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GLOBAL EXISTENCE FOR IMPULSIVE ABSTRACT PARTIAL NEUTRAL FUNCTIONAL VOLTERRA-FREDHOLM INTEGRODIFFERENTIAL EQUATIONS

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ABSTRACT

In this paper we study the existence of global solutions for a class of impulsive abstract partial neutral functional Volterra-Fredholm integrodifferential equations with unbounded delay. The results are obtained by using semigroup theory and Leray-Schauder's Alternative theorem.

Keywords: *Impulsive systems; Neutral functional Volterra-Fredholm Integrodifferential equations; Leray-Schauder's Alternative theorem; Mild solutions; Global Solutions.*

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1. INTRODUCTION

In the past decades, the theory of impulsive integrodifferential equations has become an active area of investigation due to their applications in the fields such as mechanics, electrical engineering, medicine biology, ecology and so on. For some general and recent works on the theory of impulsive differential and integrodifferential equations, we refer the reader to [3, 4, 5, 6, 7, 8, 10, 16, 22]. Neutral differential equations arise in many areas of applied mathematics. Theory of neutral differential equations has been studied by several authors in Banach Spaces [1, 12]. Balachandran and Sakthivel [2] and Balachandran and Dauer [9] studied the existence of solutions for neutral functional integrodifferential equations in Banach spaces. In [23] we study the global existence of solutions for the initial value problems for the first and second order Volterra-Fredholm type neutral impulsive functional integrodifferential equations.

In [21], the existence of solutions for impulsive neutral functional differential equations of the form,

$$\frac{d}{dt}(u(t)+F(t,u_t)) = A(t)u(t)+G(t,u_t), \quad t\in I,\ t\neq t_i$$

$$\Delta u(t_i) = I_i(t_i)$$

$$u_0=\varphi\in\mathcal{B},$$

by using Leray-Schauder's alternative theorem are obtained.

In this paper, we establish the existence of mild solutions for a class of impulsive neutral functional Volterra-Fredholm integrodifferential equations with unbounded delays such as

$$\frac{d}{dt}(u(t)+g(t,u_t)) = Au(t)+f(t,u_t,\int_0^t h(t,s,u_s)ds,\int_0^a k(t,s,u_s)ds), \quad t\in I, t\neq t_i \tag{1.1}$$

$$\Delta u(t_i) = I_i(u_{t_i}), \quad i=1,2,,...,n. \tag{1.2}$$

$$u_0=\varphi\in\mathcal{B} \tag{1.3}$$

where A is the infinitesimal generator of a C_0 semigroup of bounded linear operators $(T(t))_{t\geq 0}$ defined on a Banach space $(\mathbb{X}, \|.\|)$; Here $0< t_1 < t_2 <\ldots<t_n<a$ are prefixed numbers; the history $u_t : (-\infty,0]\to \mathbb{X}$, $u_t(\theta) = u(t+\theta)$ belongs to some abstract phase space $\mathcal{B}$ defined axiomatically; $g:I \times \mathcal{B} \to \mathbb{X}$, $f : I \times \mathcal{B} \times X \times X\to X$, $h : Q \times \mathcal{B} \to X$, $k : Q \times \mathcal{B}\to X$, $I_i : \mathcal{B} \to X$, $i = 1, 2, \ldots, n$ are appropriate functions and $Q = \{(t,s) \in I \times I ;\ s \leq t\}$ and the symbol $\Delta\xi(t)$ represents the jump of the function ξ at t, which is defined by $\Delta\xi(t) = \xi(t^+)-\xi(t^-)$.

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The aim of this paper is to study the existence of global solutions for the impulsive neutral functional Volterra-Fredhom type integrodifferential equations described in the general "abstract" form (1.1)-(1.3).

In second section, we provide the definitions and preliminary results to be used in the theorems stated and proved in this article; we review some of the standard facts on phase spaces, mild solutions and certain useful fixed point theorems. Third section deals with the local existence of mild solutions for the problem (1.1)-(1.3). In fourth section is dedicated to the study of the existence of global solutions for the problem (1.1)-(1.3). Finally in the fifth section, we give an application.

2. PRELIMINARIES

We introduce certain notations which will be used throughout the paper without any further mention. Let $(\mathbb{X}, \|.\|_X)$ and $(\mathbb{Y},\|.\|_Y)$ be Banach spaces, and $\mathcal{L}(\mathbb{Y},\mathbb{X})$ be the Banach space of bounded linear operators from $\mathbb{Y}$ into $\mathbb{X}$ equipped with its natural topology; in particular, we use the notations $\mathcal{L}(\mathbb{X})$ when $\mathbb{Y}=\mathbb{X}$.

Throughout this work, A is the infinitesimal generator of a strongly continuous semigroup of bounded linear operators $(T(t))_{t\geq 0}$ defined on a Banach space $(X,\|.\|)$ and $\widetilde{M}$ is a positive constant such that $\|T(t)\| \leq \widetilde{M}$ for every $t\in I$. For the theory of strongly continuous semigroup, refer the reader to Pazy [19].

In this paper, we employ an axiomatic definition of the phase space $\mathcal{B}$, which is similar to the one introduced in Hino *et al* [15] and suitably modified to treat retarded impulsive differential equations. More precisely, $\mathcal{B}$ is a linear space of functions mapping $(-\infty,0]$ into X endowed with a seminorm $\|.\|_B$ and we assume that $\mathcal{B}$ satisfies the following axioms:

Axiom (A): If $x : (-\infty,\sigma+a] \to \mathbb{X}$, $a>0$, $\sigma\in\mathbb{R}$ such that $x_\sigma\in \mathcal{B}$ and $x|[\sigma,\sigma+a]\in PC([\sigma,\sigma+a], \mathbb{X})$, then for every $t\in[\sigma,\sigma+a)$ the following conditions hold:

(i) x_t is in $\mathcal{B}$.

(ii) $\|x(t)\|_x \leq H\|x_t\|_B$;

(iii) $\|x_t\|_B \leq K(t-\sigma)\sup\{\|x(s)\|_X : \sigma \leq s \leq t\} + M(t-\sigma)\|x_\sigma\|_B$,

where $H > 0$ is a constant; $K, M : [0,\infty) \to [1,\infty)$, K is continuous, M is locally bounded and H, K, M are independent of x(.).

Axiom (B): The space $\mathcal{B}$ is complete.

Remark 2.1:
In impulsive functional differential systems, the map $[\sigma,\sigma+a] \to \mathcal{B}$, $t\to x_t$ is in general discontinuous. For the reason, this property has been omitted from the description of phase space $\mathcal{B}$.

Example 2.1 The Phase Space $PC_r \times L^2(g, X)$

Let $r>0$ and $g:(-\infty,-r]\to R$ be a non negative, locally Lebesgue integrable function. Assume that there is a non negative measurable, locally bounded function $\eta(.)$ on $(-\infty, 0]$ such that $g(\xi+\theta)\leq\eta(\xi)g(\theta)$ for all $\xi\in(-\infty,0]$ and $\theta\in(-\infty,-r]\setminus N_\xi$ where $N_\xi\subset(-\infty,-r]$ is a set with Lebesgue measure zero. We denote by $PC_r\, x L^2(g,X)$ the set of all functions $\varphi:(-\infty,0]\to X$ such that $\varphi|[-r,0]\in PC([-r,0],X)$ and $\int_{-\infty}^{-r} g(\theta)\|\varphi(\theta)\|_X^2\, d\theta<+\infty$. In $PC_r\, x L^2(g,X)$, we consider the seminorm defined by

$$\|\varphi\|_B=\sup_{\theta\in[-r,0]}\|\varphi(\theta)\|_X+\left(\int_{-\infty}^{-r} g(\theta)\|\varphi(\theta)\|_X^2\, d\theta\right)^{1/2}$$

From the proceeding condition, the space $PC_r\, x L^2(g,X)$, satisfies the Axioms (A) and

(B). Moreover, when r = 0, we can take H = 1, $K(t)=\left(1+\int_{-t}^{0} g(\theta)d\theta\right)^{1/2}$ and $M(t)=\eta(-t)\ for\, t\geq 0$.

Let $0 < t_1 < t_2 < \ldots < t_n < a$ be pre fixed numbers. We introduce the space $PC = PC([0, a]; \mathbb{X})$ formed by all functions $u: [0,a] \to \mathbb{X}$ such that are continuous at $t \neq t_i$, $u(t_i^-) = u(t_i)$ and $u(t_i^+)$ exists, for all i = 1,2, … n.

In this paper, we assume that PC is endowed with the norm $\|u\|_{PC} = \sup_{s\in[0,a)} \|u(s)\|_X$. It is clear that ($PC$, $\|.\|_{PC}$) is a Banach space; see [14] for details.

In what follows, we put $t_0 = 0$, $t_{n+1}=a$, and for $u\in PC$, we denote by $\tilde{u}_i \in C([t_i,t_{i+1}], \mathbb{X})$, $i=0,1,2,\dots n$, the function given by

$$\tilde{u}_i = \begin{cases} u(t), & for \ \ t \in (t, t_{i+1}], \\ u(t_i^{+}), & for \ \ t = t_i. \end{cases}$$

Moreover, for $\mathcal{B} \subset PC$, we employ the notation $\tilde{B}_i$, $i=0,1,2,\dots,n$ for the sets
$\tilde{B}_i = \{ \tilde{u}_i : u\in B\}$

Lemma 2.1 ([14]). A set $\mathcal{B} \subset PC$ is relatively compact in PC if and only if the set $\tilde{B}_i$ is relatively compact in the space $C([t_i,t_{i+1}] ; \mathbb{X})$, for every $i = 0, 1, \dots, n$.

Definition 2.1 ([20]). A function $u: (-\infty,0]\cup I\to\mathbb{X}$ is called a mild solution of the abstract Cauchy problem (1.1)-(1.3), if $u_0 = \phi\in \mathcal{B}$; $u|_I\in PC(I; \mathbb{X})$; the function $s\to AT(t-s)g(s, u_s)$ is integrable in $[0,t)$ for all $t\in I$ and

$$\begin{aligned} u(t) = T(t)(\varphi(0)+g(0,\varphi)) - g(t,u_t) - \int_0^t AT(t-s)g(s,u_s)d\ s \\ + \int_0^t T(t-s) f\left(s, u_s, \int_0^s h(s,\tau,u_\tau)d\tau, \int_0^a k(s,\tau,u_\tau)d\tau \right) d\ \ s \\ + \sum_{t_i<t} T(t-t_i) I_i(u_{t_i}) + \sum_{t_i<t} [g(t,u_t)_{|t_i^+} - g(t,u_t)_{|t_i^-}], \quad t\in I \end{aligned} \tag{2.1}$$

Motivated by the previous definition, we introduce the following assumptions. There exists a Banach space ($\mathbb{Y}$, $\|.\|_{\mathbb{Y}}$) continuously included in $\mathbb{X}$ such that

(S$_1$) The function $s \to T(t-s)\, y\in C([t, +\infty); \mathbb{Y})$, $t \geq 0$.
(S$_2$) The function $s \to AT(t-s)$ defined from $(t, +\infty)$, $t > 0$ into $\mathcal{L}(\mathbb{Y},\mathbb{X})$ is continuous and there is a function $H(.)\in L^1([0,\infty); \mathbb{R}^+)$ such that

$\|AT(t-s)\|_{L(Y,X)} \leq H(t-s)$, for all $t > s$.

Remark 2.2. Assume that (S_1) and (S_2) hold and $u\in C([0,t]; \mathbb{Y})$, then from the Bochner's criterion for integrable functions and the estimate

$$\|AT(t-s)u(s)\|_X \leq \|AT(t-s)\|_{L(Y,X)}\|u(s)\|_Y \leq H(t-s)\|u(s)\|_Y,$$

We have that the function $s\to AT(t-s)u(s)$ is integrable over $[0,t)$ for all $t > 0$. For additional remarks about these types of condition in partial neutral differential equations, see e.g. [17]. In general, we observe that, except in trivial cases the operator function $s\to AT(t-s)$ is not integrable over $[0, t]$.

To obtain our results we need the following results.

Theorem 2.1 ([11, Theorem 6.5.4]) Leray-Schauder's Alternative Theorem). Let D be a closed convex subset of a Banach space ($\mathbb{Z}$, $\|.\|_{\mathbb{Z}}$) and assume that $0\in D$. If $F: D\to D$ is a completely continuous map, then the set $\{x\in D : x = \lambda F(x),\ 0 <\lambda<1\}$ is unbounded or the map F has a fixed point in D.

Theorem 2.2 ([18, corollary 4.3.2]). Let D be a closed, convex and bounded subset of a Banach space ($\mathbb{Z}$, $\|.\|_{\mathbb{Z}}$).
If $B, C: D \to \mathbb{Z}$ are continuous functions such that
(a) $Bz+Cz\in D$ for all $z\in \mathbb{Z}$,
(b) $\overline{C(D)}$ is compact
(c) There exists $0 \leq \gamma < 1$ such that $\|Bz-Bw\| \leq \gamma \|z-w\|$ for all $z,w\in D$, then there exists $x\in D$ such that $Bx+Cx = x$.

3. LOCAL EXISTENCE

In this section, we study the existence of mild solutions for the impulsive systems (1.1)-(1.3) when I=[0,a]. To obtain our results, we introduce the following conditions.

(H_1) The function $g : I \times \mathcal{B} \to \mathbb{X}$ is completely continuous, $g(I \times \mathcal{B}) \subset \mathbb{Y}$, $g \in C(I \times \mathcal{B}, \mathbb{Y})$ and there exist positive constants c_1 and c_2 such that

$$\|g(t,\phi)\|_Y \le c_1\|\phi\|_{\mathcal{B}} + c_2,\ t\in I,\ \phi\in\mathcal{B}.$$

(H_2) The function $f: I \times \mathcal{B} \times \mathbb{X} \times \mathbb{X} \to \mathbb{X}$ satisfies the following conditions .
(a) The function $f(t, ., ., .) : \mathcal{B} \times \mathbb{X} \times \mathbb{X} \to \mathbb{X}$ is continuous for all $t\in I$.
(b) The function $f(., u, v, w) : I \to \mathbb{X}$ is strongly measurable for all $u,v\in \mathcal{B}$.
(c) There exists a continuous function $m : I \to[0,\infty)$ and a continuous non decreasing function $W: [0,\infty) \to(0,\infty)$ such that

$$\|f(t, u, v, w)\| \le m(t)W(\|u\|_B + \|v\| + \|w\|),\quad t\in I=[0,a],\ u, v, w\in \mathcal{B} \times X \times X$$

(H_3) The maps $I_i : \mathcal{B} \to \mathbb{X}$ are completely continuous and uniformly bounded, $i\in\mathbb{F} =\{1, 2, \ldots, N\}$. In what follows we use the notation $N_i =\sup(\|I_i(\phi)\| : \phi\in \mathcal{B}\}$

(H_4) There are positive constants L_i, such that $\|I_i(\psi_1)-I_i(\psi_2)\| \le L_i\|\psi_1-\psi_2\|_B$, $\psi_1, \psi_2\in \mathcal{B}$, $i\in \mathbb{F}$.

(H_5) The function $g : I \times \mathcal{B} \to \mathbb{Y}$ is Lipchitz constant, that is, there is a constant $L_g > 0$ such that

$$\|g(t,v_1)-g(t,v_2)\|_Y \le L_g\|v_1-v_2\|_B,\quad t\in I,\ v_i\in \mathcal{B},\ i=1,2.$$

(H_6) Let $y : (-\infty,0] \cup I\to\mathbb{X}$ be the extension of ϕ such that $y(t) = T(t)\phi(0)$ for all $t\in I$ and S(I) the space $S(I) = \{x : (-\infty,0] \cup I\to\mathbb{X} : x_0 = 0,\ x_{|I}\in PC(I ; \mathbb{X})\}$ endowed with uniform convergence topology in I. Then the set of functions $\{t\to g(t,x_t+y_t);\ x\in Q\}$ is equi continuous in I for all bounded sets $Q\subset S(I)$.

(H_7) The function $h : Q \times \mathcal{B} \to\mathbb{X}$ satisfies the following conditions.
(a) For every $(t,s)\in Q$, the function $h(t, s, .) : \mathcal{B} \to\mathbb{X}$ is continuous.
(b) For each $x\in\mathcal{B}$ the function $h (., ., x) : Q\to\mathbb{X}$ is strongly measurable.
(c) there exists an integrable function $p : I \to[0,\infty)$ and a constant $\gamma > 0$ such that

$$\|h(t, s, x)\| \le \gamma p(s)\Omega(\|x\|_B),$$

where $\Omega:[0,\infty) \to[0,\infty)$ is a continuous non decreasing function.

Assume that the finite bound of $\int_0^t \gamma p(s)ds$ is L_0.

(H_8) The function $k: Q \times \mathcal{B} \to\mathbb{X}$ satisfies the following conditions.
(a) For every $(t, s)\in Q$, the function $k(t, s, .) : \mathcal{B} \to\mathbb{X}$ is continuous.
(b) For each $x\in\mathcal{B}$ the function $k(., ., x) : Q\to\mathbb{X}$ is strongly measurable.
(c) there exists an integrable function $q : I \to[0,\infty)$ and a constant $\gamma_1 > 0$ such that

$$\|k(t, s, x)\| \le \gamma_1 q(s)\theta(\|x\|_B),$$

where $\theta : [0,\infty) \to[0,\infty)$ is a continuous non decreasing function.

Assume that the finite bound of $\int_0^a \gamma_1 q(s)ds$ is L_1.

Theorem 3.1. Assume that the hypotheses (S_1), (S_2), (H_1), (H_2), (H_3), (H_6), (H_7) and (H_8) are fulfilled. Suppose , in addition, that the following properties hold.

(a) For all $t,s \in[0,a]$, $t > s$ and $r > 0$, the set $\{T(t-s)f(s,x,y,z) : s\in[0,t], \|x\|_B \le r, \|y\| \le r, \|z\| \le r\}$ is relatively compact in X.

(b) $\mu = c_1 K_a \left(\|i_c\|_{L(Y,X)} + \int_0^a H(s)ds \right) < 1, \ \frac{\tilde{M}K_a}{1-\mu}\int_0^a m(s)ds < \int_c^\infty \frac{ds}{W(s+L_0\Omega(s)+L_1\theta(s))}$, where $i_c : Y \to X$ is the inclusion operator and

$$c = \frac{K_a}{1-\mu}\left[\left(\tilde{M}H + c_1\tilde{M}\|i_c\|_{L(Y,X)} + K_a^{-1}Ma\right)\|\phi\|_B + c_2\|i_c\|_{L(Y,X)}(\tilde{M}+1) + c_2\int_0^a H(s)ds + \tilde{M}\sum_{i=1}^N N_i \right]$$

where H is a constant given by Axiom(A), K_a and M_a are given by

$K_a = \sup_{0\le t\le a} K(t)$ and $M_a = \sup_{0\le t\le a} M(t)$ respectively.

Then there exists a mild solution of the initial value problem (1.1)-(1.3)

Proof: Let S(I) and $y : (-\infty,a] \to X$ be introduced in **(H_6)** and $\Gamma : S(I) \to S(I)$ be the operator defined by $(\Gamma u)_0 = 0$ and

$$\Gamma u(t) = T(t)g(0,\varphi) - g(t,u_t+y_t) - \int_0^t AT(t-s)g(s,u_s+y_s)d\ s$$

$$+ \int_0^t T(t-s) f\left(s,u_s+y_s,\int_0^s h(s,\tau,u_\tau+y_\tau)d\tau,\int_0^a k(s,\tau,u_\tau+y_\tau)d\tau\right)ds + \sum_{0<t_i<t} T(t-t_i)I_i(u_{t_i}+y_{t_i})$$

To prove that the function $\Gamma u \in S(I)$, we need to show that the following properties are satisfied

(i) The function Γu is continuous in $t \neq t_i$

(ii) The limit $\lim_{t\to t_i^-} \Gamma u(t) = \Gamma u(t_i)$, for all i=1, 2, …, N

(iii) The limit $\lim_{t\to t_i^+} \Gamma u(t)$ exists for all i=1, 2, …, N

Taking into the account that g and $t \to T(t)g(0,\phi)$ are continuous, it follows from **(S_1)** and (H_2) and the Remark 2.2 that the function $\Gamma u(t)$ is continuous in $t \neq t_i$. On the other hand, from the definition of Γ , the dominated convergence theorem and the condition **(S_2),** we have that $\lim_{t\to t_i^-} \Gamma u(t) = \Gamma u(t_i)$ and that $\lim_{t\to t_i^+} \Gamma u(t) = \Gamma u(t_i) + I_i(u_{t_i})$

Next, we show that Γ is a continuous operator. Let $(u^n)_{n\in N}$ be a sequence in S(I) and $u \in S(I)$ such that $u^n \to u$ in S(I). By using the continuity of g and conditions ((H_2)-(a)), (H_3) and (H_6)**.** We prove that $g(t,u_s^n + y_s^n) \to g(t,u_s+y_s)$ uniformly in I,

$$f\left(t,u_s^n+y_s^n,\int_0^t h(s,\tau,u_\tau{}^n+y_\tau{}^n)d\tau,\int_0^a k(s,\tau,u_\tau{}^n+y_\tau{}^n)d\tau\right) \to$$

$$f(t,u_s+y_s,\int_0^t h(s,\tau,u_\tau+y_\tau)d\tau,\int_0^a k(t,s,u_\tau+y_\tau)d\tau)$$

for all $s \in I$ and $I_i(u_t^n + y_t) \to I_i(u_t+y_t)$ uniformly on I. Using conditions((H_2)-(b,c)) and Lebesgue's Dominated Convergence Theorem, we conclude that

$$\int_0^t AT(t-s)g(s,u_s^n+y_s^n)ds \to \int_0^t AT(t-s)g(s,u_s+y_s)ds$$

$$\int_0^t T(t-s) f\left(s,u_s^n+y_s^n,\int_0^s h(s,\tau,u_\tau{}^n+y_\tau{}^n)d\tau,\int_0^a k(s,\tau,u_\tau{}^n+y_\tau{}^n)d\tau\right)ds \to$$

$$\int_0^t T(t-s) f\left(s,u_s+y_s,\int_0^s h(s,\tau,u_\tau+y_\tau)d\tau,\int_0^a k(s,\tau,u_\tau+y_\tau)d\tau\right)ds$$

as $n\to\infty$, which is clearly implies that Γ is a continuous operator.

In order to use Theorem 2.1 we need to obtain a priori bound for the solutions of the integral equations $\lambda\Gamma u = u$, $\lambda \in (0,1)$ and x^λ be a solution of $\lambda\Gamma u = u$, $0<\lambda<1$; taking into account that $\|y_t\|_B \le (K_a\tilde{M}H + M_a)\|\phi\|_B$ and using Remark 2.2, (H_2) and (H_4) we find that

$$\|\lambda\Gamma(x^\lambda(t))\| \le \tilde{M}\left(\|i_c\|_{L(Y,X)}(c_1\|\varphi\|_B + c_2)\right) + \|i_c\|_{L(Y,X)}\left(c_1\left(K_a\|x^\lambda\|_t + (K_a\tilde{M}H + M_a)\|\varphi\|_B\right) + c_2\right)$$
$$+\int_0^t H(t-s)\left(c_1\left(K_a\|x^\lambda\|_t + (K_a\tilde{M}H + M_a)\|\varphi\|_B\right) + c_2\right)ds$$
$$+\tilde{M}\int_0^t m(s)W\left(K_a\|x^\lambda\|_t + (K_a\tilde{M}H + M_a)\|\varphi\|_B + L_0\Omega\left(K_a\|x^\lambda\|_t + (K_a\tilde{M}H + M_a)\|\varphi\|_B\right)\right.$$
$$\left.+L_1\theta\left(K_a\|x^\lambda\|_t + (K_a\tilde{M}H + M_a)\|\varphi\|_B\right)\right)ds + \sum_{i=1}^N \tilde{M}N_i$$

Where we are using the following notation $\|f\|_t = \sup_{0\le s\le t}\|f(s)\|_X$. Next, putting

$\delta_\lambda(t) = K_a\|x^\lambda\|_t + \left(K_a\tilde{M}H + M_a\right)\|\phi\|_B$ it follows that

$$\|x^\lambda(t)\| \le \|i_c\|_{L(Y,X)}\left(\tilde{M}(c_1\|\phi\|_B + c_2)\right) + \|i_c\|_{L(Y,X)}\left(c_1\delta_\lambda(t) + c_2\right)$$
$$+\int_0^t H(t-s)\left(c_1\delta_\lambda(s) + c_2\right)ds + \tilde{M}\int_0^t m(s)W\left(\delta_\lambda(s) + L_0\Omega(\delta_\lambda(s)) + L_1\theta(\delta_\lambda(s))\right)ds + \sum_{i=1}^N \tilde{M}N_i$$

After some simplifications and rearrangement of terms we get

$$\delta_\lambda(t) \le K_a\left[\|i_c\|_{L(Y,X)}\left(\tilde{M}(c_1\|\varphi\|_B + c_2)\right) + \|i_c\|_{L(Y,X)}(c_1(\delta_\lambda(t) + L_0\Omega(\delta_\lambda(t)) + c_2)\right.$$
$$+\int_0^t H(t-s)(c_1(\delta_\lambda(s) + L_0\Omega(\delta_\lambda(s))) + c_2)ds$$
$$\left.+\tilde{M}\int_0^t m(s)W\left(\delta_\lambda(s) + L_0\Omega(\delta_\lambda(s)) + L_1\theta(\delta_\lambda(s))\right)ds + \tilde{M}\sum_{i=1}^N N_i\right] + \left(K_a\tilde{M}H + M_a\right)\|\varphi\|_B$$

From the hypothesis (b), we have that

$$\delta_\lambda(t) \le c + \frac{K_a\tilde{M}}{1-\mu}\int_0^t m(s)W\left(\delta_\lambda(s) + L_0\Omega(\delta_\lambda(s)) + L_1\theta(\delta_\lambda(s))\right)ds$$

Denote by $\beta_\lambda(t)$ the right hand side of the previous inequality, and observing that $\delta_\lambda(t) \le \beta_\lambda(t)$ for $t\in I$, we arrive at

$$\beta_\lambda'(t) \le \frac{\tilde{M}K_a}{1-\mu}m(t)W\left(\beta_\lambda(t) + L_0\Omega(\beta_\lambda(t)) + L_1\theta(\beta_\lambda(t))\right)$$

Hence $\displaystyle\int_{\beta_\lambda(0)}^{\beta_\lambda(t)} \frac{ds}{W(s + L_0\Omega(s) + L_1\theta(s))} \le \frac{\tilde{M}K_a}{1-\mu}\int_0^a m(s)ds \le \int_c^\infty \frac{ds}{W(s + L_0\Omega(s) + L_1\theta(s))}$

This allows us to conclude that the set of functions $\{\beta_\lambda : \lambda\in(0,1)\}$ is bounded. This implies that $\{u_\lambda : u_\lambda = \Gamma u_\lambda, \lambda\in(0,1)\}$ is bounded to S(I).

It remains to show that Γ is completely continuous. To this end, we introduce the decomposition $\Gamma=\Gamma_1+\Gamma_2$, Where $(\Gamma_i u)_0=0$, i = 1,2 and

$$\Gamma_1 u(t) = T(t)g(0,\varphi) - g(t, u_t + y_t) - \int_0^t AT(t-s)g(s, u_s + y_s)d\ s$$
$$+\int_0^t T(t-s)f\left(s, u_s + y_s, \int_0^s h(s,\tau,u_\tau + y_\tau)d\tau, \int_0^a k(s,\tau,u_\tau + y_\tau)d\tau\right)d\ s$$

$$\Gamma_2 u(t) = \sum_{0<t_i<t} T(t-t_i)I_i(u_{t_i} + y_{t_i}), \qquad t\in[0,a]$$

We now prove that Γ_1 is a completely continuous operator. We consider u in $B_r(0,S(I))$, an open ball of radius r. Applying Mean Value Theorem for the Bochner integral (see [11, Lemma 2.1.3]), we infer that

$$(\Gamma_1 u)(t) \subset T(t)g(0,\phi) - g(t,u_t+y_t) - t\overline{co\{AT(t-\theta)g(\theta,u_\theta+y_\theta),\ \theta\in[0,t], u\in B_r(0,S(I))^Y\}}$$
$$+t\overline{co\left\{T(t-\theta)f(\theta,u_\theta+y_\theta,\int_0^s h(s,\tau,u_\tau+y_\tau)d\tau,\int_0^a k(s,\tau,u_\tau+y_\tau)d\tau,\ \theta\in[0,t], u\in B_r(0,S(I))\right\}}$$

Taking the advantage of Theorem 3.1 (a) and (H_1), we conclude that $\{(\Gamma_1 u)(t): u\in B_r(0,S(I))\}$ is a compact subset of X.

Now, we prove that the set function $\{(\Gamma_1 u)(t): u\in B_r(0, S(I))\}$ is equicontinuous in I. To this end, consider $0<\varepsilon<t<t'\in[0,a]$; then there is $0<\delta<\varepsilon$ such that

$$\|T(t)g(0,\phi)-T(t')g(0,\phi)\|<\varepsilon$$
$$\|g(t,u_t+y_t)-g(t',u_{t'}+y_{t'})\|<\varepsilon$$
$$\Big\|T(t-s)f(s,u_s+y_s,\int_0^s h(s,\tau,u_\tau+y_\tau)d\tau,\int_0^a k(s,\tau,u_\tau+y_\tau)d\tau)$$
$$-T(t'-s)f(s,u_s+y_s,\int_0^s h(s,\tau,u_\tau+y_\tau)d\tau,\int_0^a k(s,\tau,u_\tau+y_\tau)d\tau))\Big\|<\varepsilon$$

For all $s\in[0,t]$, $|t-t'|<\delta$ and $u\in B_r(0,S(I))$. On the other hand, using the continuity of the map $(t,s)\to AT(t-s)$ for $t>s$, we arrive at

$$\|AT(t-s)g(s,u_s+y_s)-AT(t'-s)g(s,u_s+y_s)\|<\varepsilon$$

For all $s\in[0,t-\varepsilon]$, $|t-t'|<\delta$ and $u\in B_r(0,S(I))$. Under these conditions, for $u\in B_r(0,S(I))$, $|t-t'|<\delta$, we infer that

$$\|\Gamma_1 u(t)-\Gamma_1 u(t')\| \le \|(T(t)-T(t'))g(0,\varphi)\| + \|g(t,u_t+y_t)-g(t',u_t+y_t)\|$$
$$+\int_0^{t-\varepsilon}\|(AT(t-s)-AT(t'-s))g(s,u_s+y_s)\|ds$$
$$+\int_{t-\varepsilon}^{t}\|(AT(t-s)-AT(t'-s))g(s,u_s+y_s)\|ds$$
$$+\int_t^{t'}\|AT(t'-s)g(s,u_s+y_s)\|ds$$
$$+\int_0^t\Big\|(T(t-s)-T(t'-s))f(s,u_s+y_s,\int_0^s h(s,\tau,u_\tau+y_\tau)d\tau,\int_0^a k(s,\tau,u_\tau+y_\tau)d\tau)\Big\|ds$$
$$+\int_t^{t'}\Big\|T(t'-s)f(s,u_s+y_s,\int_0^s h(s,\tau,u_\tau+y_\tau)d\tau,\int_0^a k(s,\tau,u_\tau+y_\tau)d\tau)\Big\|ds$$

From the above estimate, one can deduce the following inequality:

$$\|\Gamma_1 u(t)-\Gamma_1 u(t')\| \le 2\varepsilon+\varepsilon(t-\varepsilon)+\varepsilon t+\left(c_1\left(K_a r+K_a\tilde{M}H+M_a\right)+c_2\right)\int_{t-\varepsilon}^{t}H(t-s)ds+\int_t^{t'}H(t-s)ds$$
$$+\tilde{M}W\left(\left(K_a r+(K_a\tilde{M}H+M_a)\|\varphi\|_B\right)+L_0\Omega\left(K_a r+(K_a\tilde{M}H+M_a)\|\varphi\|_B\right)+L_1\theta\left(K_a r+(K_a\tilde{M}H+M_a)\|\varphi\|_B\right)\right)$$
$$+\int_t^{t'}m(s)ds$$

which shows the equicontinuity at $t\in I$. So, to conclude the proof, we show that Γ_2 is completely continuous. We observe that the continuity of Γ_2 is obvious. On the other hand for $r>0$, $t\in[t_i, t_{i+1}]$, $i\geq 1$, and $u \in B_r = B_r(0, PC([0,a];X))$, we have that there exists $\tilde{r} > 0$ such that

$$\left[\overline{\Gamma_2 u}\right]_i(t) \in \begin{cases} \sum_{j=1}^{i} T(t-t_j) I_j\left(B_{\tilde{r}}(0,B)\right) & t \in (t_i, t_{i+1}), \\ \sum_{j=1}^{i} T(t_{i+1}-t_j) I_j\left(B_{\tilde{r}}(0,B)\right) & t = t_{i+1}, \\ \sum_{j=1}^{i-1} T(t_i-t_j) I_j\left(B_{\tilde{r}}(0,B)\right) + I_i\left(B_{\tilde{r}}(0,B)\right) & t = t_i \end{cases}$$

Where $B_{\tilde{r}}(0,B)$ is an open ball of radius $\tilde{r}$. From condition (H_4) it follows that $\left[\overline{\Gamma_2(B_r)}\right]_i(t)$ is relatively compact in X, for all $t\in[t_i,t_{i+1}]$, $i \geq 1$. More over, using the fact that the operators $\{I_i\}_i$ are compact and the strong continuity of $(T(t_i))_{t\geq 0}$, we conclude that for $\varepsilon>0$, there exists $\delta>0$ such that

$$\left\|T(t-t_i)z - T(s-t_i)z\right\| \leq \varepsilon, \quad z \in \bigcup_{i=1}^{N} I_i(B_{\tilde{r}}(0,B)) \tag{3.1}$$

for all t_i, $i=1,\ldots N$, t, $s\in(t_i,t_{i+1}]$ with $|t-s| < \delta$. Thus for $u\in B_r$, $t\in[t_i,t_{i+1}]$, $i \geq 0$, and $0 < |h| < \delta$ with $t+h\in[t_i,t_{i+1}]$, we have that

$$\| [\overline{\Gamma_2 u}]_i(t+h) - [\overline{\Gamma_2 u}]_i(t) \| \leq N\varepsilon, \quad i = 1, 2, ..., N.$$

We have shown that the set $\left[\overline{\Gamma_2(B_r)}\right]_i$ is uniformly equicontinuous for $i \geq 0$. Now, from lemma 2.1, we conclude that Γ_2 is completely continuous.

We have proven that Γ satisfies the conditions of theorem 2.1, which allows us infer the existence of a mild solution of the problem (1.1)-(1.3). This completes the proof of the theorem.

If the map F and I_i, i=1…N fulfill some Lipchitz condition instead of the compactness properties considered in theorem 3.1, we also can establish an existence result.

Theorem 3.2. Suppose that the assumptions (S_1), (S_2), (H_2), (H_4), (H_5), (H_7) and (H_8) are satisfied and that the Condition (a) of Theorem 3.1 is fulfilled. If

$$\left[K_a L_g\left(\| i_c \|_{L(Y,X)} + \int_0^a H(s)ds\right) + \tilde{M}K_a \sum_{i=1}^{N} L_i + \tilde{M}K_a \liminf_{r\to\infty} \frac{W((r + L_0\Omega(r) + L_1\theta(r))}{r} \int_0^a m(s)ds\right] < 1$$

Where i_c denotes the inclusion operator from Y into X, then there exists a mild solution of the impulsive problem (1.1) – (1.3)

Proof: Let Γ be the function given in the proof of theorem 3.1 and consider the decomposition $\Gamma = \Gamma_1 + \Gamma_2$ where $(\Gamma_i u) = 0$ on $(-\infty,0]$, i = 1, 2 and

$$\Gamma_1 u(t) = T(t)g(0,\phi) - g(t, u_t + y_t) - \int_0^t AT(t-s)g(s, u_s + y_s)ds + \sum_{0<t_i<t} T(t-t_i)I_i(u_{t_i} + y_{t_i}), \quad t \in [0,a]$$

$$\Gamma_2 u(t) = \int_0^t T(t-s) f(s, u_s + y_s, \int_0^s h(s,\tau,u_\tau + y_\tau)d\tau, \int_0^a k(s,\tau,u_\tau + y_\tau)d\tau)d \ ,st, s \in [0,a]$$

We claim that there exists $r > 0$ such that $\Gamma(B_r(0, S(I))) \subset B_r(0, S(I))$.

In fact, if we assume that this assertion is false, then for all $r > 0$ we can choose $x^r\in B_r=B_r(0,S(I))$ and $t^r\in[0,a]$ such that $\|\Gamma x^r(t^r)\| > r$. Observe that standard computations involving the phase space axioms yield

$$r < \| \Gamma(x^r(t^r) \| \le \| T(t) g(0,\varphi) \| + \| g(t, x_t^r + y_t) - g(t,0) \| + \| g(t,0) \|$$
$$+ \int_0^a \| AT(t-s) \| \left[\| g(s, x_s^r + y_s) - g(\tau, 0) \| + \| g(\tau,0) \| \right] ds$$
$$+ \int_0^a \| T(t-s) \| \; \| f(s, x_s^r + y_s, \int_0^s h(s,\tau,\; x_\tau^r + y_\tau) d\tau, \int_0^a k(s,\tau, x_\tau^r + y_\tau) d\tau) \| d$$
$$+ \sum_{0<t_i<a} \| T(t-t_i) \| \; \| I_i(x_{t_i}^r + y_{t_i}) - I_i(0) \| + \| I_i(0) \|$$

$$\le \tilde{M} \| g(0,\varphi) \|_Y + \| i_c \|_{L(Y,X)} \sup_{0\le t \le a} \| g(t,0) \|_Y + L_g \| i_c \|_{L(Y,X)} \left[K_a r + (K_a \tilde{M} H + M_a) \| \varphi \|_B \right]$$
$$+ \int_0^a \left[H(s) \left[L_g (K_a r + (K_a \tilde{M} H + M_a) \| \varphi \|_B \right] + \sup_{0 \le \tau \le a} \| g(\tau,0) \|_Y \right] ds$$
$$+ \tilde{M} \int_0^a m(s) W \left[K_a r + (K_a \tilde{M} H + M_a) \| \varphi \|_B + L_0 \Omega \left(K_a r + (K_a \tilde{M} H + M_a) \| \varphi \|_B \right) + L_1 \theta (K_a r + (K_a \tilde{M} H + M_a) \| \varphi \|_B) \right] ds$$
$$+ \tilde{M} \sum_{i=1}^N \left[L_i (K_a r + (K_a \tilde{M} H + M_a) \| \varphi \|_B + \| I_i(0) \| \right]$$

Dividing both sides by r and taking r→∞ we get

$$1 \le K_a L_g \left[\| i_c \|_{L(Y,X)} + \int_0^a H(s) ds \right] + \tilde{M} K_a \liminf_{r\to\infty} \frac{W((r + L_0 \Omega(r) + L_1 \theta(r))}{r} \int_0^a m(s) ds + \tilde{M} K_a \sum_{i=1}^N L_i$$

Which is contrary to our assumptions.

Let r > 0 such that $\Gamma(B_r(0, S(I)) \subset B_r(0,S(I))$. It follows from the proof of Lemma 3.1 in [13] that Γ_2 is completely continuous in $B_r(0, S(I))$. Moreover the estimate

$$\| \Gamma_1 u - \Gamma_1 v \|_\infty \le \left[K_a L_g \left(\| i_c \|_{L(Y,X)} + \int_0^a H(s) ds \right) + \tilde{M} K_a \sum_{i=1}^N L_i \right] \| u - v \|_\infty$$

u, v ∈ B_r, shows that Γ_1 is a contraction on B_r. Consequently, Γ is a condensing operator from B_r into B_r. Then the existence of a mild solution of (1.1)-(1.3) is a consequence of Theorem 2.2. This completes the proof.

4. GLOBAL SOLUTIONS

In this section, we study the existence of mild solutions for the impulsive problem

$$\frac{d}{dt}(u(t) + g(t,u_t)) = Au(t) + f(t, u_t, \int_0^t h(s,\tau,u_\tau) d\tau, \int_0^a k(s,\tau,u_\tau) d\tau), \quad t \in I = [0,a] \tag{4.1}$$

$$\Delta u(t_i) = I_i(u_{t_i}), \quad i = 1,\, 2, ..., n. \tag{4.2}$$

$$u_0 = \varphi \in \mathscr{B}_h,\; 0 < t_1 < t_2 < \ldots < t_n < \ldots \tag{4.3}$$

where $(t_i)_{i\in N}$ is an increasing sequence of positive numbers.

Let h: [0,∞) → [0,∞) be a positive, non-decreasing, continuous function such that h(0) = 1 and $\lim_{t\to\infty} h(t) = +\infty$. In addition, suppose that the map (t, s) → T (t-s) is uniformly bounded. Moreover, $PC([0,\infty); X)$ and $(PC_h)^0(X)$ denote the spaces

$$PC([0,\infty);X) = \left\{ \begin{array}{l} x:[0,\infty) \to X \quad : x_{|[0,a]} \in PC,\; \forall a \in (0,\infty) \setminus \{t_i : i \in N\}, \\ \qquad x_0 = 0 \\ \qquad \|x\|_\infty = \sup_{t\ge 0} \|x(t)\| < \infty \end{array} \right\}$$

$$(PC)_h^0(X)=\left\{x\in PC([0,\infty);X):\lim_{t\to\infty}\frac{\|x(t)\|}{h(t)}=0\right\}$$

endowed with the norms $\|x\|_\infty=\sup_{t\geq 0}\|x(t)\|$ and $\|x\|_{PC_h}=\sup_{t\geq 0}\frac{\|x(t)\|}{h(t)}$ respectively.

To get the next results, we need a very detailed knowledge of the relatively compact sets of the space $(PC)_h^0(X)$. We will use the following result.

Lemma 4.2. A bounded set $B\subset(PC)_h^0(X)$ is relatively compact in $(PC)_h^0(X)$ if and only if

(a) The set $B_a=\{u|_{[0,a]}:u\in B\}$ is relatively compact in $PC([0,a];X)$ for all $a\in(0,\infty)\setminus\{t_i : i\in N\}$

(b) $\lim_{t\to\infty}\frac{\|x(t)\|}{h(t)}=0$, uniformly for $x\in B$,

We introduce the following concept of global solution of the system (4.1)-(4.3).

Definition 4.2. A function $u : R\to X$ is called a global solution of the problem (4.1)-(4.3), if the conditions (4.2) and (4.3) are verified ; $u|_{[0,a]}\in PC([0,a];X)$ for all $a\in(0,\infty)\setminus\{t_i : i\in N\}$

$$u(t)=T(t)(\varphi(0)+g(0,\varphi))-g(t,u_t)-\int_0^t AT(t-s)g(s,u_s)ds+\int_0^t T(t-s)f\left(s,u_s,\int_0^s h(s,\tau,u_\tau)d\tau,\int_0^a k(s,\tau,u_\tau)d\tau\right)ds \tag{4.4}$$
$$+\sum_{t_i<t}T(t-t_i)I_i(u_{t_i})+\sum_{t_i<t}[g(t,u_t)_{|t_i^+}-g(t,u_t)_{|t_i^-}],\qquad t\in I=[0,\infty)$$

We have to prove the following theorem.

Theorem 4.1. Assume that the hypotheses (S_1), (S_2), (H_2), (H_3), (H_6) and the Conditions (a) of Theorem 3.1 are valid in I= [0, a] for all a > 0. Suppose, in addition, that the functions M (.) and K(.) given by Axioms (A) are bounded and that the following conditions hold.

(a) For all J > 0, $\lim_{t\to\infty}\frac{1}{h(t)}\int_0^t m(s)W(1+L_0\Omega+L_1\theta)Jh(s)ds=0$

(b) The function $g:I\times B\to Y$ is completely continuous and there exists a positive function $c_1 : [0,\infty)\to[0,\infty)$ with $c_1(t)\to 0$ when $t\to\infty$ and a positive constant d_2 such that $\|g(t,\psi)\|_Y\leq c_1(t)\|\psi\|_B+d_2$ for all t > 0, $\psi\in B$ and

$$\lim_{t\to\infty}\frac{1}{h(t)}\int_o^t H(t-s)h(s)ds=0$$

(c) $\eta=\left(\|i_c\|_{L(Y,X)}K_\infty c_{1,\infty}+\sup_{t\geq 0}K_\infty\int_o^t H(t-s)c_1(s)ds\right)<1$

$$(d)\ \frac{\tilde{M}K_\infty}{1-\eta}\int_0^\infty m(s)ds<\int_c^\infty\frac{ds}{W(s+L_0\Omega(s)+L_1\theta(s))}\quad with$$
$$c=(1-\eta)^{-1}\Big(\left(\tilde{M}K_\infty H+M_\infty+\tilde{M}K_\infty\|i_c\|_{L(Y,X)}c_{1,\infty}\right)\|\phi\|_B+\|i_c\|_{L(Y,X)}K_\infty(\tilde{M}+1)d_2$$
$$+\tilde{M}K_\infty\sum_{i=1}^\infty N_i+d_2K_\infty\int_o^\infty H(s)ds\Big)<\infty$$

where $c_{1,\infty}=\sup_{t\geq 0}c_1(t)$, $K_\infty=\sup_{t\geq 0}K(t)$ and $M_\infty=\sup_{t\geq 0}M(t)$,

Then there exists a global solution of (4.1)-(4.3)

Proof: Let $y: R\to X$ be the extension of ϕ such that $y(t) = T(t)\phi(0)$, $t\geq 0$ and $S(\infty)$ be the set defined by $S(\infty)=\{x:R\to X:x_0=0\ and\ x_{|[0,\infty)}\in(PC)_h^0(X)\}$, endowed with the norm $\|x\|_{\wp c_h}=\sup_{t\geq 0}\frac{\|x(t)\|}{h(t)}$ and

$\Gamma : S(\infty) \to S(\infty)$ be the operator defined by $(\Gamma u)_0=0$ and

$$\Gamma u(t) = T(t)g(0,\varphi) - g(t,u_t+y_t) - \int_0^t AT(t-s)g(s,u_s+y_s)d\ s$$
$$+\int_0^t T(t-s)f(s,u_s+y_s,\int_0^s h(s,\tau,u_\tau+y_\tau)d\tau,\int_0^a k(s,\tau,u_\tau+y_\tau)d\tau)d\ \ s$$
$$+\sum_{0<t_i<t} T(t-t_i)I_i(u_{t_i}+y_{t_i}),\ t\geq 0$$

$$\|\Gamma(u(t))\| \leq \|T(t)\|\|g(0,\varphi)\| + \|g(t,u_t+y_t)\| + \int_0^t \|AT(t-s)\|\|g(s,u_s+y_s)\|d\ s$$
$$+\int_0^t \|T(t-s)\| \left\| f(s,u_s+y_s,\int_0^s h(s,\tau,u_\tau+y_\tau)d\tau,\int_0^a k(s,\tau,u_\tau+y_\tau)d\tau)\right\| d$$
$$+\sum_{0<t_i<t} \|T(t-t_i)\|\|I_i(u_{t_i}+y_{t_i})\|$$

Since $\|u(t)\| \leq \|u\|_{PC_h}\ h(t)$ for all $t \geq 0$, we have that

$$\frac{\|\Gamma u(t)\|}{h(t)} \leq \frac{\tilde{M}\|g(0,\varphi)\|}{h(t)} + \frac{\|i_c\|_{L(Y,X)}[c_1(t)K_\infty\|u\|_{PC_h}\ h(t)]}{h(t)}$$
$$+\frac{\|i_c\|_{L(Y,X)}[c_1(t)(\tilde{M}K_\infty H + M_\infty)\|\varphi\|_B + d_2]}{h(t)}$$
$$+\frac{1}{h(t)}\int_0^t H(t-s)\left(c_1(s)(K_\infty\|u\|_{PC_h} + (\tilde{M}K_\infty H + M_\infty)\|\varphi\|_B) + d_2\right)h(s)ds$$
$$+\frac{\tilde{M}}{h(t)}\int_0^t m(s)W\left((K_\infty\|u\|_{PC_h} + (\tilde{M}K_\infty H + M_\infty)\|\varphi\|_B) + L_0\Omega(K_\infty\|u\|_{PC_h} + (\tilde{M}K_\infty H + M_\infty)\|\varphi\|_B)\right.$$
$$\left. + L_1\theta((K_\infty\|u\|_{PC_h} + (\tilde{M}K_\infty H + M_\infty)\|\varphi\|_B\right)h(s)ds + \frac{\tilde{M}}{h(t)}\sum_{i=1}^\infty N_i$$

So by (a) and (b) we have that $\Gamma u \in S(\infty)$.

Next we show that $\Gamma(B_r)$, where $B_r=B_r(0,(PC)^0_h(X))$, satisfies the conditions of Lemma 4.1. To do this, we show the continuity of the operator Γ. Let $(u^n)_{n\in N}$ be a sequence in $S(\infty)$ and $u\in S(\infty)$ such that $u^n \to u$ in $S(\infty)$. Take $C=\sup\{\|u^n\|_{PC_h}, \|u\|_{PC_h};\ n\in N\}$, $J=(\tilde{M}K_\infty H + M_\infty)\|\phi\|_B + CK_\infty$ and the function $\mu : [0,\infty)\to R$ defined by $\mu(t) = c_1(t)J+d_2$.

From the conditions (a), (b) and (c) there exists $L_1 > 0$ such that

$$\frac{2}{h(t)}\int_0^t H(t-s)\mu(s)h(s)ds + \frac{2\tilde{M}}{h(t)}\int_0^t m(s)W(J+L_0\Omega(J)+L_1\theta(J))h(s)ds + \frac{2\tilde{M}}{h(t)}\sum_{i=1}^\infty N_i < \frac{\varepsilon}{2}$$

for $t \geq L_1$. From the proof of Theorem 3.1, we have that $g(t, u^n_t+y_t)\to g(t, u_t+y_t)$ uniformly for $t \in [0,L_1]$, when $n\to\infty$. From Lebesgue's Dominated Convergence Theorem, we can fix positive number $N_\varepsilon > 0$ such that

$$\frac{1}{h(t)}\| g(t,u_t^n+y_t)-g(u_t+y_t)\|+\frac{1}{h(t)}\int_0^t H(t-s)\| g(s,u_s^n+y_s)-g(s,u_s+y_s)\| ds$$
$$+\frac{\tilde{M}}{h(t)}\int_0^{L_1}\Big\| f\Big(s,u_s^n+y_s,\int_0^s h(s,\tau,u_\tau^n+y_\tau)d\tau,\int_0^a k(s,\tau,u_\tau^n+y_\tau)d\tau\Big)$$
$$-f\Big(s,u_s+y_s,\int_0^s h(s,\tau,u_\tau+y_\tau)d\tau,\int_0^a k(s,\tau,u_\tau+y_\tau)d\tau\Big)\Big\| ds$$
$$+\frac{\tilde{M}}{h(t)}\sum_{t_i\le L_1}\| I_i(u_{t_i}^n+y_{t_i})-I_i(u_{t_i}+y_{t_i})\|<\frac{\varepsilon}{2}$$

for all $t\in[0,L_1]$ and $n\ge N_\varepsilon$. Using the above inequality, for $t\in[0,L_1]$ and $n\ge N_\varepsilon$, we have that

$$\sup\left\{\frac{\|\Gamma u^n(t)-\Gamma u(t)\|}{h(t)}: t\in[0,L_1],\ n\ge N_\varepsilon\right\}\le\varepsilon \tag{4.5}$$

On the other hand for $t\ge L_1$ and $n\ge N_\varepsilon$, we get

$$\frac{\|\Gamma u^n(t)-\Gamma u(t)\|}{h(t)}\le\frac{\| g(t,u_t^n+y_t)-g(t,u_t+y_t)\|}{h(t)}$$
$$+\frac{1}{h(t)}\int_0^{L_1}\| AT(t-s)(g(s,u_s^n+y_s)-g(s,u_s+y_s))\| ds$$
$$+\frac{1}{h(t)}\int_{L_1}^{t}\| AT(t-s)(g(s,u_s^n+y_s)-g(s,u_s+y_s))\| ds$$
$$+\frac{1}{h(t)}\int_0^{L_1}\Big\| T(t-s)\Big(f\Big(s,u_s^n+y_s,\int_0^s h(s,\tau,u_\tau^n+y_\tau)d\tau,\int_0^a k(s,\tau,u_\tau^n+y_\tau)d\tau\Big)$$
$$-f\Big(s,u_s+y_s,\int_0^s h(s,\tau,u_\tau+y_\tau)d\tau,\int_0^a k(s,\tau,u_\tau+y_\tau)d\tau\Big)\Big\| ds$$
$$+\frac{1}{h(t)}\int_{L_1}^{t}\Big\| T(t-s)\Big(f\Big(s,u_s^n+y_s,\int_0^s h(s,\tau,u_\tau^n+y_\tau)d\tau,\int_0^a k(s,\tau,u_\tau^n+y_\tau)d\tau\Big)$$
$$-f\Big(s,u_s+y_s,\int_0^s h(s,\tau,u_\tau+y_\tau)d\tau,\int_0^a k(s,\tau,u_\tau+y_\tau)d\tau\Big)\Big)\Big\| ds$$
$$+\frac{\tilde{M}}{h(t)}\sum_{t_i\le L_1}\| I_i(u_{t_i}^n+y_{t_i})-I_i(u_{t_i}+y_{t_i})\|+\frac{2\tilde{M}}{h(t)}\sum_{t_i\ge L_1}N_i$$
$$\le\frac{2}{h(t)}\int_0^t H(t-s)\mu(s)h(s)ds+\frac{2\tilde{M}}{h(t)}\int_0^t m(s)W(Jh(s)+L_0\Omega Jh(s)+L_1\theta Jh(s))ds$$
$$+\frac{2\tilde{M}}{h(t)}\sum_{i=1}^{\infty}N_i+\frac{\varepsilon}{2}$$
$$\le\frac{\varepsilon}{2}+\frac{\varepsilon}{2}=\varepsilon$$

And thus

$$\sup\left\{\frac{\|\Gamma u^n(t)-\Gamma u(t)\|}{h(t)}: t\ge L_1,\ n\ge N_\varepsilon\right\}\le\varepsilon \tag{4.6}$$

Using inequalities (4.5) and (4.6), we conclude that Γ is continuous.

Next, we prove that the set $\Gamma(B_r)$ is relatively compact. Let r > 0 be a positive real number $J = (\tilde{M}K_\infty H + M_\infty)\|\phi\|_B + rK_\infty$. From the proof of theorem 3.1 it follows that the set $\Gamma(B_r)|_{[0,a]} = \{\Gamma u|_{[0,a]} : u\in B_r\}$ is relatively compact in PC([0,a];X) for all $a\in(0,\infty) \setminus \{t_i, i\in N\}$. Moreover, for $x\in B_r$, we have that

$$\frac{\|\Gamma u(t)\|}{h(t)} \le \frac{1}{h(t)}\left[\tilde{M}\|i_c\|_{L(Y,X)}(c_1(0)\|\varphi\|_B + d_2)\right] + \frac{1}{h(t)}(\|i_c\|_{L(Y,X)}c_1(t)Jh(t)+d_2)$$
$$+\frac{1}{h(t)}\int_0^t H(t-s)(c_1(s)Jh(s)+d_2)ds + \frac{\tilde{M}}{h(t)}\int_0^t m(s)W(Jh(s)+L_0\Omega(Jh(s))+L_1\theta(Jh(s)))ds$$
$$+\frac{\tilde{M}}{h(t)}\sum_{i=1}^{\infty}N_i$$
$$\le \frac{1}{h(t)}\left[\tilde{M}\|i_c\|_{L(Y,X)}(c_1(0))\|\varphi\|_B + d_2\right] + \frac{Jc_{1,\infty}}{h(t)}\int_0^t H(t-s)h(s)d\,s + \frac{d_2}{h(t)}\int_0^t H(t-s)d\,s$$
$$+\|i_c\|_{L(Y,X)}\left(c_1(t)J+\frac{d_2}{h(t)}\right) + \frac{\tilde{M}}{h(t)}\int_o^t m(s)W(Jh(s)+L_0\Omega(Jh(s))+L_1\theta(Jh(s)))ds + \frac{\tilde{M}}{h(t)}\sum_{i=1}^{\infty}N_i$$

Which enables us to conclude that $\frac{\|\Gamma u(t)\|}{h(t)} \to 0, \ when \ \ t\to\infty$ uniformly for $x\in B_r$.

By lemma 4.1 we infer that $\Gamma(B_r)$ is relatively compact in $(PC)_h^0(X)$. Thus Γ is completely continuous. To finish the proof, we need to obtain an a priori estimate for the solutions of the integral equations $\lambda\Gamma u = u$, $\lambda\in(0,1)$. To this end, let u_λ be the equation $\lambda\Gamma u_\lambda = u_\lambda$. For t ≥ 0 we have that

$$\|u_\lambda(t)\| \le \|i_c\|_{L(Y,X)}\left[c_{1,\infty}((\tilde{M}(1+K_\infty H)+M_\infty)\|\varphi\|_B + K_\infty\|u_\lambda\|_t)\right] + \|i_c\|_{L(Y,X)}(\tilde{M}+1)d_2$$
$$+\int_0^t H(t-s)\left(c_1(s)((\tilde{M}K_\infty H+M_\infty)\|\varphi\|_B + K_\infty\|u_\lambda\|_s)+d_2\right)ds + \tilde{M}\sum_{i=1}^{\infty}N_i$$
$$+\tilde{M}\int_0^t m(s)W(((\tilde{M}K_\infty H+M_\infty)\|\varphi\|_B + K_\infty\|u_\lambda\|_s) + L_0\Omega((\tilde{M}K_\infty H+M_\infty)\|\varphi\|_B + K_\infty\|u_\lambda\|_s)$$
$$+L_1\theta((\tilde{M}K_\infty H+M_\infty)\|\varphi\|_B + K_\infty\|u_\lambda\|_s))ds$$

putting $\delta^\lambda(t) = (\tilde{M}K_\infty H + M_\infty)\|\phi\|_B + K_\infty\|u_\lambda\|_t$

$$\|u_\lambda(t)\| \le \|i_c\|_{L(Y,X)}c_{1,\infty}\left(\tilde{M}+\delta_\lambda(t)\right) + \|i_c\|_{L(Y,X)}\left(\tilde{M}+1\right)d_2$$
$$+\int_0^t H(t-s)c_1(s)\delta_\lambda(s)ds + \int_0^t H(t-s)d_2 ds + \tilde{M}\int_0^t m(s)W\left(\delta_\lambda(s)+L_0\Omega(\delta_\lambda(s))+L_1\theta(\delta_\lambda(s))\right)ds + \sum_{i=1}^{N}\tilde{M}N_i$$

We have that

$$\delta_\lambda(t) \le (1-\eta)^{-1}\Big((\tilde{M}K_\infty H + M_\infty + \tilde{M}K_\infty\|i_c\|_{L(Y,X)}c_{1,\infty})\|\phi\|_B + \|i_c\|_{L(Y,x)}K_\infty(\tilde{M}+1)d_2$$
$$+\tilde{M}K_\infty\int_0^t m(s)W(\delta_\lambda(s)+L_0\Omega(\delta_\lambda(s))+L_1\theta(\delta_\lambda(s)))ds + d_2K_\infty\int_0^\infty H(s)ds + \tilde{M}K_\infty\sum_{i=1}^{\infty}N_i\Big)$$
$$\le c + \frac{\tilde{M}}{1-\eta}K_\infty\int_0^t m(s)W(\delta_\lambda(s)+L_0\Omega(\delta_\lambda(s))+L_1\theta(\delta_\lambda(s)))ds$$

Where $\eta = \left(\|i_c\|_{L(Y,X)}K_\infty c_{1,\infty} + \sup_{t\ge 0}K_\infty\int_0^t H(t-s)c_1(s)ds\right)$. Denoting the right hand side of the previous expression as $\beta_\lambda(t)$ we see that

$$\beta_\lambda(t) \le c + \frac{\tilde{M}}{1-\eta} K_\infty \int_0^t m(s) W(\beta_\lambda(s) + L_0\Omega(\beta_\lambda(s)) + L_1\theta(\beta_\lambda(s)))ds$$

$$\beta_\lambda'(t) \le \frac{\tilde{M}K_\infty}{1-\eta} m(t) W(\beta_\lambda(t) + L_0\Omega(\beta_\lambda(t)) + L_1\theta(\beta_\lambda(t)))$$

And subsequently, upon integrating over [0,t], we obtain

$$\int_{\beta_\lambda(0)}^{\beta_\lambda(t)} \frac{ds}{W(s + L_0\Omega(s) + L_1\theta(s))} \le \frac{\tilde{M}K_\infty}{1-\mu} \int_0^a m(s)ds \le \int_c^\infty \frac{ds}{W(s + L_0\Omega(s) + L_1\theta(s))}$$

The above inequality together with condition (d) enables us to conclude that the set of functions $\{u_\lambda : u_\lambda = \lambda\Gamma u_\lambda\}$ is bounded in PC ([0, ∞) ;X). Note that, if $x \in PC([0, \infty); X)$, then $\| x \|_{PC_h} \le \| x \|_\infty$. This allows us to conclude that the set $\{u_\lambda : u_\lambda = \lambda\Gamma u_\lambda, \lambda \in (0, 1)\}$ is bounded in S(∞). Finally from Theorem 2.1 we infer the existence of a fixed point of Γ, and consequently the existence of a global solution of (4.1)-(4.3). This completes the proof.

If we suppose that the functions I_i, $i \in N$ are Lipschitz continuous, then we have the following result.

Theorem 4.2. Suppose that the hypotheses (S_1), (S_2), (H_2), (H_4) and (H_6) are valid for all a > 0. Assume also that condition (a) of Theorem 3.1 and conditions (b) of theorem 4.1 are satisfied and that $\sum_{i=1}^{\infty} \| I_i(0) \| < +\infty$. If

$$\begin{aligned} &c_{1,\infty} K_\infty \left(\| i_c \|_{L(Y,X)} + \int_0^\infty H(s)ds \right) + K_\infty \tilde{M} \sum_{i=1}^{\infty} L_i \\ &\quad + \liminf_{r\to\infty} \tilde{M} \int_0^\infty m(s) \frac{W(T(r,s) + L_0\Omega(T(r,s)) + L_1\theta(T(r,s)))}{r} ds < 1 \end{aligned} \tag{4.7}$$

where $T(r,s) = (\tilde{M}K_\infty H + M_\infty) \| \phi \|_B + K_\infty r h(s)$ then there exists a global solution of the problem (4.1)-(4.3).

Proof: Let Γ be the operator introduced in the Theorem 4.1 and consider the decomposition $\Gamma = \Gamma_1 + \Gamma_2$, where $(\Gamma_i u)_0 = 0$, i = 1,2 and

$$\begin{aligned} \Gamma_1 u(t) &= T(t)g(0,\varphi) - g(t, u_t + y_t) - \int_0^t AT(t-s) g(s, u_s + y_s) ds + \\ &\quad \int_0^t T(t-s) f(s, u_s + y_s, \int_0^s h(s,\tau,u_\tau + y_\tau)d\tau, \int_0^a k(s,\tau,u_\tau + y_\tau)d\tau) ds, \quad t \ge 0 \end{aligned}$$

$$\Gamma_2 u(t) = \sum_{0<t_i<t} T(t - t_i) I_i(u_{t_i} + y_{t_i}), \quad t \ge 0$$

Proceeding as in the proof of the Theorem 4.1 we infer that the map Γ_1 is completely continuous. Moreover it is easy to see that

$$\| \Gamma_2 u - \Gamma_2 v \|_{PC_h} \le \tilde{M}K_\infty \sum_{i=1}^{\infty} L_i \| u - v \|_{PC_h}, \qquad u, v \in S(\infty)$$

Which implies that Γ_2 is a contraction in S(∞). We prove that there is r > 0 such that $\Gamma B_r \subset B_r$ where $B_r = B_r(0, S(\infty))$. In fact if we assume that the assertion is false, then, for every r > 0, there exists $u^r \in B_r$ and $t^r \ge 0$ such that $\left\| \frac{\Gamma u^r(t^r)}{h(t^r)} \right\| > r$

This yields that

$$r \le \frac{\tilde{M}}{h(t^r)} \| g(0,\varphi) \| + \frac{\| g(t^r, u_{t^r}{}^r + y_{t^r}) \|}{h(t^r)}$$
$$+ \frac{1}{h(t^r)} \int_0^{t^r} \| AT(t^r - s) \|_{L(Y,X)} \| g(s, u_s{}^r + y_s) \|_Y \, ds$$
$$+ \frac{\tilde{M}}{h(t^r)} \int_0^{t^r} m(s) W \left(\| u_s{}^r + y_s \| + \| \int_0^s h(s,\tau, u_\tau{}^r + y_\tau) d\tau \| + \| \int_0^a k(s,\tau, u_\tau{}^r + y_\tau) d\tau \| \right) d$$
$$+ \frac{\tilde{M}}{h(t^r)} \sum_{i=1}^{\infty} L_i \| u_{t_i}{}^r + y_{t_i} \| + \frac{\tilde{M}}{h(t^r)} \sum_{i=1}^{\infty} \| I_i(0) \|$$

$$r \le \frac{1}{h(t^r)} \tilde{M} \| g(0,\varphi) \| + \frac{\| i_c \|_{L(Y,X)} \, c_{1,\infty} ((\tilde{M} K_\infty H + M_\infty) \| \varphi \|_B + K_\infty r h(t^r))}{h(t^r)}$$
$$+ \frac{1}{h(t^r)} \int_0^{t^r} H(t^r - s) \left[c_{1,\infty} ((\tilde{M} K_\infty H + M_\infty) \| \varphi \|_B + K_\infty r h(s)) + d_2 \right] ds$$
$$+ \frac{1}{h(t^r)} \int_0^{t^r} \tilde{M} m(s) W \Big[((\tilde{M} K_\infty H + M_\infty) \| \varphi \|_B + K_\infty r h(s)) + L_0 \Omega((\tilde{M} K_\infty H + M_\infty) \| \varphi \|_B + K_\infty r h(s))$$
$$+ L_1 \theta((\tilde{M} K_\infty H + M_\infty) \| \varphi \|_B + K_\infty r h(s)) \Big] ds + \frac{\tilde{M}}{h(t^r)} \sum_{i=1}^{\infty} \| I_i(0) \|$$

Dividing by r and take r tends to ∞ which implies that

$$1 \le c_{1,\infty} K_\infty \left(\| i_c \|_{L(Y,X)} + \int_0^\infty H(s) ds \right) + K_\infty \tilde{M} \sum_{i=1}^{\infty} L_i + \liminf_{r\to\infty} \tilde{M} \int_0^\infty m(s) \frac{W(T(r,s) + L_0 \Omega(T(r,s)) + L_1 \theta(T(r,s)))}{r} ds$$

Which is contrary to our assumptions. This prove that there exists r > 0 such that Γ is a condensing operator form B.. This completes the proof.

5. AN APPLICATION

In this section we apply some of the results established in this paper. First, we consider the partial first order differential equation of the form

$$\frac{\partial}{\partial t}[w(u,t) + g_1(t, w(x,t-r))] = \frac{\partial^2}{\partial x^2} w(u,t) + P(t, w(u,t), \int_0^t h_1(t,s,w(x,t-r)) ds, \int_0^a k_1(t,s,w(x,t-r)) ds), \tag{5.1}$$

$$w(0,t) = w(\pi,t) = 0, \quad 0 \le t \le a \tag{5.2}$$

$$w(u,t) = \varphi(x,t), \tag{5.3}$$

$$w(t_k^+, y) - w(t_k^-, y) = I_k(w(t_k^-, y)), \; k = 1,2,...,m \tag{5.4}$$

where P : [0,a] x B x R^n x R^n $\to$ R^n is a function and g_1, h_1, k_1:[0,a] x [0,a] x R $\to$ R are continuous functions. We assume that the functions P, g_1, h_1, k_1 in (5.1)-(5.4) satisfy the following conditions.

(1) There exists constants c_1 and c_2 such that $|g(t,\phi)| \le c_1|\phi| + c_2$ for $t \in [0,a]$, $\phi \in B$.

(2) There exist a nonnegative function p_1 defined on [0,a] such that

$$\left| \int_0^t h_1(t,s,x) ds \right| \le p_1(t) |x|, \; for \;\; t,s \in [0,a] \;\; and \;\; x \in B$$

(3) There exists a nonnegative function q_1 defined on [0,a] such that

$$\left| \int_0^a k_1(t,s,x) ds \right| \le q_1(t) |x|, \; for \;\; t,s \in [0,a] \;\; and \;\; x \in B$$

(4) There exist nonnegative real valued continuous function l_1 defined on [0, a] and a positive continuous increasing function W defined on R_+ such that

$| P(t,x,y,z) \le l_1(t) W(|x|+|y|+|z|)$ for t, x, y, z $\in$ [0,a] x B x R^n x R^n.

There exists constants c_k such that $| I_k(x) | \le c_k | x |$, k=1, 2, …, m for each $x \in R^n$.

Let us take X = L^2 [0,π]. Suppose that

$$\int_0^a l_1(s)(1+p_1(s)+q_1(s))ds < \int_c^\infty \frac{ds}{W(s)}$$

is satisfied. Define the function f : [0,a] x X x X x X → X, a, b :[0,a] x [0,a] → X, gw,h : [0,a] x X → X, $I_k \in$ (X , X) as follows

f(t, x, y, z) (u) = P(t, x(u,t), y(u,t), z(u,t))
k(t, s, x) (u) = k_1(t, s, x(u,t)) and
h(t, s, x) (u) = h_1(t, s, x(u,t))

for t∈[0,a], x, y, z ∈ X and 0 ≤ u ≤ π.

With these choices of the functions, the equations (5.1)-(5.4) can be modelled abstractly as non-linear mixed Volterra-Fredholm integrodifferential equations with nonlocal condition in Banach Space X:

$$\frac{d}{dt}(x(t)+g(t,x_t)) = Ax(t) + f\left(t, x(t), \int_0^t h(t,s,x(s))ds, \int_0^a k(t,s,x(s))ds\right), \quad t \in I \tag{5.5}$$

$$\Delta x|_{t=t_k} = I_k(x(t_k^-)), \quad i = 1,2,,...,m. \tag{5.6}$$

$$x_0 = \varphi \tag{5.7}$$

since all the hypotheses of the Theorem 3 are satisfied, the Theorem 3 can be applied to guarantee the solution of the nonlinear mixed Volterra-Fredholm type neutral impulsive integrodifferential equation.

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