}_{\mathrm{n}}\right)=\frac{n}{n+1}\left[2 \sqrt{n}+\sqrt{n^{2}+2 n-3}\right]$
This completes the proof of the theorem.
Observation 3.7: In the otherway of defining the modified Index, we get that $\left(\mathrm{n}_{\mathrm{i}}{ }^{*}\left(\mathrm{e}_{\mathrm{i}}\right)=(\mathrm{n}-2)\right.$ and $\left(\mathrm{n}_{2}{ }^{*}\left(\mathrm{e}_{\mathrm{i}}\right)=3\right.$ and the other relations remain the same. So,

$$
\sum_{i=1}^{n} \frac{2 \sqrt{n_{1}^{*}\left(e_{i}\right) n_{2}^{*}\left(e_{i}\right)}}{n_{1}^{*}\left(e_{i}\right)+n_{2}^{*}\left(e_{i}\right)}=\frac{2 n}{n+1}[\sqrt{3(n-2)}]
$$

Hence, $G / A-S z^{*}\left(K_{1} \vee C_{n}\right)=\left\{\begin{array}{l}\frac{2 n}{n+1}\left[\sqrt{3(n-2)}+\sqrt{\frac{n}{2}\left(\frac{n}{2}+1\right)}\right] \text { if } \mathrm{n} \text { is even, } \\ \frac{n}{n+1}\left[2 \sqrt{3(n-2)}+\sqrt{n^{2}+2 n-3}\right] \text { if } \mathrm{n} \text { is odd. }\end{array}\right.$

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