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A FIXED POINT THEOREM IN COMPLETE FUZZY METRIC SPACE THROUGH RATIONAL EXPRESSION

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ABSTRACT

Fuzzy metric space have introduced in many ways. We find some fixed point theorem in complete fuzzy metric space through rational expression. Our paper is generalization form of Binayak S. Choudhary and Krishnapada Das [1] for Fuzzy metric space motivated by Sushil Sharma [10].

1. INTRODUCTION

Fuzzy metric space has been introduced in many ways amongst specially to mention, fuzzy metric spaces were introduced by Kramosil and Michalek [7]. In this paper we use the concept of fuzzy metric space introduced by Kramosil and Michalek [7] and modified by George and Veeramani [5] to obtain Hausdorff topology for this kind of fuzzy metric space. Recently, Gregori and Sepena [6] extended Banach fixed point theorem to Fuzzy contraction mappings on complete fuzzy metric space in the sense of George and Veermani [5].

Our work demonstrates the fact that other types of contractions are possible in Fuzzy metric space.

2. PRELIMINARIES

Definition 2.: (Kramosil and Michalek 1975) A binary operation $*: [0,1] \times [0,1] \rightarrow [0,1]$ is a t-norm if it satisfies the following conditions :

(i) $*(1,a) = a$, $*(0,0) = 0$
(ii) $*(a, b) = *(b, a)$
(iii) $*(c, d) \geq *(a, b)$ whenever $c \geq a$ and $d \geq b$
(iv) $*(*(a, b), c) = *(a, *(b, c))$ where $a, b, c, d \in [0,1]$

Definition 2.2: (Kramosil and Michalek 1975) The 3-tuple (X,M, ∗) is said to be a fuzzy metric space if X is an arbitrary set ∗ is a continuous t-norm and M is a fuzzy set on $X^2 \times [0,\infty)$ satisfying the following conditions:

(i) $M(x,y,0) = 0$
(ii) $M(x, y, t) = 1$ for all $t > 0$ iff $x = y$,
(iii) $M(x, y, t) = M(y, x, t)$,
(iv) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(v) $M(x, y, .) : [0,\infty[\rightarrow[0,1]$ is left-continuous,

where $x, y, z \in X$ and $t, s > 0$.

In order to introduce a Hausdroff topology on the fuzzy metric space, in (Kramosil and Michalek 1975) the following definition was introduced.

Definition 2.3 : (George and Veermani 1994) The 3-tuple (X, M, *) is said to be a fuzzy metric space if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set on $X^2 \times [0, \infty]$ satisfying the following conditions :

(i) $M(x, y, t) > 0$
(ii) $M(x, y, t) = 1$ iff $x = y$,

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(iii) M(x, y, t) = M(y, x, t),
(iv) M(x, y, t) ∗ M(y, z, s) ≤ M(x, z, t + s),
(v) M(x, y, .) : [0,∞]→[0,1] is continuous,

where x, y, z ∈ X and t, s > 0.

Definition 2.4: (George and Veermani 1994) In a metric space (X, d) the 3-tuple (X, Md,*) where Md(x, y, t) = t / (t + d(x, y)) and a*b = ab is a fuzzy metric space. This Md is called the standard fuzzy metric space induced by d.

Remark: Definitions and prepositions from Gregori and Sepena 2002 [6], Kumar and Chugh 2001 [8] are also used to prove our theorem.

MAIN RESULT

Theorem : Let (X,M, *) be a complete fuzzy metric space in which fuzzy contractive sequences are Cauchy and T, R and S be mappings from (X, M ,*) into itself satisfying the following conditions:

$$T(X) \subseteq R(X) \text{ and } T(X) \subseteq S(X)$$

$$\frac{1}{M(T(x),T(y),t)} - 1 \leq k\left(\frac{1}{L(x,y,t)} - 1\right)$$

with 0 < k < 1 and

$$L(x,y,t) = \min\left\{\begin{matrix} M(Rx,Sy,t), M(Sx,Ry,t), M(Rx,Tx,t), \\ M(Ry,Ty,t), M(Sx,Tx,t), M(Sy,Ty,t), \\ \frac{M(Sx,Ry,t)M(Rx,Tx,t)}{M(Rx,Sy,t)}, \frac{M(Sx,Tx,t)M(Sy,Ty,t)}{M(Ry,Ty,t)} \end{matrix}\right\}$$

The pairs T, S and T, R are compatible. R, T and S are w-continuous.

Then R, T and S have a unique common fixed point.

Proof: Let $x_0 \in X$ be an arbitrary point of X. Since T(X) ⊆ R(X) and T(X) ⊆ S(X), we can construct a sequence $\{x_n\}$ in X such that

$$Tx_{n-1} = Rx_n = Sx_n$$

Now,

$$L(x_n, x_{n+1}, t) = \min\left\{\begin{matrix} M(Rx_n, Sx_{n+1}, t), M(Sx_n, Rx_{n+1}, t), M(Rx_n, Tx_n, t), \\ M(Rx_{n+1}, Tx_{n+1}, t), M(Sx_n, Tx_n, t), M(Sx_{n+1}, Tx_{n+1}, t), \\ \frac{M(Sx_n,Rx_{n+1},t)M(Rx_n,Tx_n,t)}{M(Rx_n,Sx_{n+1},t)}, \frac{M(Sx_n,Tx_n,t)M(Sx_{n+1},Tx_{n+1},t)}{M(Rx_{n+1},Tx_{n+1},t)} \end{matrix}\right\}$$

$$= \min\left\{\begin{matrix} M(Tx_{n-1}, Tx_n, t), M(Tx_{n-1}, Tx_n, t), M(Tx_{n+1}, Tx_n, t), \\ M(Tx_n, Tx_{n+1}, t)M(Tx_{n-1}, Tx_n, t), M(Tx_n, Tx_{n+1}, t), \\ \frac{M(Tx_{n-1},Tx_n,t)M(Tx_{n+1},Tx_n,t)}{M(Tx_{n-1},Tx_n,t)}, \frac{M(Tx_{n-1},Tx_n,t)M(Tx_n,Tx_{n+1},t)}{M(Tx_n,Tx_{n+1},t)} \end{matrix}\right\}$$

$$= \min\{M(Tx_{n-1}, Tx_n, t), M(Tx_n, Tx_{n+1}, t)\}$$

We now claim that $M(Tx_{n-1}, Tx_n, t) < M(Tx_n, Tx_{n+1}, t)$

Otherwise we claim that $M(Tx_{n-1}, Tx_n, t) \geq M(Tx_n, Tx_{n+1}, t)$

i.e. $L(x_n, x_{n+1}, t) = M(Tx_n, Tx_{n+1}, t)$

$$\therefore \quad \frac{1}{M(Tx_n,Tx_{n+1},t)} - 1 \leq k\left(\frac{1}{M(Tx_n,Tx_{n+1},t)} - 1\right)$$

which is a contradiction.

Hence, $$\frac{1}{M(Tx_n,Tx_{n+1},t)} - 1 \leq k\left(\frac{1}{M(Tx_{n-1},Tx_n,t)} - 1\right)$$

∴ $\{Tx_n\}$ is a fuzzy contractive sequence in (X, M,*). So $\{Tx_n\}$ is a Cauchy sequence in (X, M,*).

As X is a complete fuzzy metric space, $\{Tx_{n-1}\}$ is convergent. So, $\{Tx_{n-1}\}$ converges to some point z in X.

∴ $\{Tx_{n-1}\}$, $\{Rx_n\}$, $\{Sx_n\}$ converges to z. By w-continuity of R, S and T, there exists a point u in X such that $x_n \rightarrow$ u as and so lim Rx_n = lim Sx_n = lim Tx_{n-1} = z implies.

$$Ru = Su = Tu = z$$

Also by compatibility of pairs T, S and T, R and Tu = Ru = Su = z implies

Tz = TRu = RTu = Rz and Tz = TSu = STu = Sz

Therefore, Tz = Rz = Sz

We now claim that Tz = z.

If not $\frac{1}{M(Tx_n,Tx_{n+1},t)} - 1 \leq k\left(\frac{1}{M(Tx_{n-1},Tx_n,t)} - 1\right)$

$$L(z,u,t) = \min\left\{\begin{array}{c} M(Rz,Su,t), M(Sz,Ru,t), M(Rz,Tz,t), \\ M(Ru,Tu,t), M(Sz,Tz,t), M(Su,Tu,t), \\ \frac{M(Sz,Ru,t)M(Rz,Tz,t)}{M(Rz,Su,t)}, \frac{M(Sz,Tz,t)M(Su,Tu,t)}{M(Ru,Tu,t)} \end{array}\right\}$$

$$= \min\left\{\begin{array}{c} M(Tz,z,t), M(Tz,z,t), M(Tz,Tz,t), \\ M(z,z,t), M(Tz,Tz,t), M(z,z,t), \\ \frac{M(Tz,z,t)M(Tz,Tz,t)}{M(Tz,z,t)}, \frac{M(Tz,Tz,t)M(z,z,t)}{M(z,z,t)} \end{array}\right\}$$

$$= \min\{M(Tz,z,t), M(Tz,z,t), 1,1,1,1,1,1\}$$

$$= M(Tz,z,t)$$

$$\therefore \frac{1}{M(Tx_n,Tx_{n+1},t)} - 1 \leq k\left(\frac{1}{M(Tx_{n-1},Tx_n,t)} - 1\right)$$

which is a contradiction.

Hence Tz = z

So z is a common fixed point of R, T and S.

Now suppose v ≠ z be another fixed point of R, T and

$$\therefore \frac{1}{M(Tx_n,Tx_{n+1},t)} - 1 \leq k\left(\frac{1}{M(Tx_{n-1},Tx_n,t)} - 1\right)$$

$$L(v,z,t) = \min\left\{\begin{array}{c} M(Rv,Sz,t), M(Sv,Rz,t), M(Rv,Tv,t), \\ M(Rz,Tz,t), M(Sv,Tv,t), M(Sz,Tz,t), \\ \frac{M(Sv,Rz,t)M(Rv,Tv,t)}{M(Rv,Sz,t)}, \frac{M(Sv,Tv,t)M(Sz,Tz,t)}{M(Rz,Tz,t)} \end{array}\right\}$$

$$= \min\left\{\begin{array}{c} M(v,z,t), M(v,z,t), M(v,v,t), \\ M(z,z,t), M(v,v,t), M(z,z,t), \\ \frac{M(v,z,t)M(v,v,t)}{M(v,z,t)}, \frac{M(v,v,t)M(z,z,t)}{M(z,z,t)} \end{array}\right\}$$

$$= \min\{M(v,z,t), M(v,z,t), 1,1,1,1,1,1\}$$

$$= M(v,z,t)$$

$$\therefore \quad \frac{1}{M(Tx_n,Tx_{n+1},t)} - 1 \leq k\left(\frac{1}{M(Tx_{n-1},Tx_n,t)} - 1\right)$$

which is a contradiction. Hence v = z.

Thus R, T and S have a unique fixed point.

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