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MULTICRITERIA DECISION MAKING ON PERFORMANCE AND OTHER PARAMETERS OF AUTOMOBILES

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ABSTRACT

The analytical hierarchy process is a Multi Criteria Decision Making (MCDM) technique. When uncertainty occurs fuzzy analytical hierarchy process is used. In this paper a comparison between the traditional analytical hierarchy process and fuzzy analytical hierarchy process is discussed with a case study on performance and parameters of automobiles.

Keywords: *Analytical Hierarchy Process, Fuzzy Analytical Hierarchy Process, Multi Criteria Decision Making.*

Ams Subject Classification (2010): *62C86, 90B50.*

1.1 INTRODUCTION

A decision is a choice out of a number of alternatives in such a way that the preferred choice is the best option among the possible. Decision making is considered as one of the challenging task in human life.

Decision making involves many criteria and sub criteria used to rank the alternatives of a decision. The priorities are to be evaluated for the alternative with respect to the criteria or sub criteria and also for the criteria in terms of a higher goal or if they are dependent **o**n the alternatives.

1.2 ANALYTICAL HIERARCHY PROCESS (AHP)

The analytical hierarchy process is a Multi Criteria Decision Making (MCDM) technique proposed by Saaty in 1970. In1980 Saaty proposed AHP as a decision aid to solve unstructured problems in economics social and management sciences. AHP has been applied in a variety of contexts from the simple everyday problems of selecting a school to the complex problem of designing alternative future outcomes of a developing country, evaluating potential candidacy, allocating energy resources and so on.

The AHP enables the decision makers to structure a complex problem in the form of a simple hierarchy and to evaluate a large number of qualitative and quantitative factors.

1.3 THE SCALE OF RELATIVE IMPORTANCE ACCORDING TO SAATY (1980)

INTENSITY OF INFORMATION	DEFINITION	EXPLANATION
1.	Equal Importance	Two activities contribute equally to the each other
3.	Weak importance of one over another	Experience and judgments slightly favour one activity over another
5.	Essential or strong importance	Experience and judgments strongly favour one activity over another
7.	Demonstrated importance	An activity is strongly favored and its dominance is demonstrated in practice
9.	Absolute importance	The evidence favoring one activity over another is of the highest possible order of affirmation
2,4,6,8	Intermediate values between the two judgments	When compromise is needed
Reciprocals of the above non zero	If the activity i has one of the above non zero numbers assigned to it when compared to activity j, then j has the reciprocal value when compared with i.	

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1.4 THE FUZZY AHP

In most of the real world problems, some of the decision data can be precisely assessed while others cannot. Due to the complexity and uncertainty involved in real world decision problems and inherent subjective nature of human preference judgments, it is always unrealistic to obtain a feasible exact ratio. Essentially, the uncertainty in the preference judgments gives rise to uncertainty in the ranking of alternatives as well as difficulty in determining consistency of the preferences.

ALGORITHM TO SOLVE DECISION MAKING PROBLEM USING FUZZY ANALYTICAL HIERARCHY PROCESS:

STEP 1:

The fuzzy synthetic extent value (S_i) with respect to the i^{th} criterion is defined as equation (1).

$$S_i = \sum_{j=1}^{m} M_{g_i}^{j} \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^{j}\right]^{-1} \quad (1)$$

to obtain equation (2)

$$\sum_{j=1}^{m} M_{g_i}^{j} \quad (2)$$

Perform the "fuzzy addition operation" of m extent analysis values for a particular matrix given in equation (3) below, at the end step of calculation, new (l, m, u) set is obtained and used for the next

$$\sum_{j=1}^{m} M_{g_i}^{j} = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right). \quad (3)$$

Where l is the lower limit value, m is the most promising value and u is the upper limit value.

To obtain equation (4),

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^{j}\right]^{-1} \quad (4)$$

Perform the fuzzy addition operation of $M_{g_i}^{j}$ (j= 1, 2, 3, 4…... m) values given as equation (5)

$$\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^{j} = \left(\sum_{i=1}^{n} l_i, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i\right) \quad (5)$$

The inverse of the vector in the equation (5) is

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^{j}\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i}\right) \quad (6)$$

STEP 2:

The degree of possibility of $M_2 = (l_2,m_2,u_2) \geq (l_1,m_1,u_1)$ is defined as equation (7).

$$V(M_2 \geq M_1) = \sup_{y \geq x} [\min((x), \mu_{M_2}(y))] \quad (7)$$

x and y are the values on the membership function of each criterion. This expression can be equivalently written as given in equation below.

$$V(M_2 \geq M_1) = \begin{cases} 1, & \text{if } m_2 \geq m_1, \\ 0, & \text{if } l_1 \geq u_2, \\ \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)} & \text{otherwise} \end{cases} \quad (8)$$

Where d is the highest intersection point μ_{M_1} and μ_{M_2}

To compare M_1 and M_2 we need both the values of $V(M_2 \geq M_1)$ and $V(M_1 \geq M_2)$

STEP 3:

The degree possibility for a convex fuzzy number to be greater than k convex fuzzy numbers M_i*(i=1,2,3,4……………, k)* can be defined by

$$V(M \geq M_1, M_2, M_3, M_4, M_5, \ldots\ldots M_6) = V[(M \geq M_1) \text{ and } (M \geq M_2) \text{ and } (M \geq M_3) \text{ and } (M \geq M_4) \ldots\ldots \text{and } (M \geq M_k)]$$
$$= \min V(M \geq M_i),\ i=1, 2, 3\ldots..k.$$

Assume that equation (9) is

$$d^{l}(A_i) = \min V(S_i \geq S_k) \quad (9)$$

For $k= 1, 2, 3, 4\ldots\ldots, n; k\neq i$.

Then the weight vector is given by equation (10)

$$W^l= (d^l (A_1), d^l (A_2), d^l (A_3), d^l (A_4)\ldots d^l (A_n))^T \tag{10}$$

Where A_i (i=1, 2, 3, 4… n) are n elements.

STEP 4:
After normalization, the normalized weight factors are given in equation (11)

$$W= (d(A_1), d (A_2), d(A_3), d(A_4),\ldots\ldots d(A_n))^T \tag{11}$$

Where W is non fuzzy numbers

2. CASE STUDY

Evaluation of the performance of the best car using the Analytical Hierarchy process using various parameters

In today's situation where India is a developing economy, most of the people prefer to own a car and this is very much necessary for day to day life. Choosing a car depends upon each one's personal opinion as to what they would give importance. Some might want a car just for the purpose of travel not worrying about the features and luxury. Some might want the pleasure in driving and looks of the car to be prestigious. Some would expect performance level to be high. So the importance level may vary according to each individual. But there are few standard criteria on which a car is considered to be best. As the rate of petrol is increasing people have turned their concentration on diesel cars. 7 cars have been taken for comparison. My goal is to suggest the best car among the 7. The comparison is done using Analytical Hierarchy Process and confirmed with the uncertainty situation using Fuzzy Analytic Hierarchy Process. The general criteria on which I have compared the cars is

- PRICE
- ENGINE POWER
- MILEAGE
- SAFETY
- LUXURY

CAR	PRICE	COMFORT	SAFETY	MILEAGE	ENGINE
CAR 1	Rs.10,26,123	8 features	5 features	18/20	105Ps@4400rpm 250Nm@1500rpm
CAR 2	Rs.8,18,811	7 features.	4 features	18/21	86Ps@3750rpm 200Nm2000rpm
CAR 3	Rs.5,24,385	6 features	2 features	14/16	80Ps@2200 108Nm@4400
CAR 4	Rs.4,85,598	5 features	NIL	18/21	80Ps@6200 108Nm@4400
CAR 5	Rs5,64,731	3 features	2 features	16/18	75Ps@4000 190Nm@2000
CAR 6	Rs.15,09,240	9 features	5 features	12/14	140Ps@4000rpm 320Nm@1700rpm
CAR 7	Rs.13,04,362	8 features	6 features	16/18	160Ps@4500rpm 250Nm@1500rpm

Table: 1

Each car is compared for all the features and each feature is compared among the other features and finally the best car is chosen. The comparison is done using Analytical Hierarchy Process

CRITERIA FOR PAIR WISE COMPARISON:

	Price	Comfort	Safety	Mileage	Engine	A_{ij}	λ
Price	1	5	3	$1/5$	$1/3$	0.1344	5.200
Comfort	$1/5$	1	$1/3$	$1/9$	$1/7$	0.0348	5.089

Safety	$1/3$	3	1	$1/7$	$1/5$	0.0678	5.026
Mileage	5	9	7	1	3	0.5028	5.458
Engine	3	7	5	$1/3$	1	0.2602	5.432
						1.0000	5.241

Table: 2

CONSISTENCY TEST:
CI= 0.06
CR=0.05<0.1

COMPARISON OF ALTERNATIVES WITH THE CRITERIA:

CAR	Price	Comfort	Safety	Engine	Mileage	Weight
Car 1	0.0719	0.1552	0.2066	0.1050	0.1980	0.1555
Car 2	0.1190	0.3156	0.1368	0.0722	0.2834	0.1971
Car 3	0.2263	0.0492	0.0535	0.0489	0.0530	0.075
Car 4	0.3286	0.0337	0.0211	0.0249	0.2834	0.1954
Car 5	0.1923	0.0241	0.0535	0.0341	0.1090	0.1015
Car 6	0.0239	0.2440	0.2066	0.4290	0.0224	0.1485
Car 7	0.0380	0.1782	0.3219	0.2859	0.0509	0.1591

Table: 3

RANKING AS PER WEIGHTAGE

CAR	Weight	RANK
Car 1	0.1555	4
Car 2	0.1971	1
Car 3	0.075	7
Car 4	0.1954	2
Car 5	0.1015	6
Car 6	0.1485	5
Car 7	0.1591	3

Table: 4

Hence as a result of the analysis CAR 2 is found to be the best among all the 7 followed by, CAR 4 and CAR 7 rated to be the least preferred on comparison with the criteria taken.

2.1 FUZZY ANALYTICAL HIERARCHY PROCESS

FUZZY RATING	VERBAL JUGEMENT
(1,1,1)	Equal importance
(1,1,2)	Equal to moderate importance
(1,2,3)	Moderate importance
(2,3,4)	Moderate to strong importance
(3,4,5)	Strong importance
(4,5,6)	Strong to very strong importance
(5,6,7)	Very strong importance
(7,8,9)	Absolute importance

Table: 5

2.2 FUZZY PAIR WISE COMPARISION MATRIX

Car	Car 1	Car 2	Car 3	Car 4	Car 5	Car 6	Car 7	Fuzzy scores
Car 1	(1,1,1)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$	(3,4,5)	$(\frac{1}{3},\frac{1}{2},1)$	(1,2,3)	(1,1,2)	$(\frac{1}{2},1,1)$	(7.08,9.83,13.5)
Car 2	(2,3,4)	(1,1,1)	(5,6,7)	(1,1,2)	(2,3,4)	(3,4,5)	(1,2,3)	(15,20,26)
Car 3	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$	$(\frac{1}{7},\frac{1}{6},\frac{1}{5})$	(1,1,1)	$(\frac{1}{6},\frac{1}{5},\frac{1}{4})$	$(\frac{1}{2},1,1)$	$(\frac{1}{3},\frac{1}{2},1)$	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$	(2.54,3.36,4.11)
Car 4	(1,2,3)	$(\frac{1}{2},1,1)$	(4,5,6)	(1,1,1)	(3,4,5)	(2,3,4)	(1,1,2)	(12.5, 17, 22)
Car 5	$(\frac{1}{3},\frac{1}{2},1)$	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$	(1,1,2)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$	(1,1,1)	$(\frac{1}{2}1,1,)$	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$	(3.53,4.41,6.33)
Car 6	$(\frac{1}{2}1,1,)$	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$	(1,2,3)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$	(1,1,2)	(1,1,1)	$(\frac{1}{3},\frac{1}{2},1)$	(4.28,6.08,8.83)
Car 7	(1,1,2)	$(\frac{1}{3},\frac{1}{2},1)$	(3,4,5)	$(\frac{1}{2},1,1)$	(2,3,4)	(1,2,3)	(1,1,1)	(8.83, 12.5, 17)

Table: 6

First we calculate $\sum_{j=1}^{m} M_{g_i}^{j}$ values for each row of the matrix
And hence the values for $\sum_{j=1}^{m} M_{g_i}^{j}$ have been calculated.

$\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^{j}$ = (53.76, 73.18, and 97.77)
$[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^{j}]^{-1}$
= (0.01022, 0.01366, and 0.0186)

S_i value for each column of reciprocal matrix are calculated as
$S_i = \sum_{j=1}^{m} M_{g_i}^{j} \otimes [\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^{j}]^{-1}$
S_1 =(0.0722, 0.1342, 0.2511)
S_2= (0.1533, 0.272, 0.4836)
S_3 = (0.0259, 0.0458, 0.0764)
S_4 = (0.1275, 0.2322, 0.4092)
S_5 = (0.03607, 0.0602, 0.1177)
S_6= (0.04374, 0.0826, 0.1642)
S_7= (0.09024, 0.1707, 0.3162)

Then the values are calculated by using these vectors

$$V(M_2 \geq M_1) = \begin{cases} 1, & \text{if } m_2 \geq m_1, \\ 0, & \text{if } l_1 \geq u_2, \\ \frac{l_1-u_2}{(m_2-u_2)-(m_1-l_1)} & \text{otherwise} \end{cases}$$

Final weights (W') and normalized weights (W) are given below

CAR	(W')	(W)	Rank
Car 1	0.4151	0.1406	Fourth
Car 2	1	0.3388	First
Car 3	0	0	Seven
Car 4	0.8654	0.2932	Second
Car 5	0	0	Sixth
Car 6	0.0544	0.0184	Fifth
Car 7	0.6165	0.2088	Third

Table: 7

The weight of the seven cars and there ranking are done. They imply that Car 2 rates the first followed by Car 4 where as Car 5 and Car 3 rates the least.

3. CONCLUSION:

Thus by both analytical hierarchy process and fuzzy analytical hierarchy process, it's found to be that Car 2 rates the first and Car 3 the least. The conclusions are used to find as what may be the best. To find the truly best solution to a

MCDM problem may never be humanly possible. This method is very much simplified when compared to the statistical method where the user of this data is not a statically literate and the above results can be easily understood by an ordinary person who can come out with a non biased decision.

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