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MIXED CONVECTION FLOW IN A VERTICAL CHANNEL FILLED WITH POROUS MEDIUM WITH BOUNDARY CONDITIONS OF THIRD KIND WITH HEAT SOURCE/SINK-BRINKMAN MODEL

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ABSTRACT

The effect of mixed convection in a vertical channel with porous medium is analysed in the present study with heat source or sink using Brinkman model. The plate exchanges heat with external fluid. Both equal and of different reference temperature of the external fluid is considered. The governing equations are solved numerically by using Runge-Kutta fourth order shooting method and analytically by using pertubation method. The effect of various parameters involved in the problem are illustrated graphically. The analytical and numerical solutions agree very well for small values of pertubation parameter. In the absence of porous parameter and heat source or sink the result agree with that of Zanchini [1].

Keywords: *Mixed convection; porous medium; Runge-Kutta method; perturbation parameter; Heat source; Heat sink.*

1. INTRODUCTION

Forced and mixed convection in porous channels have been the focus of a variety of numerical studies and modeling efforts. Free and mixed convection on vertical surfaces under various thermal boundary conditions and external flow conditions remain problems of focus both experimentally and analytically. The literature on heat and mass transfer in porous media continues to expand, and several studies of a very fundamental nature capture the breadth of recent activity are published recently e.g. Nield and Bejan [2], Vafai [3], Bejan et al. [4], and Pop and Ingham [5], is motivated by numerous applications of this class of phenomena in the modern technologies. The large number of applications, such as, oil extraction, fluid flow in geothermal reservoirs, solid matrix heat exchangers, iron blast furnaces, energy efficient drying processes, ground water hydrology, solidification of casting etc. Lauriat and Prasad [6] investigated the free convection in a vertical porous layer and in a vertical enclosure filled with a porous medium. Hadim and Chen [7] carried out a numerical study of buoyancy-aided mixed convection in an isothermally heated vertical channel filled with a fluid saturated porous medium.

Recently, Ali [8] analyzed the effect of lateral mass flux on the natural convection boundary layer induced by a heated vertical plate embedded in a saturated porous medium with an exponential decaying heat generation. Ishak et al. [9] presented the problem of mixed convection boundary layer flow over a vertical surface embedded in a thermally stratified porous medium assuming that the external velocity and surface temperature with an exponential decaying heat generation. The study of heat generation in moving fluids is important in view of several physical problems such as those concerned with dissociating fluids. Possible heat generation effects may alter the temperature distribution and, therefore, the particle deposition rate. This may occur in such applications related to electronic chips and semi conductor wafers, nuclear reactor cores, fire and combustion modeling. In fact, the literature is replete with examples dealing with the heat transfer in laminar flow of viscous fluids.

Effect of the heat generation or absorption and thermophoresis on a hydromagnetic flow with heat and mass transfer over a flat plate was investigated by Chamkha and Issa [10]. Also the effects of the conjugate conduction-natural convection heat transfer along a thin vertical plate with non-uniform heat generation have been studied by Mendez and Trevino [11]. Recently Molla et al. [12] have investigated the natural convection flow with heat generation/absorption along a uniform heated vertical wavy surface. The convection in enclosures has become increasingly important in engineering applications. In particular, as high power electronic packaging and component density continue to increase substantially with the fast growth of electronic technology, effective cooling of electronic equipment has become warranted.

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The application of natural convection cooling for electronic equipment ranges from individual transistors to mainframe computers and from energy supplies to telephone switch boards. Recent technological implications have given rise to increased interest in combined free and forced convection flow in vertical channels in which the objective is to secure a quantitative understanding of a configuration having current engineering applications (Al-Hadharami et al., [13]). Parang and Keyhani [14] studied the fully developed buoyancy-assisted mixed convection in a vertical annulus by using Brinkman-extended Darcy model. Their results indicated that the Brinkman term could be neglected for lower Darcy number. Muralidar [15] performed a numerical calculation for buoyancy-assisted mixed convection in a vertical annulus by using the Darcy model. The results show that the Nusselt number increases with the Rayleigh number and/or Peclet number. Umavathi et al. [16-19] studied mixed convection in a vertical and horizontal porous channel.

In the past, the laminar forced convection heat transfer in the thermal entrance region of a rectangular channel has been analyzed either for the temperature boundary condition of the first kind, characterized by prescribed wall temperature (Wibulswas [20], Lyczkowski *et al*., [21] and Javeri, [22]) or for the boundary condition of the second kind, expressed by the prescribed wall heat flux (Hicken, [23] and Sparrow and Siegel, [24]). A more realistic condition in many applications, however, will be temperature boundary condition of third kind: the local wall heat flux is a linear function of the local wall temperature. Heat transfer in laminar region of a flat channel for the temperature boundary condition of third kind was explored by Javeri [25]. Javeri [26] investigated the influence of the temperature boundary condition of the third kind on the laminar heat transfer in the thermal entrance region of a rectangular channel. Later Zanchini [1] analyzed the effect of viscous dissipation on mixed convection in a vertical channel with boundary conditions of third kind. Kumari and Nath [27] analyzed the effect of localized cooling/heating and injection/suction on mixed convection flow on a thin vertical cylinder. Mahanti and Gaur [28] investigated the effects of the viscosity and thermal conductivity on steady free-convection flow of a viscous incompressible fluid along an isothermal vertical plate in the presence of heat sink. Umavathi and Prathap [29] found the exact solutions for the mixed convection flow of micro-polar fluid in a vertical channel with symmetric and asymmetric conditions in the presence of source or sink. Prathap Kumar *et al* [30] studied mixed convection magneto hydrodynamic and viscous fluid in a vertical channel.

Accordingly, the aim of the present paper is to explore the velocity and temperature fields in a vertical channel embedded with porous medium using boundary conditions of the third kind. Both equal and different reference temperatures of the external fluid as well as both equal and unequal Biot numbers are considered. In the absence of porous medium and heat source/sink the solutions obtained in this paper coincide with those of the Zanchini [1].

2. PROBLEM FORMULATION

Consider the steady and laminar flow of a Newtonian fluid in the fully developed region of a parallel-plate vertical channel filled with a porous material. The porous medium is isotropic and homogeneous. The X -axis lies on the axial plane of the channel, and its direction is opposite to the gravitational field. The Y -axis is orthogonal to the walls. The channel occupies the region of space $-L/2 \le Y \le L/2$. The thermal conductivity, the thermal diffusivity, the dynamic viscosity and the thermal expansion coefficient of the fluid are considered constant. As customary, the Boussineq approximation and the equation of state will be adopted.

$$\rho = \rho_0[1-\beta_t(T-T_0)] \tag{1}$$

Moreover, it will be assumed that the only nonzero component of the velocity field U is the X -component of the U . Thus, since $\nabla.U$ =0, one has so that U depends only on Y .

$$\frac{\partial U}{\partial X} = 0 \tag{2}$$

The momentum balance equation along X and Y yields (Vafai and Tien, [31]).

$$\beta_t g(T-T_0) - \frac{1}{\rho_0}\frac{\partial P}{\partial X} + \nu\frac{d^2U}{dY^2} - \frac{\nu U}{k} = 0 \tag{3}$$

$$\frac{\partial P}{\partial Y} = 0 \tag{4}$$

where $P = p + \rho_0 gX$. Since, on account of equation (4) P depends only on X , and hence equation (3) can be rewritten as

$$T - T_0 = \frac{1}{\beta_t g \rho_0}\frac{dP}{dX} - \frac{\nu}{\beta_t g}\frac{d^2U}{dY^2} + \frac{\nu U}{k\beta_t g} \tag{5}$$

From equation (5), one obtains

$$\frac{\partial T}{\partial X}=\frac{1}{\beta_t g\rho_0}\frac{d^2P}{dX^2} \tag{6}$$

$$\frac{\partial T}{\partial Y}=-\frac{\nu}{\beta_t g}\frac{d^3U}{dY^3}+\frac{\nu}{k\beta_t g}\frac{dU}{dY} \tag{7}$$

$$\frac{\partial^2 T}{\partial Y^2}=-\frac{\nu}{\beta_t g}\frac{d^4U}{dY^4}+\frac{\nu}{k\beta_t g}\frac{d^2U}{dY^2} \tag{8}$$

Both the walls of the channel will be assumed to have a negligible thickness and to exchange heat by convection with an external fluid. In particular, at $Y=-L/2$ the external convection coefficient will be considered as uniform with the value h_1 and the fluid in the region $Y<-L/2$ will be assumed to have a uniform reference temperature T_1. At $Y=L/2$ the external convection coefficient will be considered as uniform with the value h_2 and the fluid in the region $Y>L/2$ will be supposed to have a uniform reference temperature $T_2\geq T_1$. Therefore, the boundary conditions on the temperature field can be expressed as

$$-K\left.\frac{\partial T}{\partial Y}\right|_{Y=-L/2}=h_1[T_1-T(X,-L/2)] \tag{9}$$

$$-K\left.\frac{\partial T}{\partial Y}\right|_{Y=L/2}=h_2[T(X,L/2)-T_2] \tag{10}$$

On account of equation (7), equations (9) and (10) can be rewritten as

$$\left.\frac{d^3U}{dY^3}-\frac{1}{k}\frac{dU}{dY}\right|_{Y=-L/2}=\frac{\beta_t g h_1}{K\nu}[T_1-T(X,-L/2)] \tag{11}$$

$$\left.\frac{d^3U}{dY^3}-\frac{1}{k}\frac{dU}{dY}\right|_{Y=L/2}=\frac{\beta_t g h_2}{K\nu}[T(X,L/2)-T_2] \tag{12}$$

It is easily verified that equations (11) and (12) imply that $\partial T/\partial X$ is zero both at $Y=-L/2$ and at $Y=L/2$. Since equation (6) ensures that $\partial T/\partial X$ does not depend on Y, one is led to the conclusion that $\partial T/\partial X$ is zero everywhere. Therefore, the temperature T depends only on Y, i.e. $T=T(Y)$. Thus, on account of equation (6), there exists a constant A such as

$$\frac{dP}{dX}=A \tag{13}$$

For the problem under investigation, the energy balance equation in the presence of viscous dissipation and heat source/sink can be written as (Aziz and Na, [32])

$$\frac{\partial^2 T}{\partial Y^2}=-\frac{\nu}{\alpha C_p}\left(\frac{dU}{dY}\right)^2\mp\frac{Q}{K}\left(\frac{1}{\beta_t g\rho_0}\frac{dP}{dX}-\frac{\nu}{\beta_t g}\frac{d^2U}{dY^2}+\frac{\nu}{k\beta_t g}U\right)-\frac{\nu}{K\alpha C_p}U^2 \tag{14}$$

Equations (8) and (14) yield a differential equation for U, namely

$$\frac{d^4U}{dY^4}=\frac{g\beta_t}{\alpha C_p}\left(\frac{d\ U}{dY}\right)^2+\frac{1}{k}\frac{d^2U}{dY^2}+\frac{g\beta_t\mu}{\nu kK}U^2\pm\frac{Qg\beta_t}{\nu K}\left(\frac{1}{\beta_t g\rho_0}\frac{d\ P}{dX}-\frac{\nu}{\beta_t g}\frac{d^2U}{dY^2}+\frac{\nu}{k\beta_t g}U\right) \tag{15}$$

The boundary conditions on U are

$$U(-L/2)=U(L/2)=0 \tag{16}$$

Together with equations (11) and (12), which on account of equation (5), can be rewritten as

$$\left.\frac{d^3U}{dY^3}-\frac{1}{k}\frac{dU}{dY}-\frac{h_1}{K}\frac{d^2U}{dY^2}-\frac{h_1U}{Kk}\right|_{Y=-L/2}=-\frac{Ah_1}{K\mu}-\frac{\beta_t gh_1}{K\nu}(T_0-T_1) \tag{17}$$

$$\left.\frac{d^3U}{dY^3}-\frac{1}{k}\frac{dU}{dY}+\frac{h_2}{K}\frac{d^2U}{dY^2}+\frac{h_2U}{Kk}\right|_{Y=L/2}=\frac{Ah_2}{K\mu}-\frac{\beta_t gh_2}{K\nu}(T_2-T_0) \tag{18}$$

Equations (15)-(18) determine the velocity distribution. They can be written in a dimensionless form by means of the following dimensionless parameters:

$$u=\frac{U}{U_0},\ \theta=\frac{T-T_0}{\Delta T},\ y=\frac{Y}{D},\ Gr=\frac{g\beta_t\Delta TD^3}{\nu^2},\ \mathrm{Re}=\frac{U_0D}{\nu},\ Br=\frac{\mu U_0^2}{K\Delta T},\ \mathrm{Pr}=\frac{\nu}{\alpha},\ \lambda=\frac{Gr}{\mathrm{Re}},\ R_T=\frac{T_2-T_1}{\Delta T},\ Bi_1=\frac{h_1D}{K},$$
$$Bi_2=\frac{h_2D}{K},\ S=\frac{Bi_1Bi_2}{Bi_1Bi_2+2Bi_1+2Bi_2},\ \phi=\frac{QD^2}{K},\ \sigma=\frac{D}{\sqrt{k}}. \tag{19}$$

In equation (19), $D=2L$ is the hydraulic diameter, while the reference velocity U_0 and the reference temperature T_0 are given by

$$U_0=-\frac{AD^2}{48\mu};\ T_0=\frac{T_1+T_2}{2}+S\left(\frac{1}{Bi_1}-\frac{1}{Bi_2}\right)(T_2-T_1) \tag{20}$$

The reference temperature ΔT is given either by

$$\Delta T=T_2-T_1 \quad \text{if} \quad T_1<T_2 \tag{21}$$

or by $\Delta T=\dfrac{\nu^2}{C_pD^2}$ if $T_1=T_2$ (22)

Therefore, as in Barletta [33], the value of the dimensionless parameter R_T can be either 0 or 1. More precisely, R_T equals 1 for asymmetric fluid temperatures, $T_1<T_2$, and equals 0 for symmetric fluid temperatures, $T_1=T_2$. The dimensionless mean velocity $\bar{u}$ and the dimensionless bulk temperature θ_b are given

$$\bar{u}=2\int_{-1/4}^{1/4}u\,dy \tag{23}$$

$$\theta_b=\frac{2}{\bar{u}}\int_{-1/4}^{1/4}u\theta\,dy \tag{24}$$

On account of equation (13), for upward flow $A<0$, so that U_0, Re and λ are positive and for downward flow $A>0$, while U_0, Re and λ are negative. By employing the dimensionless quantities defined in equation (19), equations (15)-(18) can be rewritten as

$$\frac{d^4u}{dy^4}=\lambda Br\left(\left(\frac{du}{dy}\right)^2+\sigma^2u^2\right)+\left(\sigma^2\mp\phi\right)\frac{d^2u}{dy^2}\mp\phi\left(48-\sigma^2u\right) \tag{25}$$

$$u(-1/4)=u(1/4)=0 \tag{26}$$

$$\left.\frac{d^2u}{dy^2}+\frac{\sigma^2}{Bi_1}\frac{du}{dy}-\frac{1}{Bi_1}\frac{d^3u}{dy^3}+\sigma^2u\right|_{y=-1/4}=-48+\frac{R_T\lambda S}{2}\left(1+\frac{4}{Bi_1}\right) \tag{27}$$

$$\frac{d^2u}{dy^2}-\frac{\sigma^2}{Bi_2}\frac{du}{dy}+\frac{1}{Bi_2}\frac{d^3u}{dy^3}-\sigma^2 u\Big|_{y=1/4}=-48-\frac{R_T\lambda S}{2}\left(1+\frac{4}{Bi_2}\right) \tag{28}$$

Similarly, equations (14) and (19) yield

$$\frac{d^2\theta}{dy^2}=-Br\left(\left(\frac{du}{dy}\right)^2+\sigma^2u^2\right)\mp\frac{\phi}{\lambda}\left(48+\frac{d^2u}{dy^2}-\sigma^2u\right) \tag{29}$$

while from equations (5) and (19) one obtains

$$\theta=-\frac{1}{\lambda}\left(48+\frac{d^2u}{dy^2}-\sigma^2u\right) \tag{30}$$

where the plus sign relate to heat absorption and minus sign relates to heat generation.

A Nusselt number can be defined at each boundary, as follows:

$$Nu_1=\frac{1}{R_T[T(L/2)-T(-L/2)]+(1-R_T)\Delta T}\frac{dT}{dY}\Big|_{Y=-L/2}$$

$$Nu_2=\frac{1}{R_T[T(L/2)-T(-L/2)]+(1-R_T)\Delta T}\frac{dT}{dY}\Big|_{Y=L/2} \tag{31}$$

The Nusselt numbers Nu_1 and Nu_2 can be employed to evaluate the heat fluxes at the walls. In fact, the heat flux per unit area is given by $q_1=-K(dT/dY)\big|_{Y=-L/2}$ at the left wall, and by $q_2=-K(dT/dY)\big|_{Y=L/2}$ at the right wall. Let us first assume $R_T=1$. Then, from equation (31) one obtains

$$q_1=-\frac{KNu_1}{D}[T(L/2)-T(-L/2)]\,;\,q_2=-\frac{KNu_2}{D}[T(L/2)-T(-L/2)] \tag{32}$$

The heat flux densities q_1 and q_2 can also be expressed as

$$q_1=-h_1[T(-L/2)-T_1]\,;\quad q_2=-h_2[T_2-T(L/2)] \tag{33}$$

Equations (32) and (33) yield

$$[T(L/2)-T(-L/2)]=\left(T_2-T_1\right)\Big/\left(1+\frac{K}{D}\left(\frac{Nu_1}{h_1}+\frac{Nu_2}{h_2}\right)\right) \tag{34}$$

Let us now assume $R_T=0$. Equations (31) yields

$$q_1=-\frac{KNu_1}{D}\Delta T\ \text{ and }\ q_2=-\frac{KNu_2}{D}\Delta T \tag{35}$$

where ΔT is the reference temperature difference defined by equation (22). By employing equation (19), equation (31) can be written as

$$Nu_1=\frac{1}{R_T[\theta(1/4)-\theta(-1/4)+(1-R_T)]}\frac{d\theta}{dy}\Big|_{y=-1/4}\,;\,Nu_2=\frac{1}{R_T[\theta(1/4)-\theta(-1/4)+(1-R_T)]}\frac{d\theta}{dy}\Big|_{y=1/4} \tag{36}$$

3. SOLUTIONS

i). Separated effects of buoyancy forces and viscous dissipation

In this section, the simpler cases of either negligible viscous dissipation or negligible buoyancy forces will be solved analytically. The case of negligible viscous dissipation can be obtained by setting $Br=0$ in the dimensionless energy

Equation (36). As a consequence; the dimensionless temperature field is independent of the dimensionless velocity field u. Moreover, equations (31)-(34) can be easily solved and yield

$$u = C_1 Sinh(\sigma y) + C_2 Cosh(\sigma y) + C_3 Sinh(\sqrt{\phi} y) + C_4 Cosh(\sqrt{\phi} y) + 48/\sigma^2 \quad (37)$$

for the case of heat absorption and

$$u = C_1 Cosh(\sigma y) + C_2 Sinh(\sigma y) + C_3 Cos(\sqrt{\phi} y) + C_4 Sin(\sqrt{\phi} y) + 48/\sigma^2 \quad (38)$$

for the case of heat generation respectively.

With $Bi_1 = Bi_2 = Bi$ equations (37) and (38) can be rewritten as

$$u = C_1 Sinh(\sigma y) + C_2 Cosh(\sigma y) + C_3 Sinh(\sqrt{\phi} y) + 48/\sigma^2 \quad (39)$$

for the case of heat absorption and

$$u = C_1 Cosh(\sigma y) + C_2 Sinh(\sigma y) + C_4 Sin(\sqrt{\phi} y) + 48/\sigma^2 \quad (40)$$

for the case of heat generation, respectively.

In the limit $Bi \to +\infty$, one obtains the special case in which the boundaries of the channel are kept at the temperatures T_1 and T_2, respectively. In this limit, equation (19) yields $S \to 1$, so that equations (37) and (38) reduces to

$$u = C_1 Sinh(\sigma y) + C_2 Cosh(\sigma y) + C_3 Sinh(\sqrt{\phi y}) + 48/\sigma^2 \quad (41)$$

for the case of heat absorption and

$$u = C_1 Cosh(\sigma y) + C_2 Sinh(\sigma y) + C_4 Sin(\sqrt{\phi} y) + 48/\sigma^2 \quad (42)$$

for the case of heat generation, respectively.

By substituting equations (37) and (38) in equations (23) and (24), one obtains

$$\bar{u} = 4C_4 \left(\left(1/\sqrt{\phi}\right) Sinh\left(\sqrt{\phi}/4\right) - (1/\sigma) Cosh\left(\sqrt{\phi}/4\right) Tanh(\sigma/4) \right) + \left(48/\sigma^2\right)\left(1 - \left(4/\sqrt{\phi}\right) Tanh(\sigma/4)\right)$$

$$\theta_b = \frac{-2(\phi - \sigma^2)\{b_1 + b_2 + b_3 + b_4 + b_5\}}{\lambda b_6} \quad (43)$$

for the case of heat absorption and

$$\bar{u} = \left((4C_1/\sigma) Sinh(\sigma/4) + \left(4C_3/\sqrt{\phi}\right) Sin\left(\sqrt{\phi}/4\right) + \left(4\ /\sigma^2 8\right) \right)$$

$$\theta_b = \frac{2(\phi - \sigma^2)\left(\left(\sigma / \left(\sqrt{\phi}\left(1 + \sigma^2/\phi\right)\right)\right)\right)(b_1 + b_2 + b_3) + b_4)}{\lambda b_5} \quad (44)$$

for the case of heat generation, respectively.

Moreover, equations (30), (37) and (38) yield

$$\theta = -\frac{(\phi - \sigma^2)}{\lambda}\left(C_3 Sinh(\sqrt{\phi} y) + C_4 Cosh(\sqrt{\phi} y)\right) \quad (45)$$

for the case of heat absorption and

$$\theta = \frac{(\phi + \sigma^2)}{\lambda}\left(C_3 Cos(\sqrt{\phi} y) + C_4 Sin(\sqrt{\phi} y)\right) \quad (46)$$

for the case of heat generation, respectively.

In the absence of heat absorption/generation and porous parameter, equations (30), (36) and (37) reduces to

$$\theta = 2SR_T\, y; \qquad Nu_1 = Nu_2 = 2R_T \tag{47}$$

which are the same solutions obtained by Zanchini [1].

Plots of u versus y evaluated through equation (37), are reported in figure 1 for $\lambda = 0, \lambda = 1000$, heat absorption co-efficient ϕ and $Bi_1 = Bi_2 = 10$ for different values of porous parameter. Let us now consider the case of negligible buoyancy forces with a relevant viscous dissipation, which corresponds to $\lambda = Gr/\mathrm{Re} = 0$. Since a purely forced convection occurs in this case, the solution for velocity becomes

$$u = \frac{48}{\sigma^2}\left(1 - \frac{Cosh(\sigma y)}{Cosh(\sigma/4)}\right) \tag{48}$$

Indeed, both for symmetric and for asymmetric fluid temperatures, equation (48) is the solution of equations (25)-(30) when $\lambda = 0$. Equations (9), (10) and (19) yield the boundary conditions on θ, i.e,

$$\left.\frac{d\theta}{dy}\right|_{y=-1/4} = Bi_1\left(\frac{SR_T}{2}\left(1 + \frac{4}{Bi_1}\right) + \theta(-1/4)\right)$$

$$\left.\frac{d\theta}{dy}\right|_{y=1/4} = Bi_2\left(\frac{SR_T}{2}\left(1 + \frac{4}{Bi_2}\right) - \theta(1/4)\right) \tag{49}$$

On the account of equations (29), (48) and (49), the temperature can be expressed as

$$\theta = C_5 Sinh(\sqrt{\phi} y) + C_6 Cosh(\sqrt{\phi} y) + f_1 Cosh(2\sigma y) + f_2 Cosh(\sigma y) + f_3 \tag{50}$$

for the case of heat absorption and

$$\theta = C_5 Sin(\sqrt{\phi} y) + C_6 Cos(\sqrt{\phi} y) + f_1 Cosh(2\sigma y) + f_2 Cosh(\sigma y) + f_3 \tag{51}$$

for the case of heat generation, respectively.

Equations (31), (50) and (51) yield respectively as

$$Nu_1 = \frac{\sqrt{\phi} C_5 Cosh\left(\sqrt{\phi}/4\right) - \sqrt{\phi} C_6 Sinh\left(\sqrt{\phi}/4\right) - 2\sigma f_1 Sinh(\sigma/2) - \sigma f_2 Sinh(\sigma/4)}{R_T\, 2C_5 Sinh\left(\sqrt{\phi}/4\right) + \lambda(1 - R_T)}$$

$$Nu_2 = \frac{\sqrt{\phi} C_5 Cosh\left(\sqrt{\phi}/4\right) + \sqrt{\phi} C_6 Sinh\left(\sqrt{\phi}/4\right) + 2\sigma f_1 Sinh(\sigma/2) + \sigma f_2 Sinh(\sigma/4)}{R_T\, 2C_5 Sinh\left(\sqrt{\phi}/4\right) + \lambda(1 - R_T)} \tag{52}$$

for the case of heat absorption and

$$Nu_1 = \frac{\sqrt{\phi} C_5 Cos\left(\sqrt{\phi}/4\right) + \sqrt{\phi} C_6 Sin\left(\sqrt{\phi}/4\right) - 2\sigma f_1 Sinh(\sigma/2) - \sigma f_2 Sinh(\sigma/4)}{R_T\, 2C_5 Sin\left(\sqrt{\phi}/4\right) + \lambda(1 - R_T)}$$

$$Nu_2 = \frac{\sqrt{\phi} C_5 Cos\left(\sqrt{\phi}/4\right) - \sqrt{\phi} C_6 Sin\left(\sqrt{\phi}/4\right) + 2\sigma f_1 Sinh(\sigma/2) + \sigma f_2 Sinh(\sigma/4)}{R_T\, 2C_5 Sin\left(\sqrt{\phi}/4\right) + \lambda(1 - R_T)} \tag{53}$$

for the case of heat generation, respectively.

Plots of θ versus y for $R_T = 1 (T_1 < T_2)$, evaluated through equation (50), and are reported in figures 2 and 3, for some values of Br. Figure 2 refers to $Bi_1 = Bi_2 = 10$, while figure 3 refers to $Bi_1 = 1, Bi_2 = 10$ for different porous parameters σ.

ii). Combined effects of buoyancy forces and viscous dissipation

In this section, both buoyancy forces and viscous dissipation are considered as non-negligible. First, equations (31)-(36) are solved by a perturbation series method. Then, the dimensionless temperature field is determined by means of equation (36). Let us consider the dimensionless parameter

$$\varepsilon = \lambda Br = \mathrm{Re}\,\mathrm{Pr}\,\frac{\beta g D}{C_p} \tag{54}$$

which is independent of the reference temperature difference ΔT . The solutions of equation (31)-(36) can be expressed by the perturbation expansion

$$u\left(y\right) = u_0(y) + \varepsilon u_1(y) + \varepsilon^2 u_2(y) + . \tag{55}$$

To obtain the solutions of equations (25)-(28) with the form (55), one first substitutes equation (55) in equations (25)-(28) and collects terms having like power of ε . Then, one equates the coefficient of ε to zero (Aziz and Na, [32]). Thus, one obtains a sequence of boundary value problems which can be solved in succession and can yield the unknown functions $u_n(y)$.

The boundary value problem for $n = 0$ and $n = 1$ are

$$\frac{d^4 u_0}{dy^4} = \left(\sigma^2 \mp \phi\right)\frac{d^2 u_0}{dy^2} \mp \phi\left(48 - \sigma^2 u_0\right) \tag{56}$$

$$u_0(-1/4) = u_0(1/4) = 0 \tag{57}$$

$$\left.\frac{d^2 u_0}{dy^2} + \frac{\sigma^2}{Bi_1}\frac{du_0}{dy} - \frac{1}{Bi_1}\frac{d^3 u_0}{dy^3} + \sigma^2 u_0\right|_{y=-1/4} = -48 + \frac{R_T \lambda S}{2}\left(1 + \frac{4}{Bi_1}\right) \tag{58}$$

$$\left.\frac{d^2 u_0}{dy^2} - \frac{\sigma^2}{Bi_2}\frac{du_0}{dy} + \frac{1}{Bi_2}\frac{d^3 u_0}{dy^3} - \sigma^2 u_0\right|_{y=1/4} = -48 - \frac{R_T \lambda S}{2}\left(1 + \frac{4}{Bi_2}\right) \tag{59}$$

$$\frac{d^4 u_1}{dy^4} = \left(\frac{du_0}{dy}\right)^2 + \sigma^2 u_0^2 + \left(\sigma^2 \mp \phi\right)\frac{d^2 u_1}{dy^2} \pm \phi\sigma^2 u_1 \tag{60}$$

$$u_1(-1/4) = u_1(1/4) = 0 \tag{61}$$

$$\left.\frac{d^2 u_1}{dy^2} - \frac{1}{Bi_1}\frac{d^3 u_1}{dy^3} + \frac{\sigma^2}{Bi_1}\frac{du_1}{dy} + \sigma^2 u_0\right|_{y=-1/4} = 0 \tag{62}$$

$$\left.\frac{d^2 u_1}{dy^2} + \frac{1}{Bi_2}\frac{d^3 u_1}{dy^3} - \frac{\sigma^2}{Bi_2}\frac{du_1}{dy} - \sigma^2 u_1\right|_{y=1/4} = 0 \tag{63}$$

where plus sign relates to heat absorption and minus sign relates to heat generation.

The solutions of equations (56)-(63) are given by

$$u_0 = C_1 Sinh\left(\sigma y\right) + C_2 Cosh\left(\sigma y\right) + C_3 Sinh\left(\sqrt{\phi} y\right) + C_4 Cosh\left(\sqrt{\phi} y\right) + 48/\sigma^2 \tag{64}$$

$$\begin{aligned} u_1 = {} & C_5 Sinh\left(\sigma y\right) + C_6 Cosh\left(\sigma y\right) + C_7 Sinh\left(\sqrt{\phi} y\right) + C_8 Cosh\left(\sqrt{\phi} y\right) + l_5 Cosh\left(2\sigma y\right) + l_6 Cosh\left(2\sqrt{\phi} y\right) + l_7 Sinh\left(2\sigma y\right) + \\ & l_8 Sinh\left(2\sqrt{\phi} y\right) + l_9 Cosh\left(\left(\sigma + \sqrt{\phi}\right) y\right) + l_{10} Cosh\left(\left(\sigma - \sqrt{\phi}\right) y\right) + l_{11} Sinh\left(\left(\sigma + \sqrt{\phi}\right) y\right) + l_{12} Sinh\left(\left(\sigma - \sqrt{\phi}\right) y\right) + \\ & l_{13} y Cosh\left(\sigma y\right) + l_{14} y Sinh\left(\sigma y\right) + l_{15} y Cosh\left(\sqrt{\phi} y\right) + l_{16} y Sinh\left(\sqrt{\phi} y\right) + l_{17} \end{aligned} \tag{65}$$

for the case of heat absorption and

$$u_0 = C_1 Cosh(\sigma y) + C_2 Sinh(\sigma y) + C_3 Cos(\sqrt{\phi} y) + C_4 Sin(\sqrt{\phi} y) + \frac{48}{\sigma^2} \tag{66}$$

$$\begin{aligned} u_1 = {} & C_7 Cosh(\sigma y) + C_8 Sinh(\sigma y) + C_9 Cos(\sqrt{\phi} y) + C_{10} Sin(\sqrt{\phi} y) + l_1 Cos(\sqrt{\phi} y) + l_2 Cos(2\sqrt{\phi} y) \\ & + l_3 Sinh(2\sigma y) + l_4 Sin(2\sqrt{\phi} y) + l_5 Cosh(\sigma y) + l_6 Sinh(\sigma y) Sin(\sqrt{\phi} y) + l_7 Cosh(\sigma y) Sin(\sqrt{\phi} y) \\ & + l_8 Sinh(\sigma y) Cos(\sqrt{\phi} y) + l_9 y Sinh(\sigma y) + l_{10} y Cosh(\sigma y) + l_{11} y Sin(\sqrt{\phi} y) + l_{12} y Cos(\sqrt{\phi} y) + l_{13} \end{aligned} \tag{67}$$

for the case of heat generation, respectively.

The right-hand side of equations (64) and (66) coincides with that of equations (37) and (38) respectively and gives the dimensionless velocity for the case of $Br = 0$.

The dimensionless temperature θ can be written in the form

$$\theta = -\frac{(\phi - \sigma^2)}{\lambda}\left((C_3 + \varepsilon C_7) Sinh(\sqrt{\phi} y) + (C_4 + \varepsilon C_8) Cosh(\sqrt{\phi} y)\right) - \frac{(l_{27} - \sigma^2 l_{26})}{\lambda(\phi - \sigma^2)} \tag{68}$$

for the case of heat absorption and

$$\theta = \frac{1}{\lambda}\left((\phi + \sigma^2)\left((C_3 + \varepsilon C_9) Cos(\sqrt{\phi} y) + (C_4 + \varepsilon C_{10}) Sin(\sqrt{\phi} y)\right) + l_{14}\right) \tag{69}$$

for the case of heat generation, respectively.

Using the equations (68) and (69) yields the following expressions of Nu_1 and Nu_2 as

$$Nu_1 = \frac{(\phi - \sigma^2)\sqrt{\phi}\left((C_3 + \varepsilon C_7) Cosh(\sqrt{\phi}/4) - (C_4 + \varepsilon C_8)\phi\sqrt{\phi} Sinh(\sqrt{\phi}/4)\right) + \varepsilon l_{29}}{R_T\left(2(\phi - \sigma^2)(C_3 + \varepsilon C_7) Sinh(\sqrt{\phi}/4) + l_{32} - l_{31}\right) + \lambda(1 - R_T)}$$

$$Nu_2 = \frac{(\phi - \sigma^2)\sqrt{\phi}\left((C_3 + \varepsilon C_7) Cosh(\sqrt{\phi}/4) + (C_4 + \varepsilon C_8)\phi\sqrt{\phi} Sinh(\sqrt{\phi}/4)\right) + \varepsilon l_{29}}{R_T\left(2(\phi - \sigma^2) C_3 Sinh(\sqrt{\phi}/4) + \varepsilon\left(2(\phi - \sigma^2) C_7 Sinh(\sqrt{\phi}/4)\right) + l_{32} - l_{31}\right) + \lambda(1 - R_T)} \tag{70}$$

for the case of heat absorption and

$$Nu_1 = \frac{-(\phi + \sigma^2)\sqrt{\phi}\left((C_3 + \varepsilon C_9) Sin(\sqrt{\phi}/4) + (C_4 + \varepsilon C_{10}) Cos(\sqrt{\phi}/4)\right) + \varepsilon(AN1)}{R_T\left(-2(\phi + \sigma^2)(C_4 + \varepsilon C_{10}) Sin(\sqrt{\phi}/4) + AD1\right) + \lambda(1 - R_T)}$$

$$Nu_2 = \frac{-(\phi + \sigma^2)\sqrt{\phi}\left((C_3 + \varepsilon C_9) Sin(\sqrt{\phi}/4) - (C_4 + \varepsilon C_{10}) Cos(\sqrt{\phi}/4)\right) + \varepsilon(AN1)}{R_T\left(-2(\phi + \sigma^2)(C_4 + \varepsilon C_{10}) Sin(\sqrt{\phi}/4) + AD1\right) + \lambda(1 - R_T)} \tag{71}$$

for the case of heat generation, respectively.

Equations (23), (55) and (64), (65), (68) yield the expression of the mean dimensionless velocity for the case of heat absorption as

$$\begin{aligned} \bar{u} = {} & (4/\sqrt{\phi})\left(C_4 + \varepsilon(C_6 + C_8)\right) Sinh(\sqrt{\phi}/4) - (4C_4/\sigma) Cosh(\sqrt{\phi}/4) Tanh(\sigma/4) \\ & + (48/\sigma^2)\left(1 - (4/\sigma) Tanh(\sigma/4)\right) + \varepsilon ub1 \end{aligned} \tag{72}$$

Similarly equations (23), (55) and (66), (67), (69) yield the expression of the mean dimensionless velocity for the case of heat generation as

$$\bar{u} = \left(4(C_1 + \varepsilon C_7)/\sigma\right) Sinh\,(\sigma/4) + (48/\sigma^2) + \left(4(C_3 + \varepsilon C_9)\Big/\sqrt{\phi}\right) Sin\,(\sqrt{\phi}/4) + \varepsilon ub1 \tag{73}$$

4. NUMERICAL SOLUTION

The analytical solutions obtained in the preceding section include only two terms of the series, which is not applicable for large values of λ i.e., for large values of buoyancy force. In many practical problems the values of λ are usually large. Therefore a numerical scheme is used to solve non-linear boundary value problem using Runge-Kutta shooting method. The analytical solutions and numerical solutions are found and it is observed that this agreement is good enough to justify the validity of the numerical scheme for small values of the perturbation parameter.

5. RESULTS AND DISCUSSION

The problem of mixed convection flow and heat transfer in a vertical channel filled with porous medium with boundary conditions of third kind is investigated. The analytical solutions are found using regular perturbation method with product of mixed convection parameter $\lambda(Gr/\mathrm{Re})$ and Brinkman number (Br) as the perturbation parameter. The analytical solutions are valid only for small values of the perturbation parameter. The restriction on the perturbation parameter ε to be small is relaxed by finding the solutions of governing equations numerically using Runge-Kutta shooting method. The flow is also analyzed depending on the thermal characteristics of the parameters such as heat source/sink. Results are depicted graphically in figures 1-8 for heat sink and in figure 9 for heat source. In the absence of viscous dissipation, for equal Biot number there is a flow reversal near the cold wall for large values of mixed convection parameter λ $(\lambda = 1000)$ as seen in figure 1. In the absence of both viscous dissipation and mixed convection parameter, there is no effect of heat sink on the velocity (figure 1). As the heat sink parameter increases velocity decreases which is also observed from figure 1.

In the absence of mixed convection parameter λ and viscous dissipation, the temperature field is linear indicating that the heat transfer is purely by conduction as can be seen in figure 2. The temperature field increases with an increase in Brinkman number for all values of heat sink parameter ϕ. The temperature decreases substantially as heat sink parameter ϕ increases in the presence of viscous dissipation as observed in figure 2 for equal Biot number. Similar results are also observed for unequal Biot number and hence not depicted graphically.

Figures 3a and 3b shows the variations of velocity and temperature fields for different values of perturbation parameter ε for downward $(\varepsilon < 0)$ and upward $(\varepsilon > 0)$ flows. For upward flow velocity and temperature are increasing function of ε. The effect of ε on velocity field is stronger while that on the temperature field is weaker. For downward flow velocity field is a decreasing function of ε where as the temperature field is an increasing function of ε. It is also seen from figure 3a that flow reversal occurs at both the left $(\varepsilon > 0)$ and $(\varepsilon < 0)$ right walls. This is because the perturbation parameter ε implies the enhancement of viscous dissipation which results in higher values of temperature which intern enhances the buoyancy force. Therefore increase in buoyancy force increases the fluid flow for $\lambda > 0$ and decreases the fluid flow for $\lambda < 0$. Figures 1-3 also prove the good agreement between numerical and analytical solutions for small perturbation parameter ε and the difference increases as ε increases.

The effect of heat absorption coefficient ϕ is to decrease the velocity field near the hot wall for upward flow ($\lambda > 0$) where as it increases the velocity field near the hot wall for downward flow ($\lambda < 0$) as seen in figure 4a. Similar effect is there on the temperature field as seen in figure 4b.

The effect of porous parameter on the flow is shown in figure 5. As the porous parameter σ increases the velocity decreases for both upward and downward flow as seen in figure 5a. The effect of porous parameter σ is to reduce the temperature field but the effect is almost invariant as seen in figure 5b.

The Nusselt number Nu_1 is an increasing function of $|\varepsilon|$, while Nusselt number Nu_2 is an decreasing function of $|\varepsilon|$ for fixed parameters ϕ and σ which are the similar results obtained by Zanchini [1] and hence not shown graphically. The Nusselt number at the cold wall is an increasing function of heat absorption coefficient ϕ for upward and downward flow for any values of porous parameter σ as seen in figure 6a. The rate of heat transfer is more for smaller values of mixed convection parameter λ at the left wall. The Nusselt number is also an increasing function of heat absorption coefficient ϕ

at the hot wall as seen in figure 6b. However the magnitude of the Nusselt number at the hot wall is less for smaller values of mixed convection parameter λ.

The effect of mixed convection parameter λ for various values of $|\varepsilon|$ is similar to the results obtained by Zanchini [1] and hence not presented. As the heat absorption coefficient ϕ increases, the average velocity decreases for upward flow where as increases for downward flow as seen in figure 7.

The effect of perturbation parameter ε on the flow field for unequal Biot number for fixed values of heat absorption coefficient ϕ and porous parameter σ is again the similar results obtained by Zanchini [1]. That is as ε increases both velocity and temperature increases but the effect of ε on temperature is more operative at the cold wall. For symmetric wall heating conditions, the heat absorption coefficient ϕ and porous parameter σ reduced the velocity and temperature fields for upward flow as seen in figures 8a and 8b respectively. Reversal effect is noticed on the flow field for downward flow (figures 8a and 8b) for equal Biot number. Similar results are obtained for the effects of ϕ, σ and ε for unequal Biot numbers.

The effect of heat generation coefficient ϕ on the flow field is qualitatively same as that in case of heat absorption however we discuss the results where there are deviations from heat absorption. The effect of heat generation coefficient ϕ is to increase the velocity and temperature field as ϕ increases for upward flow at both the walls, where as velocity and temperature decreases at both the walls for downward flow as seen in figures 9a and 9b respectively. The flow reversal is observed at the left wall and at the right wall for upward flow which is the similar result obtained for heat absorption coefficient ϕ.

6. CONCLUSIONS

The effect of mixed convection parameter, porous parameter and heat source/sink on fully developed mixed convection in a vertical channel has been studied with boundary condition of third kind. Both the cases of asymmetric ($R_T = 1$) and symmetric ($R_T = 0$) wall conditions with equal and different Biot number have been considered. The effect of heat sink is to reduce the flow field and heat source promotes the flow field. The effect of porous parameter reduces the flow field for both heat source and sink. The numerical and analytical solutions agree very well for small values of perturbation parameter. The results agree very well with that of Zanchini [1] in the absence of porous parameter and heat source/sink coefficient.

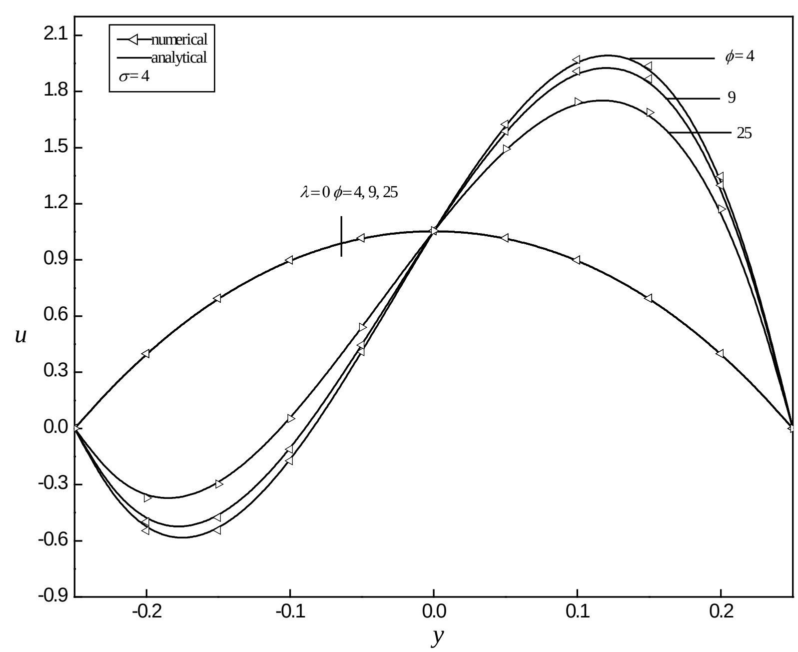

Fig. 1: Plots of u Vs. y in the case $R_T = 1$, for some value of λ, $Br = 0$ and $Bi_1 = Bi_2 = 10$.

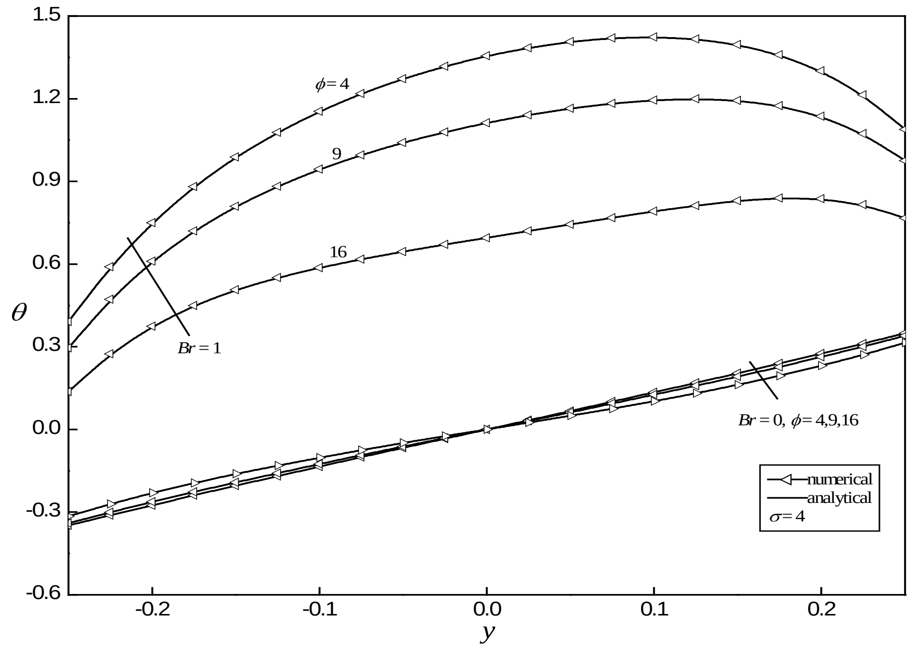

Fig. 2: Plots of θ *Vs.* y in the case $R_T = 1$, for some value of $Br, \lambda = 0$ and $Bi_1 = Bi_2 = 10$.

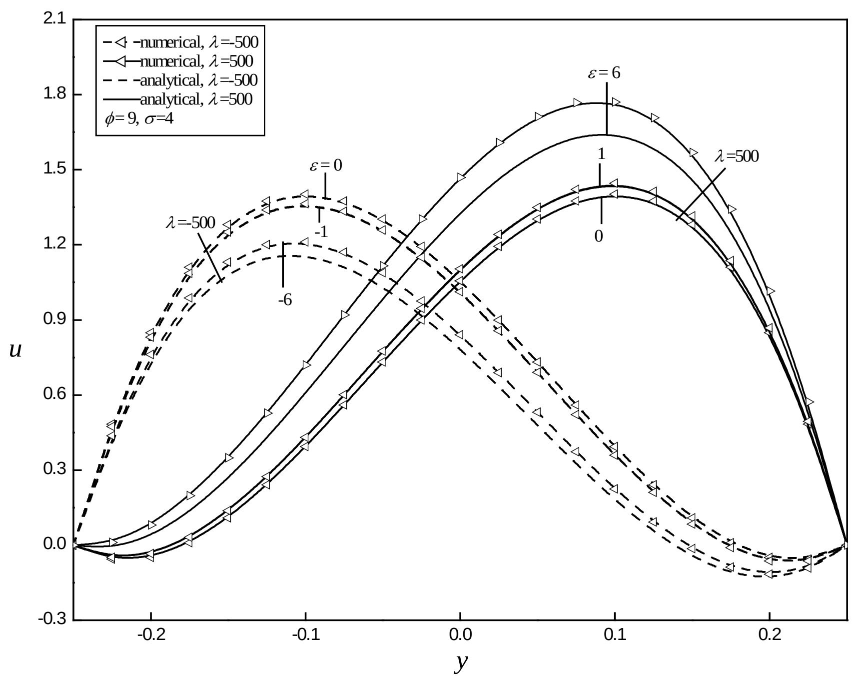

Fig. 3(a): Plots of u *vs.* y in the case $R_T = 1$, for some value of $\lambda, \varepsilon = 0$ and $Bi_1 = Bi_2 = 10$.

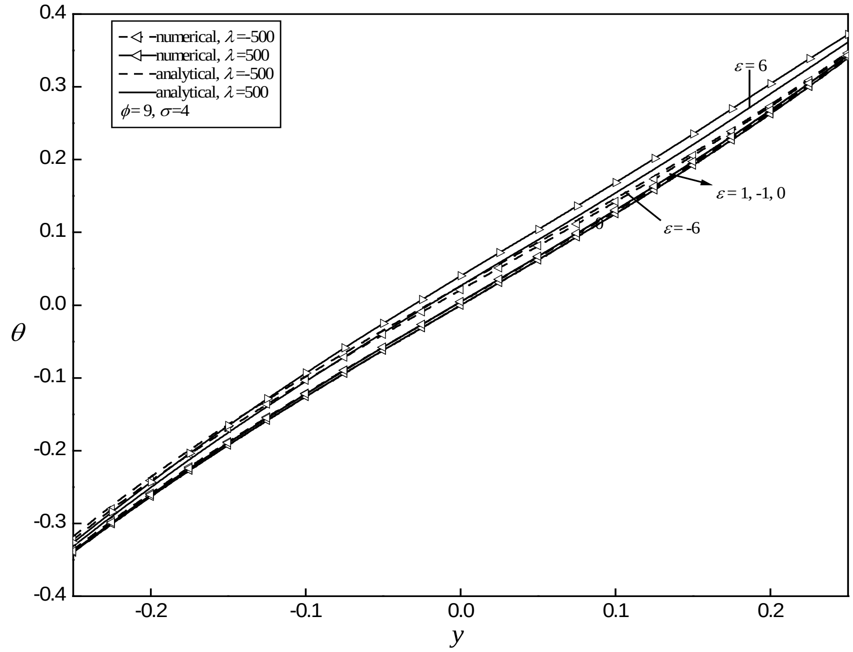

Fig. 3(b): Plots of θ *Vs.* y in the case $R_T = 1$, for some value of $\lambda, \varepsilon = 0$ and $Bi_1 = Bi_2 = 10$.

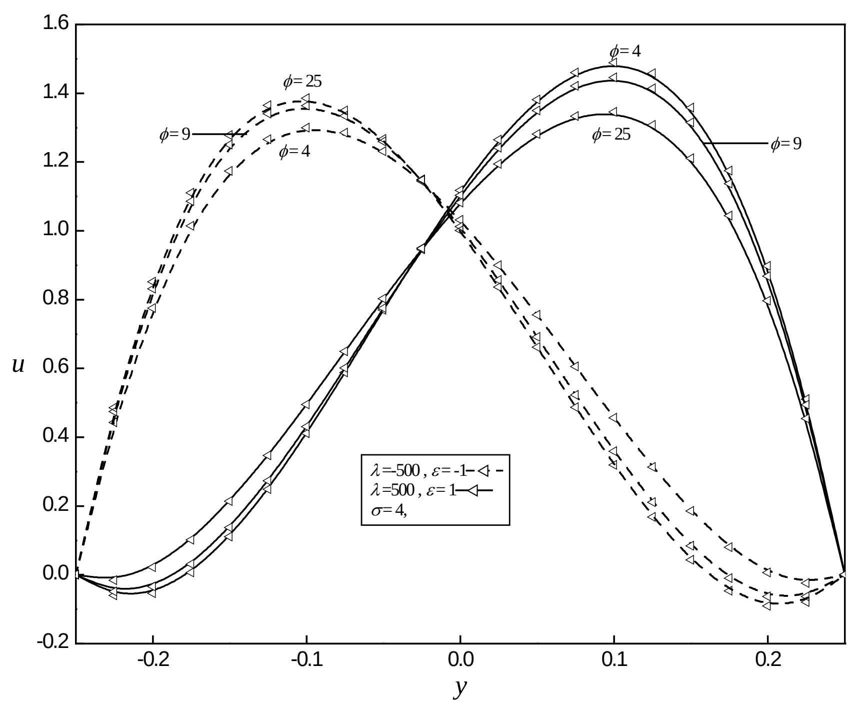

Fig. 4(a): Plots of u *Vs.* y in the case $R_T = 1$, for some value of $\lambda, \varepsilon = 0$ and $Bi_1 = Bi_2 = 10$.

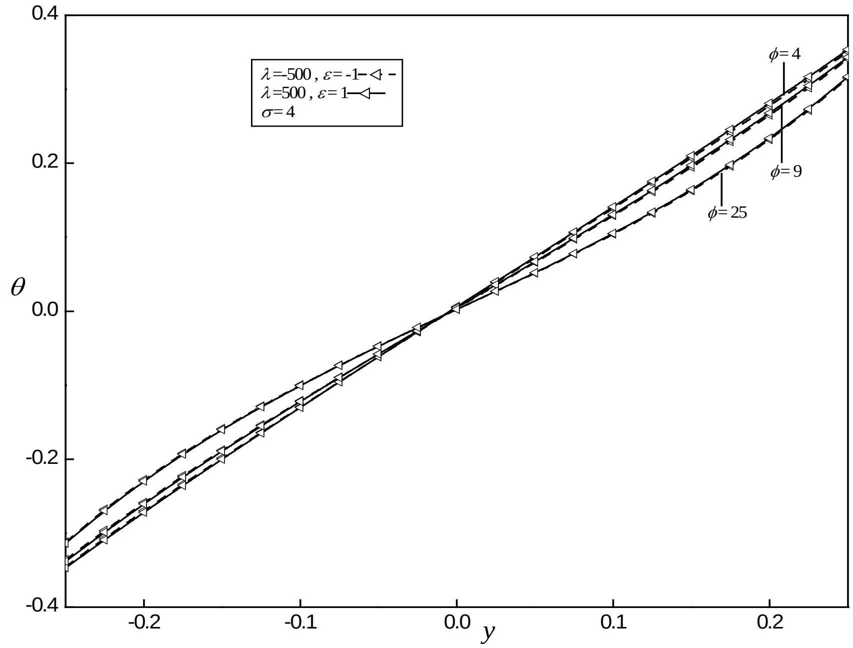

Fig. 4(b): Plots of θ *Vs.* y in the case $R_T = 1$, for some value of $\lambda, \varepsilon = 0$ and $Bi_1 = Bi_2 = 10$.

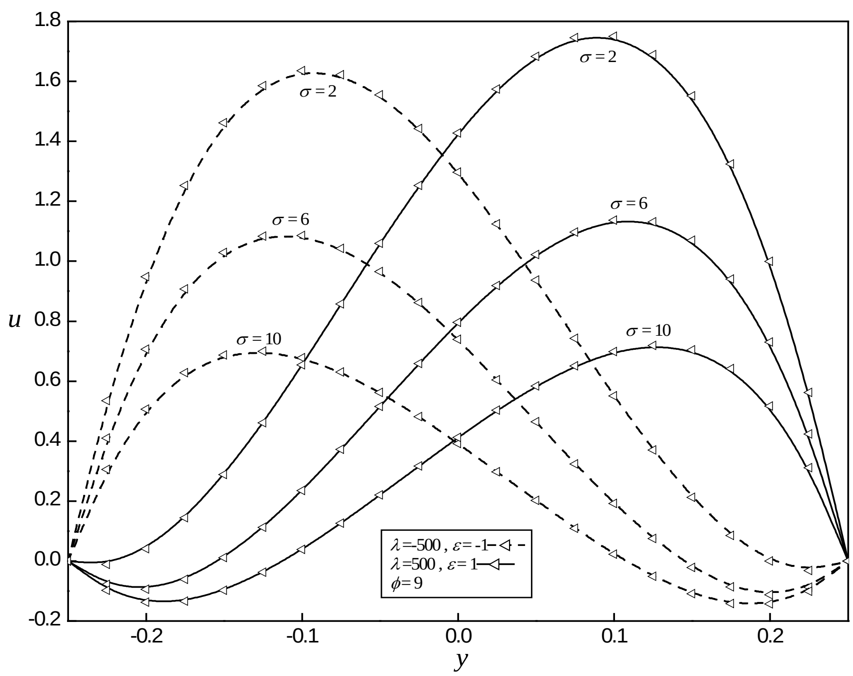

Fig. 5(a): Plots of u *Vs.* y in the case $R_T = 1$, for some value of $\lambda, \varepsilon = 0$ and $Bi_1 = Bi_2 = 10$.

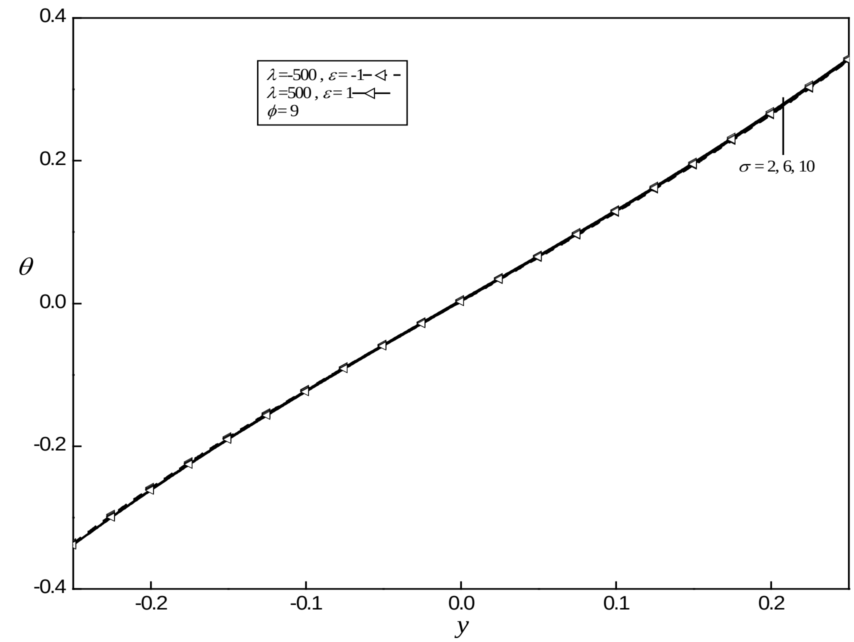

Fig. 5(b): Plots of θ *Vs.* y in the case $R_T = 1$, for some value of $\lambda, \varepsilon = 0$ and $Bi_1 = Bi_2 = 10$.

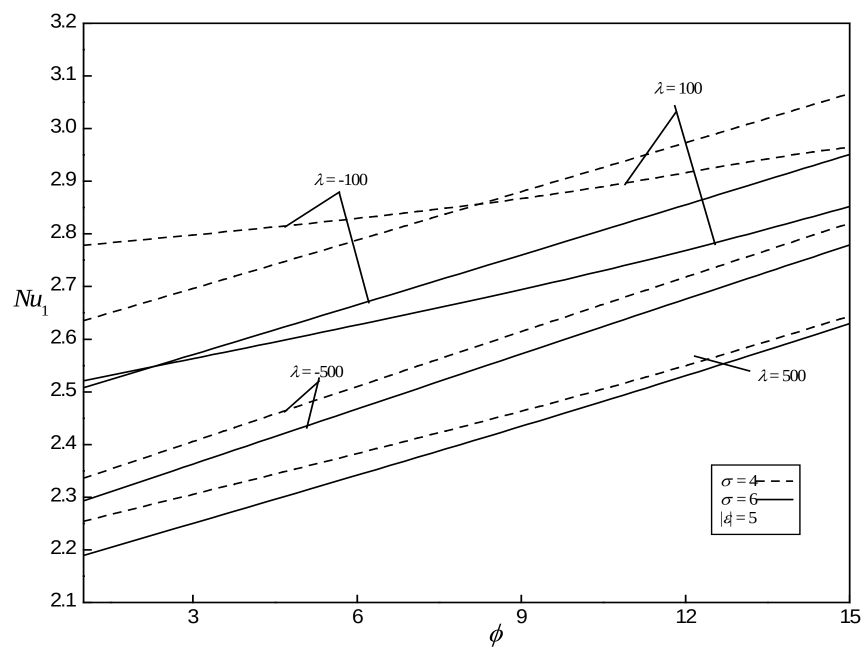

Fig. 6(a): Plots of Nu_1 *Vs.* ϕ in the case $R_T = 1$, for some value of λ and $Bi_1 = Bi_2 = 10$.

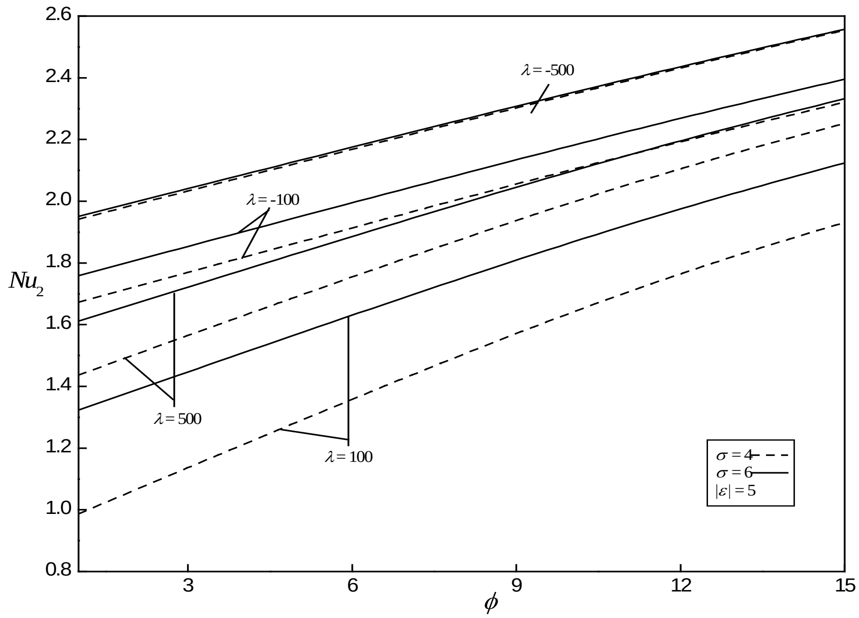

Fig. 6(b): Plots of Nu_2 *Vs.* ϕ in the case $R_T = 1$, for some value of λ and $Bi_1 = Bi_2 = 10$.

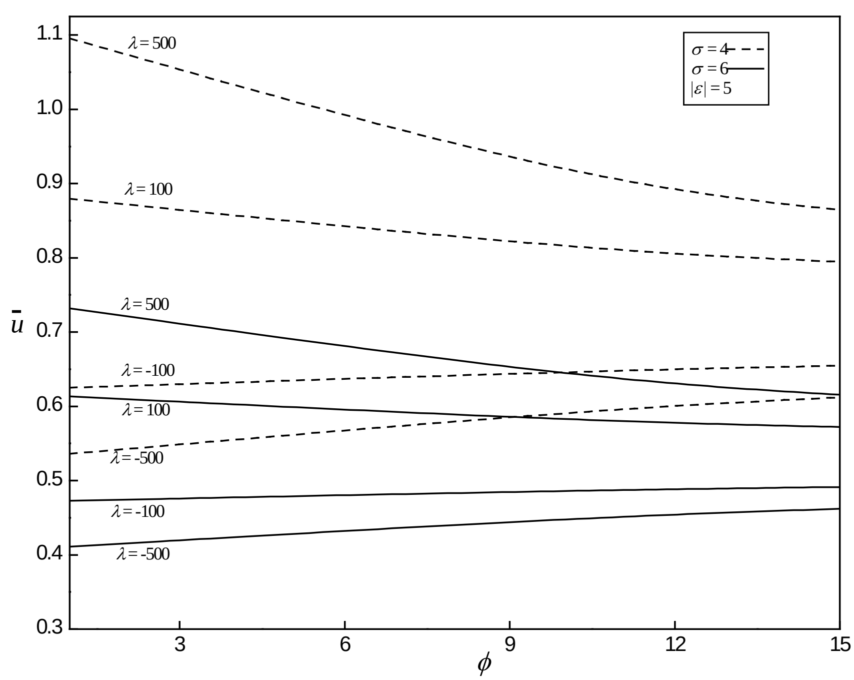

Fig. 7: Plots of $\bar{u}$ *Vs.* ϕ in the case $R_T = 1$, for some value of λ and $Bi_1 = Bi_2 = 10$.

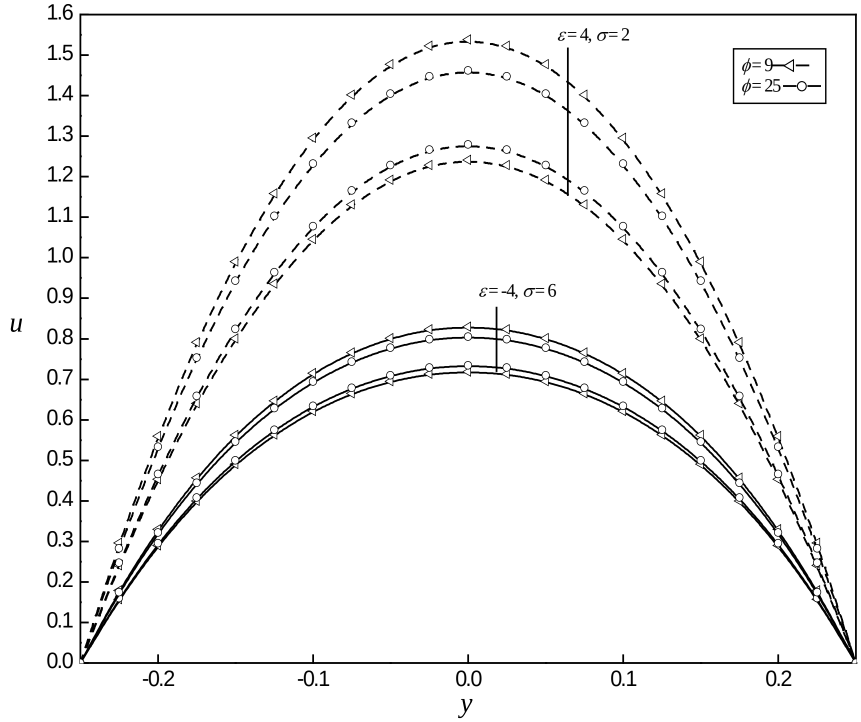

Fig. 8(a): Plots of u *Vs.* y in the case $R_T = 0$, for some value of λ and $Bi_1 = Bi_2 = 10$.

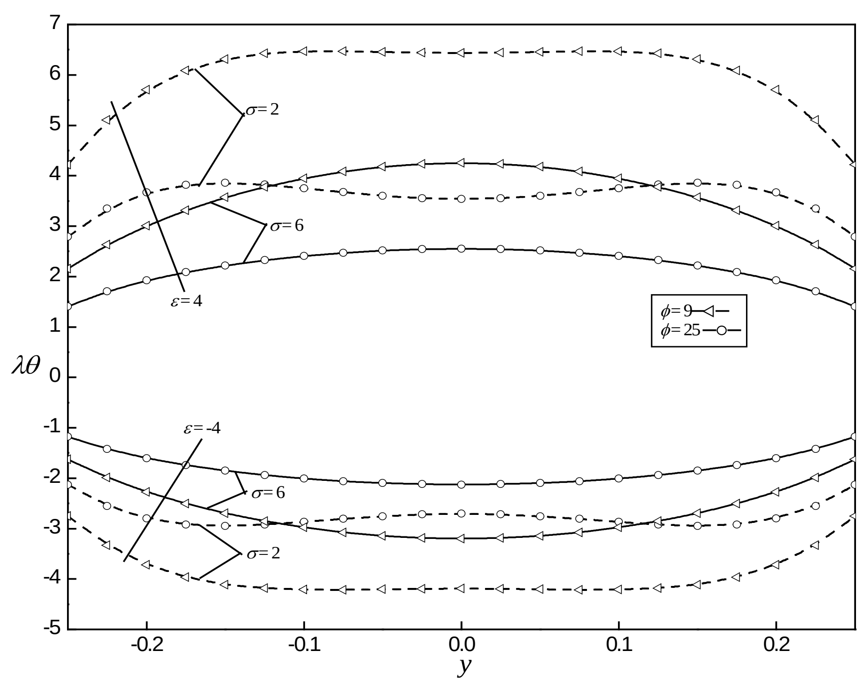

Fig. 8(b): Plots of $\lambda\theta$ *Vs.* y in the case $R_T = 0$, for some value of λ and $Bi_1 = Bi_2 = 10$.

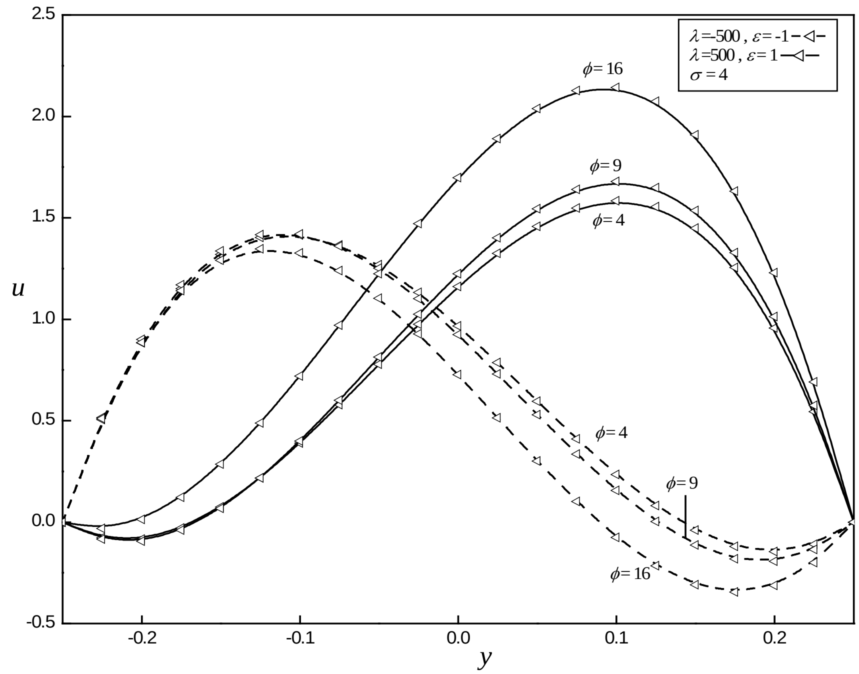

Fig. 9(a): Plots of u *vs.* y in the case of ,for some values of λ and $Bi_1 = Bi_1 = 10$.

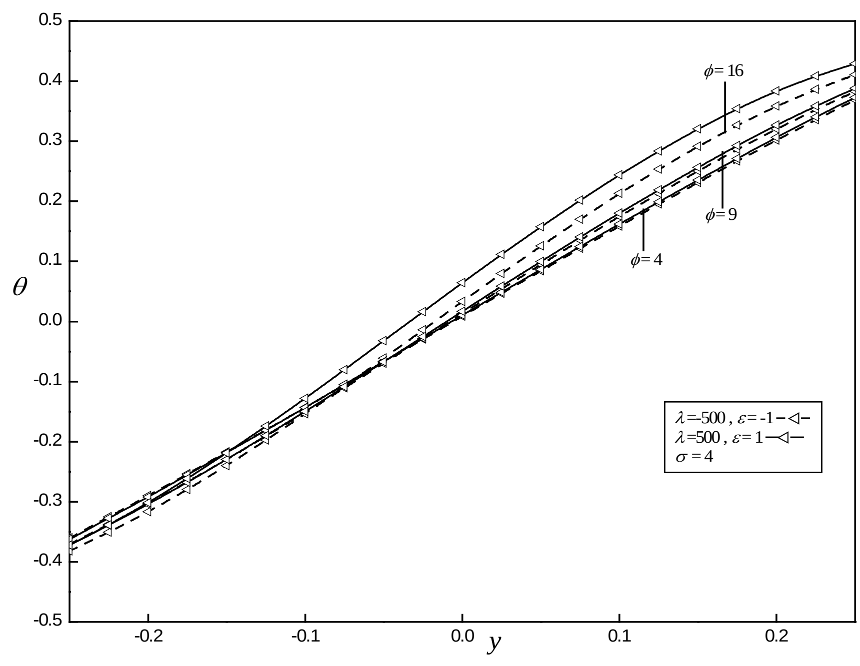

Fig. 9(b): Plots of θ *vs.* y in the case of ,for some values of λ and $Bi_1 = Bi_1 = 10$.

Table1 (a). Velocity for different values of ε and $R_T = 1$, $Bi_1 = Bi_2 = 10$, $\lambda = |500|$, $M = 4$, $\phi = 9$

y	$\varepsilon = -7$		$\varepsilon = 0$		$\varepsilon = 7$	
	Analytical	Numerical	Analytical	Numerical	Analytical	Numerical
-0.25	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
-0.2	0.720528	0.750370	-0.049284	-0.049259	0.049315	0.122398
-0.15	1.058924	1.110638	0.109423	0.109456	0.292143	0.426375
-0.1	1.119512	1.185404	0.393593	0.393623	0.643000	0.824353
-0.05	0.987947	1.061000	0.731178	0.731199	1.026612	1.239359
0.0	0.737931	0.811743	1.055837	1.055845	1.373743	1.600883
0.05	0.435744	0.504411	1.302470	1.302464	1.616993	1.840432
0.1	0.144185	0.202267	1.402846	1.402828	1.686180	1.886589
0.15	-0.073297	-0.030681	1.281102	1.281079	1.503280	1.659753
0.2	-0.147882	-0.124898	0.848901	0.848882	0.977274	1.067185
0.25	0.000000	0.000001	0.000000	0.000000	0.000000	0.000000

Table1 (b). Temperature for different values of ε and $R_T = 1$, $Bi_1 = Bi_2 = 10$, $\lambda = |500|$, $M = 4$, $\phi = 9$

y	$\varepsilon = -7$		$\varepsilon = 0$		$\varepsilon = 7$	
	Anaytical	Numerical	Anaytical	Numerical	Anaytical	Numerical
-0.25	-0.314290	-0.320101	-0.339598	-0.339597	-0.329839	-0.322000
-0.2	-0.230811	-0.238342	-0.262923	-0.262923	-0.248243	-0.236372
-0.15	-0.158931	-0.166802	-0.192175	-0.192175	-0.172401	-0.156851
-0.1	-0.092767	-0.100498	-0.125760	-0.125760	-0.101223	-0.082803
-0.05	-0.029885	-0.037335	-0.062179	-0.062179	-0.033838	-0.013409
0.0	0.030884	0.023806	0.000000	-0.000000	0.030884	0.052626
0.05	0.090520	0.083972	0.062179	0.062179	0.094474	0.117068
0.1	0.150297	0.144529	0.125760	0.125760	0.158753	0.181878
0.15	0.211950	0.207256	0.192175	0.192175	0.225419	0.248566
0.2	0.277603	0.274201	0.262923	0.262923	0.295036	0.316782
0.25	0.349356	0.347207	0.339598	0.339597	0.364905	0.381522

NOMENCLATURE

A constant
Bi_1 Biot number, $(h_1 D)/K$
Bi_2 Biot number, $(h_2 D)/K$
Br Brinkman number, $(\mu U_0^2)/(K\Delta T)$
C_P specific heat at constant pressure
D hydraulic diameter, $2L$
g acceleration due to gravity
Gr Grashof number, $(g\beta\Delta T D^3)/\nu^2$
h_1, h_2 external heat transfer coefficients
K thermal conductivity
k permeability of porous media
L channel width
Nu_1, Nu_2 Nusselt numbers
p non-dimensional pressure gradient
P difference between the pressure and the hydrostatic pressure, $p + \rho_0 g X$
Pr Prandtl number, $(\mu C_P)/K$
Re Reynolds number, $(U_0 D)/\nu$
R_T temperature difference ratio
Q rate of internal heat absorption/generation
T temperature
T_1, T_2 reference temperatures of the external fluid
T_0 reference temperature
u dimensionless velocity in the X -direction
$\overline{u}$ mean value of u
U_0 reference velocity
U velocity component in the X -direction
x dimensionless stream wise coordinate
X stream wise coordinate
y dimensionless transverse coordinate
Y transverse coordinate

GREEK SYMBOLS

α thermal diffusivity, $K/(\rho_0 C_p)$

β thermal expansion coefficient

ε dimensionless parameter, λBr

ϕ dimensionless parameter of the heat absorption/generation, $\left(QD^2\right)/K$

ΔT reference temperature difference

θ dimensionless temperature, $\left(T-T_0\right)/\Delta T$

θ_b dimensionless bulk temperature

μ viscosity

ν kinematics viscosity, μ/ρ_0

λ dimensionless parameter, Gr/Re

ρ mass density,

ρ_0 value of the mass density when $T=T_0$

σ porous parameter $D/\sqrt{k}$

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