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ON TOPOLOGICAL h^* - QUOTIENT MAPS

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ABSTRACT

In this paper, we introduce h^* –quotient map and its strong form, comparison of h^* quotient map and α- quotient map and the applications in this concern.

1. Introduction

Levine [7] offered a new and useful notion in Genaral Topology, that is the notion of a genaralised closed set. This notion has been studied extensively in resent years by many topologists. The investigation of genaralised closed set had led to several new and interesting concepts. After the introduction of genaralised closed sets there are many research papers which deal with different types of genaralised closed sets. J. Antony Rex *et. al* [1] have introduced h^* - closed sets and studied their properties, In this paper h^* - quotient map is introduced. Using these new types of functions several characterzations and its properties have been obtained.

2. Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z, η) represent topological spaces on which no separation axiom is defined unless otherwise mentioned. For a subset A of a space (X ,τ), cl(A) and int(A) denote the closure of A and the interior of A in X respectively . *We define the following* ***definition*** *which are useful in the sequel.*

Definition 2.1: *A subset A of a space (X, τ) is called*

1. a semi-open set [8] if A ⊂ cl(int(A)) and a semi closed set if int(cl(A)) ⊂ A
2. a α-open set [6] if A ⊂ int(cl(int(A)))and a α- closed set if cl(int(cl(A))) ⊂ A

Definition 2.2: A subset A of a space (X, τ) is called a

1. a generalized closed (briefly g-closed) set [7] if cl(A) ⊂ U whenever A⊂ U and U is open in(X ,τ); the compliment of a g- closed set is called a g-open set.
2. a ω - closed set [11,12] if cl(A) ⊂ U whenever A⊂ U and U is a semi -open set in (X, τ); the compliment of a ω-closed set is called a ω-open set.
3. a h^* - closed set [1] if scl(A) ⊂ int (U) whenever A⊂ U and U is a ω open set in (X, τ); the compliment of a ω-closed set is called a ω-open set.

Definition 2.3: A function f: (X, τ) → (Y, σ) is called
1. h^* – continuous [1] if f^{-1} (V) is a . h^* – closed set of (X, τ) for each closed set V of (Y, σ).
2. strongly h^* – continuous [1] if f^{-1}(V) is a closed set of (X, τ) for each . h^* closed set V of (Y, σ).

Definition 2.4: A function f: (X, τ) → (Y, σ) is called
1. a h^* –irresolute [1] if f^{-1} (V) is h^* – open set of (X, τ) for each h^* – open set V of (Y, σ).
2. a α –irresolute [6] if f^{-1} (V) is α – open set of (X, τ) for each α – open set V of (Y, σ).

Definition 2.5: A surjective map f: (X, τ) → (Y, σ) is said to be
1. a quotient map [4], provided a subset U of (Y, σ) is open in (Y, σ) if and only if f^{-1} (U) is open in (X, τ) .
2. an α- quotient map [13], if f is α- continuous and f^{-1} (V) is open in (X, τ) implies V is an α- open set in (Y,σ)
3. an α^* - quotient map [13], if f is α- irresolute and f^{-1} (V) is an α-open in (X, τ) implies V is an open set in (Y, σ)

3. h^*- Quotient maps

Definition 3.1: A surjective map f: (X, τ) → (Y, σ) is said to be a h^* – quotient map if f is h^*-continuous and f^{-1} (V) is open in (X, τ) implies V is a h^*- open set in (Y, σ).

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Example 3.2 : Let X = Y = {a, b, c}, τ ={φ,{a}, X }, σ = { φ,{a} {b} {a, b},Y}.We have h^* O(X) = { φ,{a}, {b}, {c},{a, b},{a, c},X}. h^* O(Y) = { φ,{a}, {b}, {a, b},{b, c}, {c, a},X}. The identify map f: (X, τ) → (Y, σ) is a h^* -quotient map.

Definition 3.3: A map f : (X, τ)→ (Y, σ) is said to be h^* - open if f (U) is h^*-open in (Y, σ) for each open set U in (X, τ).

Example 3.4: Let X = Y = {a, b, c}. τ = {φ, {a}, {b} {a, b},X}. σ = { φ, {a},Y}. We have h^* O(Y) = {φ, {a}, {b}, {c}, {a, b}, {a, c}, X}. Define f: (X, τ) → (Y, σ) by f(a) = b, f (b) = a , f (c) = c. Then f is a h^* – open map.

Definition 3.5: A map f: (X, τ) → (Y, σ) is said to be strongly h^* -open if f(U) is h^*- open in (Y,σ) for each h^* - open set U in (X, τ).

Example 3.6: Let X = Y = {a, b, c}. τ ={φ, {a, b},X}, σ = {φ,{a},{b},{a, b},Y }. Then h^* O(X) = {φ,{a}, {b},{a, b}, X} h^* O(Y) = {φ, {a},{b},{a, b},{b, c},{c, a},Y}. Define f: (X, τ) → (Y, σ) by f (a) = b, f (b) = a, f (c) = c .Then f is strongly h^* -open.

Proposition 3.7: If a map f: (X, τ) →(Y, σ) is surjective, h^* -continuous and h^*–open, then f is a h^* -quotient map.

Proof: We only need to prove that f^{-1} (V) is open in (X, τ) implies V is a h^*-open set in (Y, σ). Let f^{-1}(V) be open in (X, τ).Then f (f^{-1} (V)) is a h^* -open set, since f is h^*-open. Hence V is a h^*-open set, as f is surjective, f(f^{-1} (V) = V. Thus f is a h^*-quotient map.

Proposition 3.8: Let f: (X, τ^{h^*})→ (Y, σ^{h^*}) be a quotient map. Then f: (X, τ) →(Y, σ) is a h^*- quotient map.

Proof: Let V be any open set in (Y, σ).Then V is a h^* – open set in (Y, σ) and V ∈ σ^{h^*}.Since f is a quotient map f^{-1} (V) is a h^* –open set in (X, τ) . Hence f is h^* - continuous. Suppose f^{-1} (V) is open in (X, τ), that is f^{-1} (V) ∈ τ^{h^*}. Since f is a quotient map, V ∈ σ^{h^*} and V is a h^* -open set in (Y, σ). This shows that f: (X, τ) →(Y, σ) is a h^*-quotient map.

4. Strong form of h^*- quotient maps

Definition 4.1: Let f: (X, τ) → (Y, σ) be a surjective map. Then f is called a strongly h^*-quotient map provided a set U of (Y, σ) is open in Y if and only if f^{-1} (U) is h^*-open in (X, τ).

Example 4.2: Let X = Y={a, b, c}, τ = {φ,,{a, b},X}, σ={φ,{a},{b},{a, b},Y}. Then h^* O(X)={φ,{a},{b},{a, b},X} h^* O(Y) ={φ, {a}, {b},{a, b},{b, c},{c, a},Y}. The identify map f: (X, τ) → (Y, σ) is a strongly h^*-quotient map.

Theorem 4.3: Every strongly h^*–quotient map f: (X, τ) → (Y, σ) is a h^* -open map.

Proof: Let f: (X, τ) → (Y, σ) be a strongly h^* -quotient map. Let V be an open set in (X, τ). Since every open set is h^* -open V is h^* -open in (X, τ) .That is f^{-1} (f(V)) is h^* -open in (X, τ) since f is strongly h^* -quotient, f(V) is open and hence h^* -open in (Y, σ) . This shows that f is h^* -open.

Remark 4.4: The converse of theorem 4.3 need not be true.

Example 4.5: Let X =Y = {a, b, c}. τ = {φ, {a}, {a, b}, {a, c}, X}. σ = {φ,{a},{b},{a, b},Y}. Then h^* O(X)={φ,{a},{a, b},{a, c},X } h^* O(Y) ={ φ, {a}, {b},{a, b},{b, c},{c, a},Y}. The map f: (X, τ) → (Y, σ) defined as f(a) =a, f(b) = c, f(c) = b is h^*-open but not strongly h^*-quotient since f^{-1} ({b}) = {c} is not h^*-open in (X, τ) for the open set{b} in (Y, σ).

Theorem 4.6 : Every strongly h^*- quotient map f : (X, τ) → (Y, σ) is strongly h^*-open.

Proof: Let f: (X, τ) → (Y, σ) be a strongly h^* – quotient map. Let V be a h^* -open set in (X, τ). That is f^{-1}(f(V)) is h^* – open in (X, τ).Since f is strongly h^*-quotient, f(V) is open and hence h^*-open in (Y, σ).This shows that f is strongly h^*-open.

Remark 4.7: The converse of theorem 4.6 need not be true.

Example 4.8: Let X = Y = {a, b, c}, τ = {φ, {a}, {a, b}, {a, c}, X} σ = {φ, {a},{b},{a, b},Y}. Then h^* O(X)={φ,{a},{a, b},{a, c},X } h^* O(Y) ={ φ, {a}, {b},{a, b},{b, c},{c, a},Y}. Define f: (X, τ) → (Y, σ) as f(a) = a, f(b) = c, f(c) = b
The map f is strongly h^*-open, but not strongly h^* - quotient because f^{-1}({b}) = {c} is not h^*- open in (X, τ) for the open set {b} in (Y, σ).

Definition 4.9: Let f: (X, τ)→(Y, σ) be a surjective map. Then f is called completely h^* - quotient map if f is h^* - irresoulte and f^{-1} (U) is h^* – open in (X, τ) implies U is open in (Y, σ).

E*xample 4.10:* Let X = {a, b, c, d}, τ = {φ, {a},{a, b},{a, c, d},X}, Y = {p, q, r}, σ = {φ,{p},{p, q},{p, r},Y} h^*O(X)={φ,{a},{c},{d},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d},X } h^* O(Y) ={ φ, {p}, {p, q},{p, r},Y} Define f: (X, τ)→ (Y, σ) as f (a) = p, f(b) = r f(c)= f(d) = q . Then f is h^* –irresolute and also f^{-1}(U) is h^*- open in (X, τ) implies $\bigcup$ is open in(Y, σ). Therefore f is a completely h^* - quotient map.

Proposition 4.11: Every completely h^* - quotient map is h^* -irresolute.

Proof: Follows from the definition.

Remark 4.12: The converse of proposition 4.11 need not be true.

Example 4.13: Let X = Y = {a, b, c}. τ = { φ,{a},{a, b},{a, c},X},σ ={ φ,{a, b},Y}. h^* O(X)={φ,{a},{a, b},{a, c},X }, h^* O(Y) ={φ, {a},{a, b},{a, c},Y}. The identify map f: (X, τ)→ (Y, σ) is h^*- irresolute but f is not completely h^* - quotient. Since f^{-1} ({a, c}) = {a, c} is h^* -open in (X, τ) but {a, c} is not open in (Y, σ).

Theorem 4.14: If f: (X, τ) → (Y, σ) is a completely h^*-quotient map then f is strongly h^* -open.

Proof: Let f: (X, τ)→(Y, σ) be a completely h^* - quotient map. Let V be a h^*-open set in (X, τ).Then f^{-1}(f(V)) is h^* – open in (X, τ).Since f is completely h^* - quotient f(V) is open in (Y, σ) and thus h^*-open in (Y, σ). Hence f is strongly h^*-open.

Remark 4.15: The converse of Theorem 4.14 need not be true

Example 4.16 : Take X, Y, τ, σ, f as in example 3.6 .Here f is a strongly h^* -open map but not a completely h^*-quotient map since f^{-1}({b, c}) = {a, c} is not a h^* -open set in (X, τ) but {b, c } is a h^* - open set in (Y, σ) .

5. Comparison

Proposition 5.1:

1. Every quotient map is a h^* - quotient map.
2. Every α- quotient map is a h^* - quotient map.

Proof: Since every continuous and α- continuous map is h^* - continuous and every open set and α -open set is a h^* - open set [1] the theorem follows.

Remark 5.2: The separate converse of proposition 5.1 need not be true.

Example 5.3: Let X = Y = {a, b, c}, τ = {φ, {a}, {a, b}, {a, c}, X} σ = {φ, {a, b}, Y}. Then h^* O(X)={φ,{a},{a, b},{a, c},X }, h^* O(Y) ={ φ, {a},{a, b},{c, a},Y}. The identify map f: (X, τ) → (Y, σ) is h^* - quotient but not quotient (α- quotient). Since f^{-1} ({a, c}) is open (α-open) (X, τ) but not open (α-open) in (Y, σ).

Proposition 5.4: If f : (X, τ) → (Y, σ) is a strongly h^*-quotient map then f is h^*-quotient but not conversely.

Proof: Let V be an open set in (Y, σ). Since f is strongly h^* -quotient f^{-1} (V) is a h^* - open set in (X, τ). Thus f is h^* - continuous. Let f^{-1} (V) be open in (X, τ). Then f^{-1} (V) is h^*-open in (X, τ). Since f is strongly h^*-quotient, V is open in (Y, σ) and hence V is h^*-open in (Y, σ). This shows that f is a h^*-quotient map.

Example 5.5: Take X, Y, τ, σ as in example 3.2. Then f : (X, τ)→ (Y, σ) is a h^*-quotient map but noy strongly h^*-quotient .Since f^{-1} ({b}) = {b} with{b} h^*-open in (Y, σ) but {b} is not open in (X, τ).

Remark 5.6: α^* - quotient map and completely h^* - quotient map are independent of each other.

Example 5.7: Let X =Y = {a, b, c} . τ = { φ,{a},X}. σ = {φ,{a},{a, b},{a, c},Y}. Then αO(X, τ}={ φ,{a},{a, b},{a, c},X}, h^* O(X)={φ,{a},{b},{c},{a, b},{a, c},X }, αO(Y, σ}={ φ,{a},{a, b},{a, c},Y}, h^* O(Y, σ)={φ,{a},{a, b},{a, c},Y }, The identify map f : (X, τ)→ (Y, σ) is α^* - quotient but not h^* - quotient (α- quotient). Since f is not even h^* - irresolute.

***Example* 5.8 :** Let X =Y= {a, b, c, d}, τ={ φ, {b},{c},{b, c}, {b, c, d},X} and *σ = { φ, {a},{b},{a, b},{a, b, c},{a, b, d},Y}.* h^* O(X) = { φ,{b}, {c} {d},{a, b}, {a, c}, {b, c},{b, d}, {c, d},{a, b, c},{a, b, d},{a, c, d},{b, c, d},X}. αO(X) = *{ φ, {b},{c},{b, c},{a, b, c}, {b, c, d},X}.* O(Y) ={φ,{a},{b},{a, b},{a, c},{a, d},{b, c},{b, d},{a, b, c},{a, b, d}, {a, c, d},{b, c, d},Y}. αO(Y) = *{ φ, {a},{b},{a, b},{a, b, c},{a, b, d},Y}.* Define f: (X, τ) → (Y, σ) as f(a) = c, f(b) = a, f(c) =

b f(d) =d. The map f is a α^* - quotient but not h^* - quotient. Since for the h^* -open set {a, b, c} in (Y, σ) f^{-1}({a, b, c}) = {a, b, c} is not h^* -open in(X, τ), hence f is not h^* – irresolute. Therefore f is not a h^* - quotient map.

Proposition 5.9: If f: (X, τ) → (Y, σ) is a completely h^*-quotient map then f is strongly h^*-quotient.

Proof: Let V be an open set in (Y, σ).Then it is a h^*-open set in (Y, σ).Since f is h^*- irresolute f^{-1} (V) is a h^*- open set (X, τ). Thus V is a open set in (Y, σ) implies f^{-1} (V) is a h^* -open set (X, τ).Conversely if f^{-1} (V) is a h^*-open set in (X, τ), since f is a completely h^*- quotient map, V is open set in (Y, σ).Hence f is a strongly h^* - quotient map

Remark 5.10: The converse of proposition 5.9 need not be true.

Example 5.11: *Take* X, Y, τ, σ and f as in example 4.2 .Then f is a strongly h^*-quotient map. The set {b, c} is h^* -open in (Y, σ) but f^{-1}({b, c}) = {b, c} is not h^* -open in (X, τ). Therefore f is not h^*– irresolute. Hence f is not a completely h^*-quotient map.

Remark 5.12: The concepts of quotient maps and strongly h^*-quotient maps are independent of each other.

Example 5.13 : Let X = Y = {a, b, c},τ={φ,{a},{a, c},X}, σ={φ,{a},{c},{a, b},{a, c},Y}. h^* O(X) ={φ,{a},{c}, {a, b},{a, c},X}, h^* O(Y) ={φ,{a},{c},{a, b},{a, c},Y}.

Let f: (X, τ) → (Y, σ) be an identify map. Then f is a strongly ^{h*}- quotient map but f is not a quotient map because the set {a, b} open in (Y, σ) but f^{-1}({a, b}) = {a, b} is not open in (X, τ).

Example 5.14 : Let X = {a, b, c, d}, Y= {a, b, c}.τ={φ,{a},{a, d},X},σ={φ,{a},{a, c},Y}. h^* O(X)={ φ,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},X} h^* O(Y) ={φ,{a},{b},{a, b},{a, c},Y} .Define f : (X, τ) → (Y, σ) as f(a)=a, f(b)=b=f(c),f(d)=c. Then f is a quotient map .But f is not a strongly ^{h*}- quotient map since f^{-1}({a, b}) = {a, b, c} is h^* -open in (X, τ) but not open in (Y, σ).

Theorem 5.15: If f: (X, τ) → (Y, σ) is a completely h^*-quotient map then f is h^*-quotient.

Proof: Let f be a completely h^*-quotient map. Then f is h^* -irresoulte. Therefore f is h^*-continuous .Let f^{-1}(V) be an open set in (X, τ). Then f^{-1}(V) is a h^* -open set in (X, τ).Since f is a completely h^*-quotient map, V is open in (Y, σ) and hence V is h^*-open in (Y, σ).Therefore f is a h^*-quotient map.

Remark 5.16: The converse of theorem 5.15 need not be true.

Example 5.17: Let X = Y = {a, b, c} τ = {φ, X, {a}} σ = {φ, Y {a}, {b}, {a, b}}. h^* O(X) = {φ,{a},{b},{c},{a, b}, {a, c },X}, h^* O(Y) ={φ,{a},{b},{a, b},{b, c},{a, c},Y}.

Let f: (X, τ) → (Y, σ) be the identity map. Then f is a h^*-quotient map but f is not a completely h^*-quotient map since f^{-1}({a, c}) = {a, c} is a h^* –open set in (X, τ). But {a, c} is not open in (Y, σ).

6. Applications

Proposition 6.1: Let f: (X, τ) → (Y, σ) be an open surjective h^*-irresolute map and g: (Y, σ) → (Z, τ) be a h^*- quotient map. Then their composition g o f: (X, τ) → (Z, η) is a h^* -quotient map.

Proof: Let V be any open set in (Z, τ) then g^{-1} (V) is a h^* - open set in (Y, σ) since g is a h^* - continuous map. Since f is h^* -irresolute f^{-1} (g^{-1}(V)) = $(g \circ f)^{-1}$ (V) is a h^* -open set in (X, τ).This implies (go f) $^{-1}$ (V) is a h^* - open set in (X, τ). This shows that g o f is a h^*- continuous map. Assume that (g o f) $^{-1}$ (V) is open in (X, τ) for $V \subseteq Z$, that is (f^{-1} (g^{-1} (V)) is open in (X, τ). Since f is open f (f^{-1}(g^{-1} (V))) is open in (Y, σ). If follows that g^{-1} (V) is open in (Y, σ), because f is a surjective. Since g is a h^*- quotient map V is a h-open set in (Z, η). Thus g o f: (X, τ) → (Z, η) is a h^*- quotient map.

Proposition 6.2:If h : (X, τ) → (Y, σ) is a h^*-quotient map and g : (X, τ) →(Z, η) is a continuous map that is constant on each set f^{-1} ({y}), for each y ∈ Y, then g induces a h^* -quotient map f : (Y, σ)→ (Z, η) such that f o h = g.

Proof: Since g is constant on h^{-1}({y}) for each y ∈ Y, the set g (h^{-1}({y})) is a one point set in (Z, η) . If f(y) denote this point, then it is clear that f is well-defined and for each x ∈ X, f (h(x)) = g(x).We claim that f is h^*- continuous. Let V be any open set in (Z, η) then g^{-1} (V) is an open set in (X, τ) as g is continuous but g^{-1}(V) =h^{-1}(f^{-1}(V)) is open (X, τ). Since h is h^*-quotient map f^{-1}(V) is a h^*- open set in (Y, σ). Hence f is h^* -continuous.

Proposition 6.3 : Let f: (X, τ) → (Y, σ) be an strongly h^* – open surjective and h^*- irresoulte map and g: (X, τ) → (Z, η) be a strongly h^*-quotient map then g o f : (X, τ) → (Z, η) is a strongly h^* -quotient map.

Proof: Let V be any open set in (Z, η). Then g^{-1}(V) is a h^* – open set in (Y, σ) (since g is strongly h^*- quotient). Since f is a h^* - irresolute (f^{-1} (g^{-1} (V)) is a h^* -open set in (X, τ). Conversely, assume that $(go\ f)^{-1}$ (V) is a h^* -open set (X, τ) for V ⊆ Z. Then (f^{-1}(g^{-1} (V)) is a h^*--open set in (X, τ). Since f is strongly h^*-open, f (f^{-1}(g^{-1} (V))) is a h^*- open set in (Y, σ). If follows that (g^{-1} (V)) is a h^*-open set in (Y, σ). Thus V is an open set in (Z, η) (since g is strongly h^*- quotient) Thus g o f is a strongly h^*-quotient map.

Definition 6.4: [1] A space (X, τ) is called a Th^* space if every h^* – closed set in is closed.

Theorem 6.5: Let p: (X, τ) →(Y, σ) be a h^*-quotient map where (X, τ) and (Y, σ) are Th^* – spaces. Then f: (Y, σ) → (Z, η) is strongly h^*- continuous if and only if the composite map f o p : (X, τ)→ (Z, η) is strongly h^*-continuous.

P***roof:*** Let f be strongly h^*-continuous and U be any h^* -open set in(Z ,η) . Then f^{-1}(U) is open in (Y, σ). Then $(f\ o\ p)^{-1}$ (U) = p^{-1} (f^{-1}(U)) is h^*–open in (X, τ)

Since (X, τ) is a Th^* – space p^{-1}(f^{-1} (U)) is open in (X, τ).Thus f o p is strongly h^*- continuous. Conversely let the composite map f o p be strongly h^* – continuous. Then for any h^*- open set U in (Z, η), p^{-1}(f^{-1}(U)) is open in (Z, η). Since p is a h^* – quotient map, it implies that f^{-1}(U) is h^*-open in (Y, σ). Since (Y, σ) is a Th^* – space, f^{-1}(V) is open in (Y, σ).Hence f is strongly h^* -continuous.

Theorem 6.6 : Let f: (X, τ) → (Y,σ) be a subjective , strongly h^* – open and h^* -irresolute map and g: (Y, σ) → (Z, η) be a completely h^* - quotient map then g o f is completely h^* - quotient map.

Proof: Let V be a h^* –open set (Z, η). Then g^{-1}(V) is a h^* -open set a (Y, σ), because g is a completely h^* - quotient map. Since f is h^* -irresolute f^{-1}(g^{-1}(V)) is a h^* - open set in (X, τ).Hence g o f is a h^* - irresolute. Suppose $(g\ o\ f)^{-1}$ (V) is a h^* – open set in(X, τ) for a subset V ⊆ Z. that is f^{-1}(g^{-1}(V)) is a h^*-open set in (X, τ). Since f is strongly h^* – open, f (f^{-1}(g^{-1}(V))) is a h^* - open set. Thus g^{-1} (V) is a h^* – open set in (Y, σ). Since g is a completely h^* - quotient map V is an open set in (Z, η). Hence g o f a completely h^* - quotient map.

Proposition 6.7: Let f : (X, τ)→ (Y, σ) be an strongly h^*-quotient , h^*-irresolute map and g: (Y, σ) → (Z, η) be a completely h^* - quotient map then g o f is a completely h^* - quotient map .

Proof: Let V be any h^* – open set in (Z, η) then g^{-1}(V) is a h^* –open set in (Y, σ) (since g is a completely h^* - quotient map) We have f^{-1}(g^{-1} (V)) is also h^* –open set (X, τ) (since f is a h^*– irresolute. Thus $(g\ of)^{-1}$(V) is h^* – open set in (X, τ). Hence g o f is h^*-irresolute. Let $(g\ o\ f)^{-1}$ (V) be a h^* -open set in (X, τ) for V ⊆ Z. That is f^{-1}(g^{-1} (V)) is h^* -open set (X, τ). Then g^{-1}(V) is a open set in (Y, σ) because f is a strongly h^* - quotient map .This means that g^{-1}(V) is a h^* –open set in (Y, σ).Since g is a completely h^* - quotient map ,V is an open set in(Z, η).Thus g o f is a completely h^* - quotient map.

Theorem 6 8: The composition of two completely h^* - quotient maps is a completely h^* - quotient map. Assume (Y, σ) to be a locally indiscrete space.

Proof: Let f: (X, τ) →(Y, σ) and g: (Y, σ) → (Z, η) be two completely h^*-quotient maps. Let V be a h^* –open set in (Z, η).Since g is a completely h^* - quotient map g^{-1} (V) is a h^*-open set in (Y, σ). Since f is a completely h^* - quotient map f^{-1}(g^{-1} (V)) is a h^*-open set in (X, τ). That is $(g\ o\ f)^{-1}$ (V) is a h^* –open set in (X, τ). Hence g o f is h^*-irresolute.

Let $(g\ o\ f)^{-1}$(V) be a h^*-open set in (X, τ).Then f^{-1}(g^{-1} (V)) is h^*-open set (X, τ). Since f is completely h^*-quotient map g^{-1}(V) is an open set in (Y, σ).That is g^{-1}(V) is a h^*- open set in (Y, σ).

Since g is a completely h^*-quotient map V is a open set in (Z, η). Hence g o f is a completely h^* - quotient map.

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