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# ANALYSIS OF CONVECTIVE DIFFUSION IN A PULSATILE FLOW OF A NON-NEWTONIAN VISCO ELASTIC FLUIDS IN A CHANNEL

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***ABSTRACT***

*The paper presents an exact analysis of passive contaminant molecules released in an oscillatory flow of a non-Newtonian visco-elastic fluid between two parallel plates under the influence of periodic pressure gradient, using the Aris-Barton method of moments which is valid for all time after the injection of the solute, the dispersion coefficients of a contaminant clouds are obtained separately for three different cases, plane poiseuille flow, periodic and for comparison the combined effect of steady and periodic currents. Here it is shown how the injected material disperses due to the shear effect caused by the combined effects of flow and lateral diffusion about its mean position. The analysis leads to the interesting results that the dispersion coefficients consist of a steady part and a fluctuating part due to the pulsatility of the flow.*

**Keywords:** *Unsteady, pulsatile, convective diffusion, Visco-elastic fluid, contaminant, Moments,*

## 1. INTRODUCTION

The dispersion of a cloud of soluble matter injected into a pipe has been extensively studied by many researchers following the classical work of Taylor (1953). Taylor was the first among the others to study of dispersion of a soluble matter in fluids. He showed that when a small quantity of an immiscible solute is injected into a fluid moving slowly and steadily through a circular tube, the eventual dispersion of the solute is augmented by the flow of the fluid. In fact the non-uniform distribution of flow velocity across the tube causes transverse diffusion to be effective in dispersing the solute. But this conceptual model of dispersion is valid for large time. Gill and Sankarasubramamian (1970, 1971) gave a dispersion model for the above steady flow which holds for all time after the solute injection by allowing the diffusion coefficients to vary with time. The effective longitudinal dispersion coefficient of a solute in pulsating flow through a tube was studied by Aris (1960). He showed that after the transients have died out, the ultimate mean speed of the centre of gravity of the solute is the mean flow speed and the variance of the solute distribution increases linearly with time such one-half the mean rate of growth of variance defines the apparent longitudinal diffusion coefficient. However the analysis of Aris is valid for large time after the introduction of the solute and does not throw light on the instantaneous variation of the dispersion with time immediately after the injection of the solute. Fan and Hwang (1965) calculated the time-asymptotic longitudinal dispersion coefficient in the steady laminar flow of Ostwald -dewaele power law fluid in a tube. Since axial dispersion is enhanced by larger velocity gradients across the tube, flatter profiles for pseudo-plastic fluids result in decrease in longitudinal dispersion coefficient, Taylor's intuitive approach was also used by Fan and Wang (1966) to study dispersion of solute in flows of Bingham plastic fluids. The exact method of analysis of convective diffusion developed in Gill and Sankarasubramamian (1970) was extended by Sankarasubramamian and Gill (1971) to include the characteristics of non-Newtonian flows. Results were given for the specific case of dispersion of solute in steady laminar flow of a non-Newtonian power law fluid which shows that the constant coefficient Taylor dispersion model is inadequate for describing the average concentration distribution for small values of time. The dispersion of solute in time dependent flow of a non-Newtonian fluid in a channel does not seem to have received any attention. This provides the motivation for the present study where we discuss dispersion of a contaminant solute in a non-Newtonian visco-elastic fluid flowing in a parallel plate channel in the presence of a pulsating pressure gradient. The study is likely to have important bearing on dispersion of tracers in blood flowing through large arteries where the flow is oscillatory. It was observed by Fakada and kaibara (1980) Thurston (1972) and stoltz and Lucices (1981) that under certain conditions, blood displays visco-elastic properties of the individual red cells and the internal structures formed by cellular interaction. In our present study we consider dispersion of solute in oscillatory flow of a certain non-Newtonian visco-elastic fluid between two parallel plates under the influence of

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periodic pressure gradient. In contrast to the work of Bandyopadhayay and mazumder (1999). The governing partial differential equations are dimensionlised by using non-dimensionlisation, initially solved by analytical method to find plane poiseuille flow, periodic and for comparison the combined effect of steady and periodic currents. The solution was based on the method of separation of variables, which depend upon a certain Eigen-values. They combined effect of steady and periodic flow within a tube, and also solved numerically by adopting a finite difference scheme based on the crank-Nicholson implicit method.

## 2. MATHEMATICAL FORMULATION

Consider a two dimensional fully developed laminar flow of a viscous incompressible non - Newtonian visco - elastic fluid between two parallel plates of distance $2L$ apart. A Cartesian co-ordinate frame is embedded in the lower plate with $x^*$- axis coinciding with the direction of the flow and $y^*$- axis perpendicular to the flow and the plates are at $y^* = \pm L$. The flow is caused by a periodic axial pressure gradient with a non –zero mean given by

$$\frac{-1}{\rho}\frac{\partial p}{\partial x^*} = p_{x^*}(1+\varepsilon e^{i\omega t^*}) \tag{2.1}$$

where $\rho$ is the density of the (fluid assumed to be homogeneous), $p_{x^*}$ is the mean pressure gradient, $\varepsilon p_{x^*}$ and $\omega$ are respectively the amplitude and frequency of the pressure pulsation.

The velocity distribution $u^*(y^*,t^*)$ parallel to the $x^*$-axis satisfies the Navier-Stokes equation

$$\frac{\partial u^*}{\partial t^*} = \frac{-1}{\rho}\frac{\partial p}{\partial x^*} + \gamma\left(\frac{\partial^2 u^*}{\partial y^{*^2}}\right) + \beta\left(\frac{\partial^3 u^*}{\partial y^*\partial t^{*^2}}\right) \tag{2.2}$$

Using equation (2.1) in (2.2), we get

$$\frac{\partial u^*}{\partial t^*} = p_{x^*}(1+\epsilon\, e^{iwt}) + \gamma\left(\frac{\partial^2 u^*}{\partial y^{*2}}\right) + \beta\left(\frac{\partial^3 u^*}{\partial y^*\partial t^{*3}}\right) \tag{2.3}$$

With no slip conditions at the boundary $u^*(\pm L,t)$=0. $\gamma$ is the Kinematics viscosity of the fluid and $\beta$ is the visco-elastic of the fluid.

When a contaminant of constant molecular diffusivity $D$ is injected into the above time-dependent flow, the concentration $c(x,y,t)$ contaminant satisfies dimensionless convective-diffusion equation of the form

$$\frac{\partial c}{\partial t} + p_e u(y,t)\frac{\partial c}{\partial x} = \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)c \tag{2.4}$$

where the dimensionless quantities are given by

$$x = \frac{x^*}{L} \quad y = \frac{y^*}{L} \quad t = \frac{Dt^*}{L^2} \quad u = \frac{u^*}{U} \quad p_e = \frac{UL}{D}$$

U is the reference velocity. The Peclet number $p_e$ introduced here, measures the relative characteristic time of the diffusion process $\left(\frac{L^2}{D}\right)$ to the convection process $\left(\frac{L}{U}\right)$.

The initial and boundary conditions for the contaminant input are

$$c(x,y,0) = c(x,y)$$

$$\frac{\partial c}{\partial y} = 0 \quad \text{at} \quad y = \pm 1 \tag{2.5}$$

Since c is finite at all points, require

$$x^n c \to 0 \text{ and } x^m\frac{\partial^n c}{\partial x^n} \to 0 \text{ as } |x| \to \infty \quad m,n = 0,1,2,\ldots$$

$$\frac{1}{2}\int_{-1}^{1}\int_{-\infty}^{\infty} c\,dx\,dy = 1$$

 

Equations (2.3) is solved by using the no-slip condition $u(\pm 1, t) = 0$ at the boundary $y = \pm 1$ is given by

$$u(y,t) = u_0(y) + u_1(y,t) \tag{2.6}$$

Substituting equations (2.6) in (2.3) and solving for $u_0(y)$ and $u_1(y, t)$

$$u_0 = \frac{1}{2}(1 - y^2) \tag{2.7}$$

$$u_1(y,t) = -\mathrm{Re}\left[\frac{i\varepsilon}{\alpha}\left(1 - \frac{\cosh\left(\sqrt{\frac{\alpha^2\beta + i\alpha}{1+\alpha^2\beta^2}}\right)y}{\cosh\left(\sqrt{\frac{\alpha^2\beta + i\alpha}{1+\alpha^2\beta^2}}\right)}\right)e^{i\alpha st}\right] i = \sqrt{-1} \tag{2.8}$$

Substituting equation (2.7) and (2.8) in (2.6) we get,

$$(y,t) = \frac{1}{2}(1 - y^2) - Re\left[\frac{i\varepsilon}{\alpha}\left(1 - \frac{\cosh\left(\sqrt{\frac{\alpha^2\beta + i\alpha}{1+\alpha^2\beta^2}}\right)y}{\cosh\left(\sqrt{\frac{\alpha^2\beta + i\alpha}{1+\alpha^2\beta^2}}\right)}\right)e^{i\alpha st}\right]_i = \sqrt{-1} \tag{2.9}$$

$u = \frac{u^*}{U}$ is the dimensionless axial velocity (U being the time average axial velocity $\frac{p_{x^*} L^2}{4\gamma}$ ), $\alpha = \frac{\omega L^2}{\gamma}$ is the dimensionless frequency parameter or oscillation Reynolds number, $s = \frac{\gamma}{D}$ is the Schmidt number .The first term of the right hand side of (2.9) corresponds to the plane Poiseuille flow and second term corresponds to the flow due to pulsation, Here, of course, the physical significance is attributed only to the real part, and the real part of $u(y,t)$ is given by

$$u(y,t) = \frac{1}{2}(1-y^2) - \frac{\varepsilon}{\alpha(k_1^2 + k_2^2)}$$

$$\left[\begin{aligned}&\left\{k_1(\cosh(r^{\frac{1}{2}}\cos\frac{\theta}{2}y)\cos(r^{\frac{1}{2}}\sin\frac{\theta}{2}y) - k_1) + k_2(\sinh(r^{\frac{1}{2}}\cos\frac{\theta}{2}y)\sin(r^{\frac{1}{2}}\sin\frac{\theta}{2}y) - k_2)\right\}\sin\alpha st \\ &+\left\{k_1(\sinh(r^{\frac{1}{2}}\cos\frac{\theta}{2}y)\sin(r^{\frac{1}{2}}\sin\frac{\theta}{2}y) - k_2\cosh(r^{\frac{1}{2}}\cos\frac{\theta}{2}y)\cos(r^{\frac{1}{2}}\sin\frac{\theta}{2}y)\right\}\cos\alpha st\end{aligned}\right] \tag{2.10}$$

where

$$k_1 = \cosh(r^{\frac{1}{2}}\cos\frac{\theta}{2})\cos(r^{\frac{1}{2}}\sin\frac{\theta}{2})$$

$$k_2 = \sinh(r^{\frac{1}{2}}\cos\frac{\theta}{2})\sin(r^{\frac{1}{2}}\sin\frac{\theta}{2})$$

Following Aris (1956, 1960) we define the $n^{\text{th}}$ moment of the concentration distribution through y at time *t* as

$$c_n(y,t) = \int_{-\infty}^{\infty} x^n c(x,y,t)\, d\, x \tag{2.11}$$

and the $n$ th moment of the concentration over the cross section of the channel as

$$M_n(t) = \frac{1}{2}\int_{-1}^{1} c_n(y,t) dy = \overline{c_n} \tag{2.12}$$

Using equation (2.11) and (2.12) in equation (2.4) and (2.5) we have the following moment equation

$$\frac{\partial c_n}{\partial t} = \frac{\partial^2 c_n}{\partial y^2} + n(n-1)c_{n-2} + np_e u(y,t)c_{n-1} \tag{2.13}$$

with $c_n(y,0) = c_n(y)$ $\qquad \frac{\partial c_n}{\partial y} = 0$ at $y = \pm 1$ (2.14) and $$\frac{\partial M_n}{\partial t} = np_e \overline{xc_{n-1}} + n(n-1)M_{n-2} \tag{2.15}$$

$$M_n(0) = \overline{c_n} \tag{2.16}$$

Here an over bar denotes the cross sectional mean. Also $M_0(t) = 1$, since *c* has cross -sectional mean unity for all time and $M_1(t)$ is the mean of the distribution and $M_n(0) = 0$ for $n > 0$.

The $n^{\text{th}}$ central moment of the concentration distribution can be defined as

$$v_n(t) = \frac{1}{2M_0} \int_{-1}^{1} \int_{-\infty}^{\infty} \left(x - \overline{x_g}\right)^n cdxdy \tag{2.17}$$

where $\overline{x_g} = \frac{1}{2M_0} \int_{-1}^{1} \int_{-\infty}^{\infty} xcdxdy = \frac{M_1}{M_0}$ can be regarded as the centroid of the containment distribution which measures the location of centre of gravity of the cloud movement with the mean velocity of the fluid, initially located at the source and the second central moment $(v_2)$ can be related to the dispersion of diffusing substance about its mean position. Thus we have the expression for central moments

$$v_2(t) = \frac{M_2}{M_0} - \overline{x}_g^{\,2}$$

$$v_3(t) = \frac{M_3}{M_0} - 3\overline{x}_g v_2(t) - \overline{x}_g^{\,3}$$

$$v_4(t) = \frac{M_4}{M_0} - 4\overline{x}_g v_3(t) - 6\overline{x}_g^{\,2} v_2 - \overline{x}_g^{\,4} \tag{2.18}$$

Though the third and fourth moments are also important factors during the initial stage, the present study is concentrated only to the dispersion effect (variance).

The aim of the analysis is to solve the moment equations (2.13) and (2.15) subject to the initial and boundary conditions (2.14) and (2.16) for n = 0, 1, 2, 3……The method of solution adopted here is the Aris (1960) method of moment as modified by Barton [1983] for steady flow and later by Mukherjee and Mazumder (1988) for oscillatory flow. The Eigen value equation

$$\left(\frac{\partial^2}{\partial y^2} - \frac{\partial}{\partial t} + \mu_i\right) f_i = 0 \tag{2.19}$$

$$\frac{\partial f_i}{\partial y} = o \text{ at } y = \pm 1, \quad f_i \text{ is finite} \tag{2.20}$$

where *i* runs over the positive integral values.

This gives us a discrete set of Eigen-values $\mu_i = \frac{(i\pi)^2}{1+\varepsilon}$ and the corresponding Eigen function

$f_i(y,t) = \sqrt{2}\cos i\pi y e^{-\mu_i \varepsilon t}$ *i* = 1, 2, 3…….. so that

$$\overline{f_i} = 0$$

$$\overline{f_i f_j} = 0 \quad \text{if } i \neq j$$

$$\overline{f_i f_j} = h_i \quad \text{if } i = j \tag{2.21}$$

where $h_i$ is the function of *t* alone and $h_i(0) = 1$. To complete the set of Eigen functions we augment this set by setting $f_0 = 1$ corresponding to $\mu_0 = 0$.

 

Following Mukherjee and Mazumder (1988), the expression for variance $v_2(t)$ is given by

$$v_2(t) = 2t - 2p_e^2 \sum_i a'_{0i}(0) \int \overline{uf_i}\, e^{-\mu s}\, dt$$
$$+2p_e^2 \sum_i \int a'_{0i}\, \overline{uf_i}\, dt + 2p_e^2 \sum_i a'_{0i}(0)\left[\int \overline{uf_i}\, e^{-\mu s}\, dt\right]_{t=0} - 2p_e^2 \sum_i \int \left[a'_{0i}\overline{uf_i}\, dt\right] \quad (2.22)$$

where $a'_{oi}(t) = e^{-\mu_i t} \int e^{\mu_i t}\, \overline{uf_i} h^{-1}{}_i(t)\, dt$ .

The rate of growth of variance which indicates the degree of dispersion effect at any time is given by

$$\frac{1}{2}\frac{dv_2}{dt} = 1 + p^2{}_e D_a \quad (2.23)$$

Where $D_a$ is the apparent dispersion coefficient depending on parameters $s, \alpha, \varepsilon, t$ and $\delta$ .The first term on the right hand side comes from longitudinal diffusion and second term represents the interaction between the convection and lateral diffusion. The analysis is confined to study the behavior of variance $v_2$ and the dispersion $D_a$ due to shear effects of steady, oscillatory and periodic currents.

**PLANE POISEUILLE FLOW**

The velocity distribution of the plane poiseuille flow through a parallel plate channel is given (Putting $\varepsilon = 0$ in (2.10)) by

$$u(y) = \frac{1}{2}\left(1 - y^2\right) \quad (2.24)$$

The corresponding results for the plane poiseuille flow may be retrieved from the Eigen value problem (2.19) by putting $\frac{\partial f_i}{\partial t}$ equal to zero which is same as given by Barton(1983). The Eigen values and the corresponding normalized Eigen functions for this steady flow are therefore given by

$$\mu_i = (i\pi)^2$$

$$f_i(y) = \sqrt{2}\cos i\pi y \qquad i = 1, 2, 3 \ldots \quad (2.25)$$

The corresponding expression for $v_2$ is

$$v_2(t) = 2t\left[1 + p_e{}^2 \sum \frac{8}{(i\pi)^6}\right] - 2p_e{}^2 \sum_i \frac{8}{(i\pi)^8}\left(1 - e^{-\mu it}\right) \quad (2.26)$$

The rate of change of variance $\frac{1}{2}\frac{dv_2}{dt}$ is proportional to the sum of a constant quantity one due to longitudinal diffusion and the apparent dispersion coefficient $D_a$ given by

$$D_a\Big|_{Steady} = \sum_i \frac{8}{(i\pi)^8}\left(1 - e^{-\mu_i t}\right) \quad (2.27)$$

For large time $(t \to \infty)$ the longitudinal dispersion coefficient $D_a$ can be written as

$$D_a\Big|_{t\to\infty} = \frac{1}{2}\frac{dv_2}{dt}\bigg|_{t\to\infty} = 1 + Pe^2 \sum_i \frac{8}{(i\pi)^6} \quad (2.28)$$

This is consistent with the asymptotic results of Chat win(1970) and Barton(1983)

**PERIODIC FLOW**

If we consider the flow to be unsteady only due to the periodic pressure gradient, the velocity of the fluid can be obtained from equation (2.6) by setting $u_0 = 0, \varepsilon \neq 0$ and is given by

$$u = u_1(y,t) \quad (2.29)$$

where $u_1(y,t)$ is given by equation (2.8), Using the general form of $v_2(t)$ from equation (2.22) we can easily derive the explicit expression for variance $v_2(t)$ taking into account the Eigen values and the corresponding Eigen functions of the Eigen value problem (2.19).

$$
\begin{aligned}
v_2(t) = {} & 2[1+p_e^2\sum_i \frac{b_i(D_{1i}^2+D_{2i}^2)}{2}]t \\
& -2p_e^2\sum_i b_i^2(D_{2i}-\frac{\alpha s D_{1i}}{i^2\pi^2})[(D_{2i}\frac{\alpha s}{i^2\pi^2}-D_{1i})\sin\alpha st(D_{2i}\frac{\alpha s}{i^2\pi^2}+D_{1i})cos\alpha st]e^{-i2\pi^2 t} \\
& +2p_e^2\sum_i \frac{b_i}{4\alpha s}[(D_{2i}^2-D_{1i}^2-2D_{1i}D_{2i}\frac{\alpha s}{i^2\pi^2})\sin 2\alpha st \\
& +\{2D_{1i}D_{2i}+\frac{\alpha s}{i^2\pi^2}(D_{2i}^2-D_{1i}^2)\}(1-\cos 2\alpha st)] \\
& -2p_e^2\sum_i (D_{2i}-\frac{\alpha s}{i^2\pi^2}D_{1i})(D_{2i}+\frac{\alpha s}{i^2\pi^2}D_{1i})b_i^2
\end{aligned} \tag{2.30}
$$

where $b_i = \dfrac{(i\pi)^2}{(\alpha s)^2+(i\pi)^4}$

$$D_{1i} = \frac{-\varepsilon\sqrt{2}}{\alpha[k_1^2+k_2^2]}[k_1R_2+k_2R_1]$$

$$D_{2i} = \frac{-\varepsilon\sqrt{2}}{\alpha[k_1^2+k_2^2]}[k_1R_1-k_2R_2]$$

$$R_1 = \frac{\{[\cosh(\sqrt{r}\cos\frac{\theta}{2})][(\sqrt{r}\cos\frac{\theta}{2})][\sin(\sqrt{r}\sin\frac{\theta}{2}+i\pi)]\}-\{[\sinh(\sqrt{r}\cos\frac{\theta}{2})][\sqrt{r}\sin\frac{\theta}{2}+i\pi][\cos(\sqrt{r}\sin\frac{\theta}{2}+i\pi)]\}}{(\sqrt{r}\cos\frac{\theta}{2})^2+(\sqrt{r}\sin\frac{\theta}{2}+i\pi)^2}$$
$$+\frac{\{[\cosh(\sqrt{r}\cos\frac{\theta}{2})][(\sqrt{r}\cos\frac{\theta}{2})][\sin(\sqrt{r}\sin\frac{\theta}{2}-i\pi)]\}-\{[\sinh(\sqrt{r}\cos\frac{\theta}{2})][\sqrt{r}\sin\frac{\theta}{2}-i\pi][\cos(\sqrt{r}\sin\frac{\theta}{2}-i\pi)]\}}{(\sqrt{r}\cos\frac{\theta}{2})^2+(\sqrt{r}\sin\frac{\theta}{2}-i\pi)^2}$$

$$R_2 = \frac{\{[\sinh(\sqrt{r}\cos\frac{\theta}{2})][(\sqrt{r}\cos\frac{\theta}{2})][\cos(\sqrt{r}\sin\frac{\theta}{2}+i\pi)]\}+\{[\cosh(\sqrt{r}\cos\frac{\theta}{2})][\sqrt{r}\sin\frac{\theta}{2}+i\pi][\sin(\sqrt{r}\sin\frac{\theta}{2}+i\pi)]\}}{(\sqrt{r}\cos\frac{\theta}{2})^2+(\sqrt{r}\sin\frac{\theta}{2}+i\pi)^2}$$
$$+\frac{\{[\sinh(\sqrt{r}\cos\frac{\theta}{2})][(\sqrt{r}\cos\frac{\theta}{2})][\cos(\sqrt{r}\sin\frac{\theta}{2}-i\pi)]\}+\{[\cosh(\sqrt{r}\cos\frac{\theta}{2})][\sqrt{r}\sin\frac{\theta}{2}-i\pi][\sin(\sqrt{r}\sin\frac{\theta}{2}-i\pi)]\}}{(\sqrt{r}\cos\frac{\theta}{2})^2+(\sqrt{r}\sin\frac{\theta}{2}-i\pi)^2}$$

The apparent dispersion coefficient $D_a$ can be obtained from (2.22) as

$$D_a\big|_{Steady} = \sum_i \frac{b_i}{2}[(D_{1i}^2+D_{2i}^2)+A_1\cos 2\alpha st+A_2\sin 2\alpha st]-\sum_i b_i[(D_{2i}-\frac{\alpha s}{i\pi}D_{1i})(D_{2i}\cos\alpha st+D_{1i}\sin\alpha st]e^{-i^2\pi^2 t} \tag{2.31}$$

where $A_1 = D_{2i}^2-D_{1i}^2-2D_{1i}D_{2i}\dfrac{\alpha s}{i^2\pi^2}$ and $A_2 = 2D_{1i}D_{2i}+\dfrac{\alpha s}{i^2\pi^2}(D_{2i}^2-D_{1i}^2)$

For a large time after the release the expression for $D_a\big|_{Steady}$ reduces to the form

$$D_a\big|_{Steady} \approx A_0+[A_1\cos 2\alpha st+A_2\sin 2\alpha st] \tag{2.32}$$

where the constants $A_0$, $A_1$ and $A_2$ depends on $\varepsilon,\alpha,\delta$ and $s$. This result is consistent with the work of Chat win. It is observed that equation (2.31) consists of a steady $A_0$ and a fluctuating part within parenthesis due to the periodicity in the flow.

 

As the parameters $\varepsilon, \alpha, \delta$ and $S$ are involved in the expression of velocity $u$, the variance $V_2$ and $D_a$ are much more important from the physical point of view. The parameter $\varepsilon$ indicates the extent to which the velocity profile deviates from the Poiseuille profile after the perturbation is introduced in the steady flow. On the other hand the parameter $\alpha\left(=\frac{\omega L^2}{\nu}=\frac{L^2}{\nu}:\frac{1}{\omega}\right)$ is a measure of the ratio of the time required for viscous force to diffuse across the channel width $\left(\frac{L^2}{\nu}\right)$ to the period of imposed oscillation $\left(\frac{1}{\omega}\right)$, and the Schmidt number $S$ is a measure of the ratio of the intensities of viscous diffusion and the molecular diffusion. Therefore $\alpha S\left(=\frac{\omega L^2}{\nu}.\frac{\nu}{D}\right)$ can be regarded as the ratio of the time taken for transverse variations in concentration to be smoothed out by molecular diffusion $\left(\frac{L^2}{D}\right)$ to the period of imposed oscillation. The effect of the oscillation parameter on the variance $v_2$ and the apparent dispersion coefficient will be discussed.

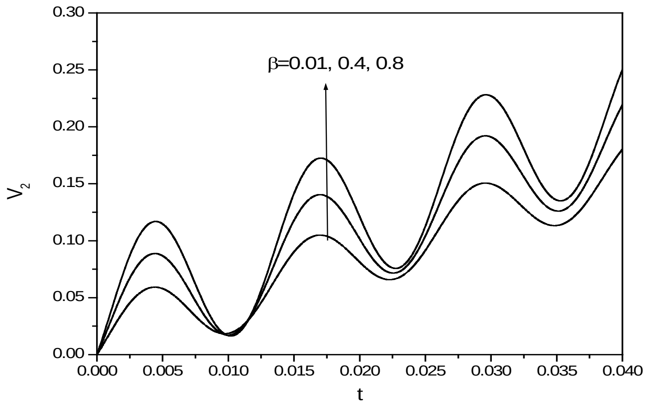


**Fig. 1:** The temporal variation of variance ($V_2$) due to periodic flow for S = Pe = $10^3$, ε = 1.5; small time when α = 0.5

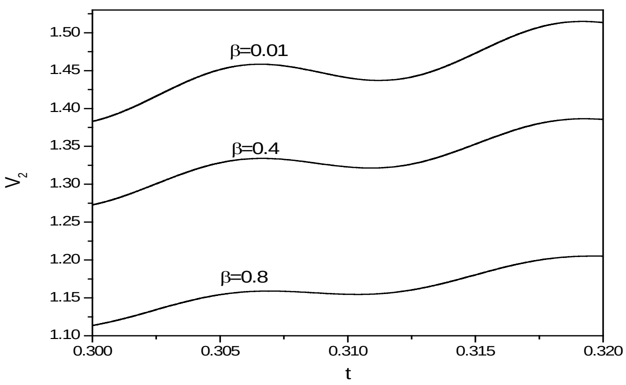


**Fig. 2:** *The temporal variation of variance ($V_2$) due to periodic flow for S =Pe = $10^3$, ε = 1.5; large time when α = 0.5*

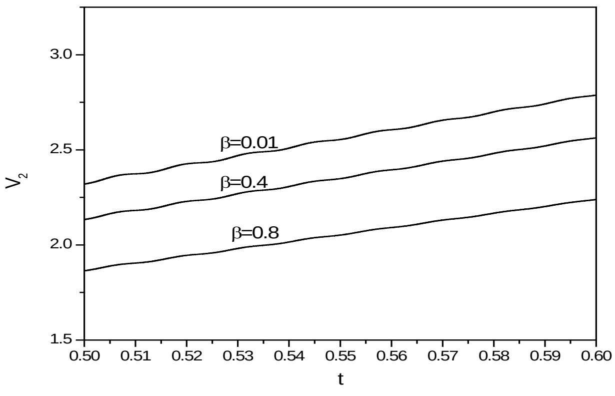


**Fig. 3:** The temporal variation of variance ($V_2$) due to periodic flow for S = $P_e$ = $10^3$, ε = 1.5; large time when α = 0.5

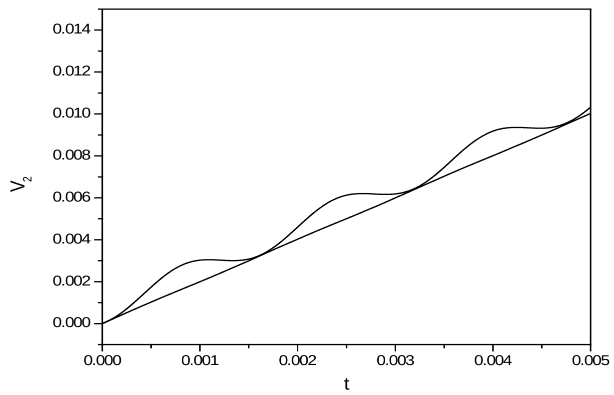


**Fig. 4:** The temporal variation of variance ($V_2$) due to periodic flow for S = Pe = $10^3$, ε = 1.5; small time when α = 4

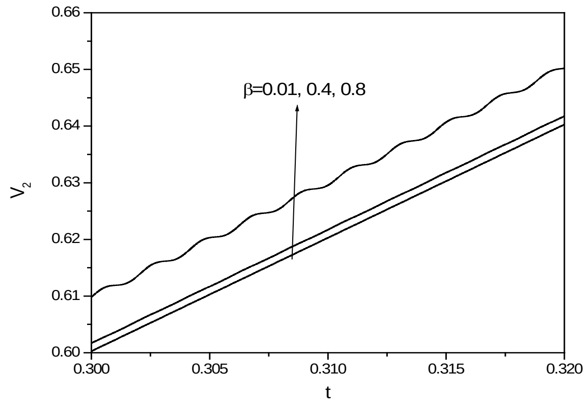


**Fig. 5:** The temporal variation of variance($V_2$) due to periodic flow for S = Pe = $10^3$, ε =1.5 large time when α = 4

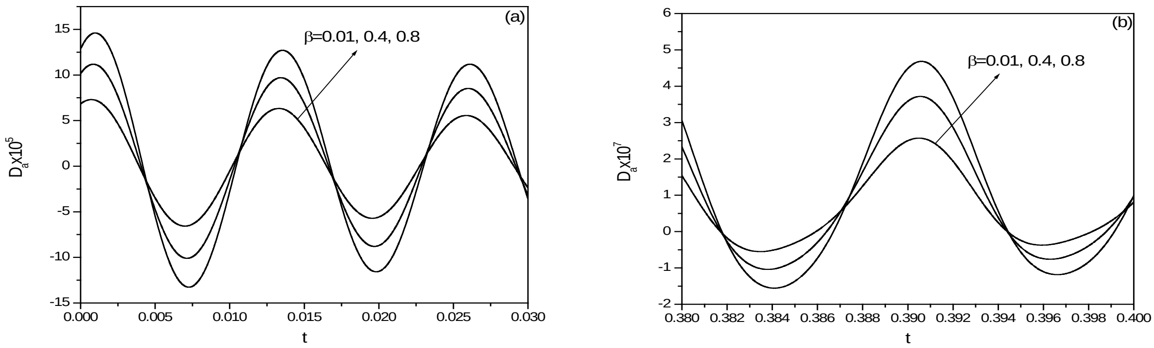


**Fig. 6:** The dispersion coefficient $D_a$ due to periodic flow (a) small time (b) Large time s = $10^3$, ε = 1.5, α = 0.5

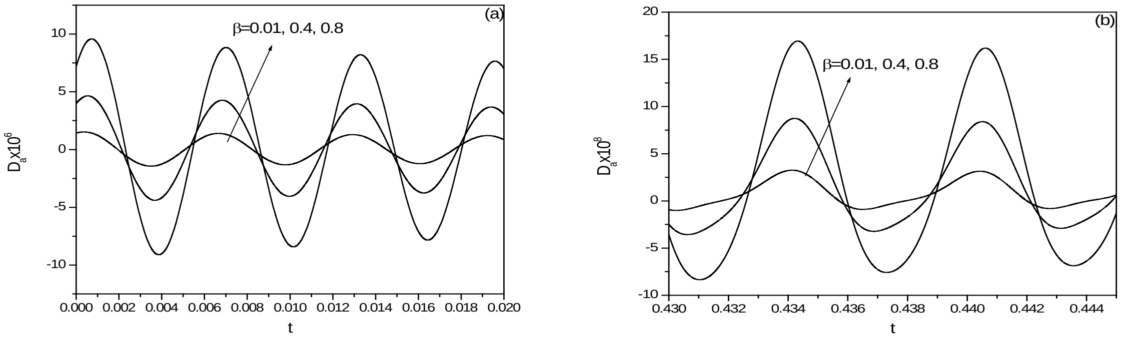


**Fig. 7:** The dispersion coefficient $D_a$ due to periodic flow (a) small time (b) Large time s = $10^3$, ε = 1.5, α = 1

 

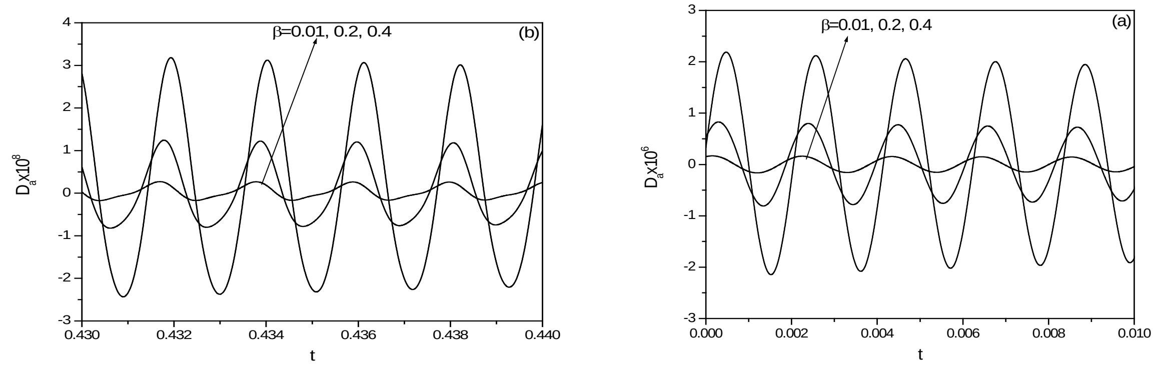


**Fig. 8:** The dispersion coefficient $D_a$ due to periodic flow (a) small time (b) Large time $s = 10^3$, $\varepsilon = 1.5$, $\alpha = 3$

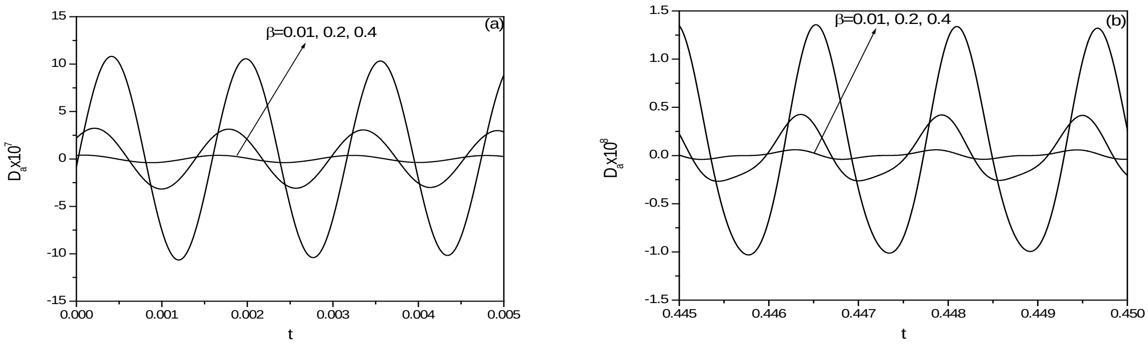


**Fig. 9:** The dispersion coefficient $D_a$ due to periodic flow (a) small time (b) Large time $s = 10^3$, $\varepsilon = 1.5$, $\alpha = 4$

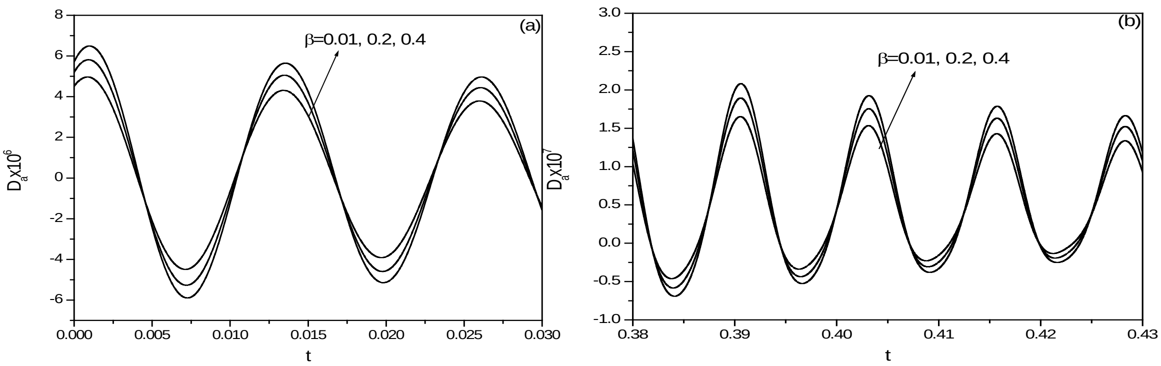


**Fig.10:** The dispersion coefficient $D_a$ due to periodic flow (a) small time (b) Large time $s = 10^3$, $\varepsilon = 1$, $\alpha = 0.5$

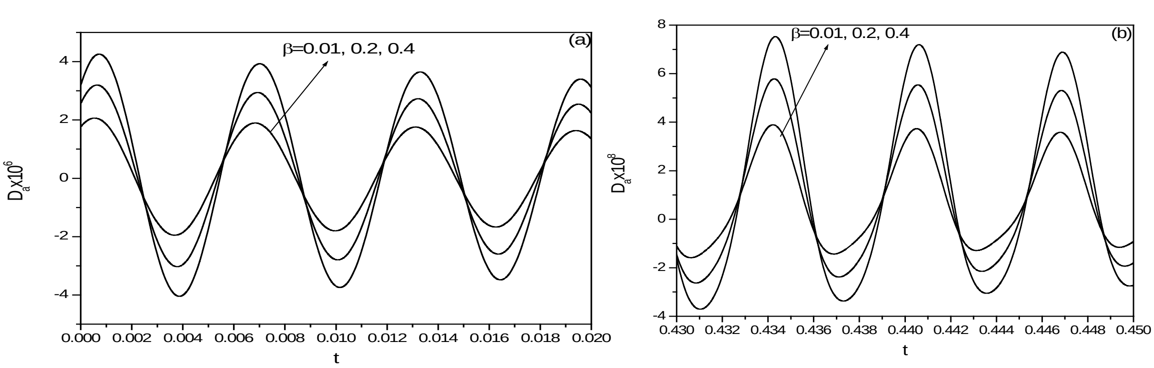


**Fig. 11:** The dispersion coefficient $D_a$ due to periodic flow (a) small time (b) Large time $s = 10^3$, $\varepsilon = 1$, $\alpha = 1$

 

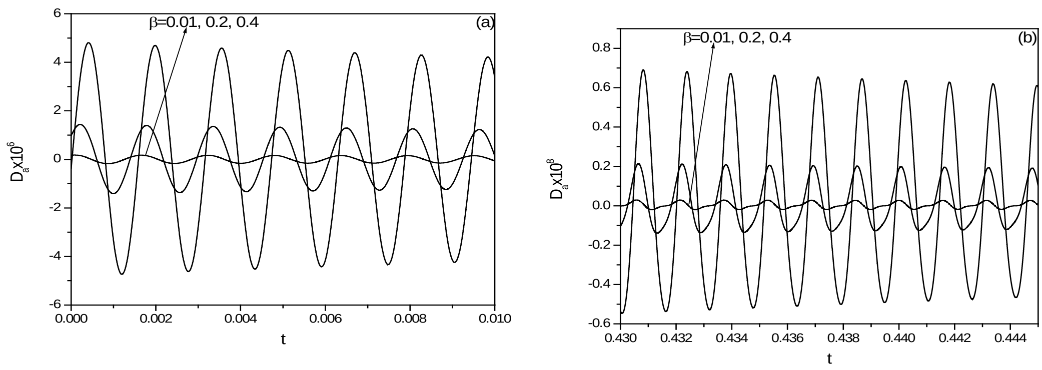


**Fig.12:** The dispersion coefficient $D_a$ due to periodic flow (a) small time (b) Large time $s = 10^3$, $\varepsilon = 1$, $\alpha = 4$

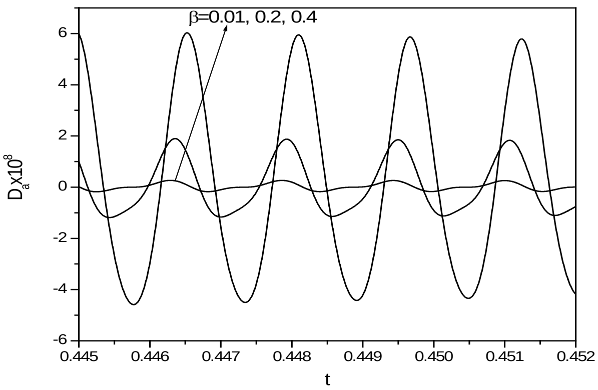


**Fig. 13:** The dispersion coefficient $D_a$ due to periodic flow for Large time $s = 10^3$, $\varepsilon = 1$, $\alpha = 4$

## 3 RESULTS AND DISCUSSION

An exact analysis of unsteady convective diffusion of passive contaminant molecules released in an incompressible viscous fluid flowing through a channel under the influence of a periodic pressure gradient is studied using the Aris-Barton method of moments. The problem brings into focus the effect of pulsatility and the visco-elastic parameter $\beta$ on the temporal variation of variance $v_2$ on the most dominant dispersion coefficient $D_a$ the results are in good agreement with those of Bandyopadhayay and Mazumder (1999).

Figures (6.1 - 6.5) are plots of the variance $v_2$ of the longitudinal concentration distribution against the dispersion time t for low and high frequency of oscillation $\alpha$= 0.5,4.0 due to oscillatory current and the visco-elastic parameter $\beta$=0.01,0.4,0.8. In steady current variance $v_2$ increases rapidly with dispersion time t which agrees well with that of yasuda (1984) in which he has shown the temporal changes of the vertically averaged variance. In oscillatory flow for small frequency $\alpha$=0.5, it is seen that the variance increases with time in a wavy nature. In one complete period, variance changes periodically with a double frequency from the figure 6.1 and it reaches a stable state after a certain time (~ t > 0.30) whereas for high frequency oscillation in the variance almost vanishes from the figures ( 6.4- 6.5) and its increases with a wavy nature. From these observations it may be concluded that the variance $v_2$ due to the oscillatory current was found to be much smaller than that due to the periodic flow. That is the pulsatility of the flow arising out of a periodic pressure gradient reduces the value of $v_{2.}$

Aris (1956) we have already described the apparent longitudinal coefficient $D_a$ on a function of $\alpha$, $\varepsilon$, s and the dispersion time. The dispersion coefficient $D_a$ will be discussed for each velocity distribution and different frequency of oscillation. In the case of a steady flow $u=u_0$ the dispersion coefficient $D_a$ increases with time t and asymptotically reaches a steady state (t~ 0.0023) at dimensionless time (t~ 0.34). It is interesting to note that the dispersion coefficient for steady flow through a channel is much smaller than that of the flow through a tube. Smith (1982) pointed out that in steady flow the apparent longitudinal dispersion coefficient can exceed molecular diffusivities are only achieved after the solute has been mixed right across the flow. The variation of the apparent dispersion coefficient Da with dispersion time t in the oscillatory flow $u = u_1 (y, t)$ is plotted in figures (6.6 - 6.9) for $\alpha$ = 0.5, 1.0, 3.0, 4.0 with visco-elastic parameter $\beta$ = 0.01, 0.4. 0.8.

From figure 6.6 it can be seen that dispersion coefficient Da changes periodically with a double frequency period in oscillatory flow and after a certain time, it reaches a stationary state. At low frequency $\alpha$ = 0.5 of oscillation the amplitude of $D_a$ are approximately equal for all time, from the figure 6.6(a) whereas in the case of high frequency $\alpha$ = 3.0, 4.0. $D_a$ varies periodically with the same frequency of oscillation as the periodic current during the initial stages and then it fluctuates with a double frequency oscillation. It is observed from the figure 6.6(a) that for small $\alpha$=0.5 the dispersion coefficient $D_a$ reaches the steady state earlier than that for high frequency, $D_a$ is more significant during the first half of the period than the second one. The solute disperses at a fairly uniform rate after a certain time (t ~ 0.4) that means $D_a$ oscillates steadily Yasuda (1984). The fluctuations in the velocity profile induce the positive and negative dispersion during the period of oscillation. A negative dispersion coefficient has been obtained due to the reversing flow of oscillation currents Smith (1983) at a particular level and $D_a$ decreases with increasing $\alpha$ which shows that due to the high frequency of oscillation $D_a$ becomes negligible although for steady and quasi-steady flow it is more significant it can be seen that a fixed instant the amplitude of $D_a$ increases in the amplitude of pressure pulsation.

Figures (6.10 - 6.13) are plots dispersion coefficient Da with dispersion time t in the oscillatory flow when $\varepsilon$ =1,
$\alpha$ = 0.5, 1.0, 3.0, 4.0 with visco - elastic parameter $\beta$ = 0.01, 0.2, 0.4 it is observed from the figures (6.10 -6.13) the amplitude of $D_a$ increases in the amplitude of pressure pulsation.

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