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STATISTICAL QUALITY CONTROL OF MULTI-ITEM EOQ MODEL WITH VARYING LEADING TIME VIA LAGRANGE METHOD

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ABSTRACT

The goal of this paper is to confirm that the production process is in control using statistical quality control process with subgroup ranges. The analytical solution of the economic order quantity model of multiple items with varying leading time using Lagrangian multipliers is derived. The varying leading time crashing cost is considered to be continuous function of leading time. The model is restricted to the budget inventory investment. The optimal order quantity is deduced as a decision variable. Finally the model is illustrating by applied example and the average of the subgroup ranges approach is used to confirm that the production process is in control.

***Keywords**: Inventory model, Lagrangian multipliers, economic order quantity (EOQ), multiple items, leading time, statistical quality control, budget inventory investment.*

1. INTRODUCTION

Many researchers have considered the single–item economic order quantity inventory models under one or two constraints such as the maximum budget inventory investment, the maximum warehouse space or the average inventory level. They solved these problems either by the classical Lagrangian multipliers method or the method of "equal order interval for each item" as suggested by Page and Paul [15]. They proposed a heuristic approach for allocating n products over m groups. In fact their method is always significantly good except for large number of products as given in Goyal [4, 5, 6]. Shawky and Abou-El-Ata [16] solved a constrained production lot size model with trade policy by Geometric Programming and Lagrangian methods. Other related works were written by Kun-Chan [12], Juneau and Eyler [7], Das and Maiti [2], Mandal and Pal [13], Mehta and Shah [14] and Teng [17].

The problem of inventory models involving lead time as a decision variable have been succinctly described by Ben-Daya and Abdul Raouf [1].

Recently, Kotb and Fergany [9, 10] derived the analytical solution of the economic order quantity model of multiple items with demand-dependent unit cost, leading time and varying holding cost using geometric programming approach.

The statistical quality control for probabilistic and deterministic models is studied by Kotb [8], El-Wakeel [3] and Kotb et al [11].

The aim of this paper is to discuss statistical quality control and inventory policy of deterministic multi-item economic order quantity model with varying leading time under linear constraint which is assumed bending. The optimal order quantity of each item was obtained as decision variable using Lagrangian method. In the final the average of the subgroup ranges approach was used to confirm that the production process is in control.

2. BASIC NOTATIONS AND ASSUMPTIONS

To construct the model of this problem, we define the following variables:

C_{hr} = Unit holding (inventory carrying) cost per item per unit time, For the r^{th} item.

C_{or} = Ordering cost, For the r^{th} item.

C_{pr} = Unit purchase (production) cost, For the r^{th} item.

CL = Control limit.

D_r = Annual demand rate, For the r^{th} item.

K = Limitation of budget inventory investment.

L_r = Leading rate time, For the r^{th} item.

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LCL = Lower control limit.
n = Number of different items carried in inventory.
Q_r = Production (order) quantity batch (a decision variable), For the r^th item.
Q_r^* = Optimal order quantity, For the r^th item.
R_i = Subgroup ranges.
$\overline{R}$ = Average of subgroup ranges (CL).
S_R^2 = Variance of subgroup ranges.
SS = $k\sigma\sqrt{L_r}$ = Safety stock, where k is known as safety factor and σ is standard deviation.
TC(Q_r) = Average annual total cost. For the r^th item.
UCL = Upper control limit.

In addition, the following basic assumptions about the model are made:
(1) Demand rate D_r is uniform over time.
(2) Time horizon is finite.
(3) Shortages are not allowed.
(4) Lead time crashing cost is related to the lead time by a function of the form:

$$R(L_r) = \alpha\, L_r, \quad r = 1, 2, 3, \cdots, n \tag{1}$$

where $\alpha > 0$ is real constant selected to provide the best fit of the estimated cost function.

(5) Our objective is to confirm that the production process is in control.

3. MATHEMATICAL MODEL:
The annual relevant total cost was composed of four components (production, order, inventory carrying and lead time crashing costs) according to the basic notations and assumptions of the EOQ model is:

$$TC(Q_r) = \sum_{r=1}^{n}\left[D_r C_{pr} + \frac{D_r}{Q_r} C_{or} + \left(\frac{Q_r}{2} + k\sigma\sqrt{L_r}\right) C_{hr} + \frac{D_r}{Q_r} R(L_r)\right] \tag{2}$$

To determine the optimal values of Q_r we Substitute (1) in (2) obtaining:

$$TC(Q_r) = \sum_{r=1}^{n}\left[D_r C_{pr} + \frac{D_r}{Q_r} C_{or} + \left(\frac{Q_r}{2} + k\sigma\sqrt{L_r}\right) C_{hr} + \frac{D_r}{Q_r} \alpha L_r\right] \tag{3}$$

The following constraint can be stated as:

$$\sum_{r=1}^{n} C_{pr}\, Q_r \le K, \quad Q_r > 0 \;\; for\ all\ r \tag{4}$$

where K is the limitation of budget inventory investment.

In order to solve this primal function which was a convex programming problem, it can be rewritten in the following form:

$$\text{Min } TC(Q_r) = \sum_{r=1}^{n}\left[D_r C_{pr} + \frac{D_r}{Q_r} C_{or} + \left(\frac{Q_r}{2} + k\sigma\sqrt{L_r}\right) C_{hr} + \frac{D_r}{Q_r} \alpha L_r\right] \tag{5}$$

$$\text{subject to } \sum_{r=1}^{n} C_{pr}\, Q_r \le K$$

This is a minimization problem in n variables with single constraint. Hence it can be solved by the Lagrangian multiplier approach. The Lagrangian function associated with problem (5) can be restated as:

$$L(Q_r, \lambda) = \sum_{r=1}^{n}\left[D_r C_{pr} + \frac{D_r}{Q_r} C_{or} + \left(\frac{Q_r}{2} + k\sigma\sqrt{L_r}\right) C_{hr} + \frac{D_r}{Q_r} \alpha L_r\right] + \lambda\left(K - \sum_{r=1}^{n} C_{pr} Q_r\right) \tag{6}$$

where λ is non-positive Lagrangian multiplier.

Taking the partial derivative of Lagrange function with respect to Q_r at $Q_r = Q_r^*$, and then setting equal to zero, gives:

$$\left.\frac{\partial L}{\partial Q_r}\right|_{Q_r=Q_r^*} = -\frac{D_r}{Q_r^{*2}} C_{or} + \frac{1}{2} C_{hr} - \frac{D_r}{Q_r^{*2}} \alpha L_r - \lambda C_{pr} = 0 \tag{7}$$

Simplifying equation (7), we obtain the optimal order quantity:

$$Q_r^* = \sqrt{\frac{2D_r(C_{or} + \alpha L_r)}{C_{hr} - 2\lambda C_{pr}}} \tag{8}$$

We shall verify that the Q_r^* minimize $TC(Q_r)$. This is done by showing that the second derivative of Lagrange function with respect to Q_r is always positive:

$$\left.\frac{\partial^2 L}{\partial Q_r^2}\right|_{Q_r=Q_r^*} = \frac{D_r}{Q_r^{*3}} C_{or} + \frac{D_r}{Q_r^{*3}} \alpha L_r > 0 \tag{9}$$

Substituting the values of Q_r^* in relation (3), we deduce the minimum total cost as:

$$\text{Min TC}(Q_r) = \sum_{r=1}^{n}\left(D_r C_{pr} + \sqrt{\frac{1}{2} D_r (C_{or} + \alpha L_r)(C_{hr} - 2\lambda C_{pr})} + \left[\sqrt{\frac{D_r(Cor + \alpha L_r)}{2(C_{hr} + 2C_{pr})}} + k\sigma\sqrt{L_r}\right] C_{hr} \right) \tag{10}$$

As special case, we assume $L_r = 0 \Rightarrow R(L) = 0$ and $K \to \infty$. This is the unconstrained classical multi-item inventory model.

4. STATISTICAL QUALITY CONTROL

The decision variables Q_r^* should be computed hose values are to be determined to minimize the total cost and to confirm that the production process is in control for three items (n = 3). The parameters of the model are shown in **TABLE 1**:

r	D_r	L_r	C_{or}	C_{pr}	C_{hr}	α_r
1	25 Units	8 Weak	\$ 200	\$ 10	\$ 0.8	\$ 1
2	25 Units	4 Weak	\$ 140	\$ 08	\$ 0.5	\$ 2
3	25 Units	2 Weak	\$ 100	\$ 05	\$ 0.3	\$ 3

TABLE 1

Assume that the standard deviation $\sigma = 6$ units/weak and k = 2.

The optimal results of the production batch quantity Q_r^*, the minimum annual total cost and subgroup ranges are given in **TABLE 2** for each values of λ:

Q_r^* \ λ	0.00000	- 00900	- 0.00910	- 0.00913	- 0.00914	- 0.00915
Q_1^*	114.0175425	103.0157507	102.9107932	102.8793685	102.868900	102.8584347
Q_2^*	121.6552506	107.1946045	107.0616911	107.0219134	107.008664	106.9954195
Q_3^*	132.9169136	116.5750556	116.4258872	116.3812482	116.366380	116.3515174
Min TC	732.8435354	744.1045510	744.7352781	744.7722876	744.6846213	744.7969535
R_i	018.8984711	013.5593049	013.5150940	013.5018797	013.4974800	013.493080

TABLE 2

In order to study the statistical quality control of the model, applying control limit (CL) approach when σ is unknown as:

The average of the subgroup ranges (CL) is:

$$\overline{R} = \frac{\sum_{i=1}^{6} R_i}{6} = \frac{86.4653097}{6} = 14.41088495$$

The standard deviation of the subgroup ranges is:

$$S_R = \sqrt{\frac{\sum_{i=1}^{6}\left(R_i - \overline{R}\right)^2}{6}} = \sqrt{4.027990753} = 2.006985489$$

The lower control limit is $LCL = \overline{R} - 3S_R = 8.389928483$, the average of the subgroup ranges is $\overline{R} = 14.41088495$ and the upper control limit is $UCL = \overline{R} + 3S_R = 20.43184142$. It is clear that $LCL < \overline{R} < UCL$. Therefore the production process is in control.

5. CONCLUSION

This paper is devoted to study statistical quality control for multi-item inventory model that considers order quantity as decision variable. An analytical solution of the EOQ model with varying leading time and one restriction is derived using Lagrangian approach. Finally, we used the optimal order quantity of each of the 3 items and subgroup ranges method to investigate quality control of the production process.

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