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INTUITIONISTIC FUZZY SUB COMMUTATIVE - IDEALS OF BCI-ALGEBRAS

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ABSTRACT

The notions of intuitionistic fuzzy sub commutative-ideals in BCI-algebras are introduced. The characterization properties of intuitionistic fuzzy sub commutative-ideals are obtained. We investigate the relations between intuitionistic fuzzy sub commutative-ideals and other intuitionistic fuzzy ideals, between intuitionistic fuzzy sub commutative-ideals and BCI-algebras and show that an intuitionistic fuzzy subset of a BCI-algebra is an intuitionistic fuzzy sub implicative – ideal if and only if it is both an intuitionistic fuzzy sub commutative – ideal and an intuitionistic fuzzy BCI-positive implicative ideal.

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1. INTRODUCTION

BCK-algebras and BCI-algebras are two classes of logical algebras, which were initiated by K. Iseki [3, 4]. The notion of fuzzy sets, invented by L.A. Zadeh [18], has been applied to many field. In 1991, O.G. Xi [17] applied it to BCK-algebras. Since then fuzzy BCI/BCK-algebras have been extensively investigated by several researchers. For BCK-algebras, Y.B. Jun et al. [6, 9] introduced the notions of fuzzy positive implicative ideals and fuzzy commutative ideals, J. Meng et al. [14] introduced the notion of fuzzy implicative ideals. For BCI-algebras, Y.B. Jun et al. [5, 7, 8] introduced the notion of fuzzy q-ideals (i.e., fuzzy quasi-associative ideals), fuzzy p-ideals and fuzzy BCI-commutative ideals, Y. L. Liu et al. [11, 12] introduced the notions of fuzzy BCI-positive implicative ideals, fuzzy BCI-implicative ideals and fuzzy a-ideals.

The idea of "intuitionistic fuzzy set" was first published by Atanassov [1, 2] as a generalization of the notion of fuzzy sets. After that many researchers considered the intuitionistic fuzzification of ideals and subalgebras in BCK/BCI-algebras. The aim of this paper is to introduce the notions of intuitionistic fuzzy sub commutative - ideals and discuss their properties. The characterization properties of intuitionistic fuzzy sub commutative - ideals are obtained. We investigate the relations between intuitionistic fuzzy sub commutative - ideals and other intuitionistic fuzzy ideals, between intuitionistic fuzzy sub commutative - ideals and BCI-algebras and show that an intuitionistic fuzzy subset of a BCI-algebra is an intuitionistic fuzzy sub implicative - ideal if and only if it is both an intuitionistic fuzzy sub commutative – ideal and an intuitionistic fuzzy BCI-positive implicative – ideal.

2. PRELIMINARIES

Let us recall that an algebra $(X, *, 0)$ of type (2, 0) is called a BCI-algebra if it satisfies the following conditions:

1. $((x*y)*(x*z))*(z*y)=0$,
2. $(x*(x*y))*y=0$,
3. $x*x=0$,
4. $x*y=0$ and $y*x=0$ imply $x=y$, for all $x, y, z \in X$.

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In a BCI-algebra, we can define a partial ordering "$\leq$" by $x \leq y$ if and only if $x * y = 0$. In a BCI-algebra X, the set $M = \{x \in X / 0 * x = 0\}$ is a subalgebra and is called the BCK-part of X. A BCI-algebra X is called proper if $X - M \neq \phi$. otherwise it is improper. Moreover, in a BCI-algebra the following conditions hold:

5. $(x * y) * z = (x * z) * y$,
6. $x * 0 = 0$,
7. $x \leq y$ imply $x * z \leq y * z$ and $z * y \leq z * x$,
8. $0 * (x * y) = (0 * x) * (0 * y)$,
9. $x * (x * (x * y)) = (x * y)$,
10. $(x * z) * (y * z) \leq x * y$.

An intuitionistic fuzzy set A in a non-empty set X is an object having the form

$$A = \{\langle x, \mu_A(x), \upsilon_A(x)\rangle \setminus x \in X\},$$

where the functions $\mu_A : X \to [0,1]$ and $\upsilon_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\upsilon_A(x)$) of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \upsilon_A(x) \leq 1$ for all $x \in X$.

Such defined objects are studied by many authors (see for Example two journals: 1. Fuzzy Sets and 2. Notes on Intuitionistic Fuzzy Sets) and have many interesting applications not only in mathematics (See Chapter 5 in the book [2]).

For the sake of simplicity, we shall use the symbol $A = \langle \mu_A, \upsilon_A \rangle$ for the intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \upsilon_A(x)\rangle \setminus x \in X\}$.

Throughout this paper X always means a BCI-algebra without any specification.

Definition 2.1: A non empty subset I of X is called an ideal of X if

(I_1) $0 \in I$,

(I_2) $x * y \in I$ and $y \in I$ imply $x \in I$.

Definition 2.2 [13]: A non empty subset I of X is called an positive implicative ideal (i.e., weakly positive implicative ideal) of X if it satisfies (I_1) and (I_3) $((x * z) * z) * (y * z) \in I$ and $y \in I$ imply $x * z \in I$.

Definition 2.3 [10]: A non empty subset I of X is called an sub implicative - ideal of X if it satisfies (I_1) and (I_4) $((x * (x * y)) * (y * x)) * z \in I$ and $z \in I$ imply $y * (y * x) \in I$.

Definition 2.4 [10]: A non empty subset I of X is called an sub commutative - ideal of X if it satisfies (I_1) and (I_5) $(y * (y * (x * (x * y)))) * z \in I$ and $z \in I$ imply $x * (x * y) \in I$.

Definition 2.5: An IFS $A = \langle \mu_A, \upsilon_A \rangle$ in a BCI-algebra X is called an intuitionistic fuzzy ideal of X if it satisfies:

$(F1)\, \mu_A(0) \geq \mu_A(x)\ \&\ \upsilon_A(0) \leq \upsilon_A(x)$,

$(F2)\, \mu_A(x) \geq \min\{\mu_A(x * y), \mu_A(y)\}$,

$(F3)\, \upsilon_A(x) \leq \max\{\upsilon_A(x * y), \upsilon_A(y)\}$, *for all* $x, y \in X$.

Definition 2.6: An IFS $A = \langle \mu_A, \upsilon_A \rangle$ in a BCI-algebra X is called an intuitionistic fuzzy positive implicative ideal of X if it satisfies $(F1)$ and

$(F4)\, \mu_A(x * z) \geq \min\{\mu_A(((x * z) * z) * (y * z)), \mu_A(y)\}$,

$(F5)\, \upsilon_A(x * z) \leq \max\{\upsilon_A(((x * z) * z) * (y * z)), \upsilon_A(y)\}$, *for all* $x, y, z \in X$.

Definition 2.7: An IFS $A=\langle \mu_A, \upsilon_A \rangle$ in a BCI-algebra X is called an intuitionistic fuzzy p-ideal of X if it satisfies $(F1)$ and

$$(F6)\ \mu_A(x) \geq \min\{\mu_A((x*z)*(y*z)), \mu_A(y)\},$$
$$(F7)\ \upsilon_A(x) \leq \max\{\upsilon_A((x*z)*(y*z)), \upsilon_A(y)\},\ \textit{for all}\ x, y, z \in X.$$

Definition 2.8: Let $A=\langle \mu_A, \upsilon_A \rangle$ be an intuitionistic fuzzy set of a BCI-algebra X. For $t, s \in [0,1]$, the set $U(x;t)=\{x \in X / \mu_A(x) \geq t\}$ is called the upper t-level of A and the set $L(x;s)=\{x \in X / \upsilon_A(x) \leq s\}$ is called the lower s-level of A.

Theorem 2.9: Every intuitionistic fuzzy ideal A of X, μ_A is order reversing and υ_A is order preserving.

Theorem 2.10: Let A be an intuitionistic fuzzy ideal of X. Then $x*y \leq z$ implies

$$\mu_A(x) \geq \min\{\mu_A(y), \mu_A(z)\} \text{ and } \upsilon_A(x) \leq \max\{\upsilon_A(y), \upsilon_A(z)\} \text{ for all } x, y, z \in X.$$

3. INTUITIONISTIC FUZZY SUB COMMUTATIVE - IDEALS OF BCI-ALGEBRAS

Definition 3.1: An intuitionistic fuzzy subset A of X is called an intuitionistic fuzzy sub commutative - ideal (briefly, IFSC-ideal) of X if it satisfies $(F1)$ and

$$(F8)\ \mu_A(x*(x*y)) \geq \min\{\mu_A((y*(y*(x*(x*y))))*z), \mu_A(z)\},$$
$$(F9)\ \upsilon_A(x*(x*y)) \leq \max\{\upsilon_A((y*(y*(x*(x*y))))*z), \upsilon_A(z)\}, \text{ for all } x, y, z \in X.$$

Example 3.2: Let $X=\{0, 1, 2, 3\}$ be a BCI-algebra with Cayley table as follows:

$*$	0	1	2	3
0	0	0	0	0
1	1	0	0	1
2	2	1	0	2
3	3	3	3	0

Define $A: X \to [0,1]$ by $\mu_A(0)=\mu_A(3)=0.8$ and $\mu_A(1)=\mu_A(2)=0.2;\ \upsilon_A(0)=\upsilon_A(3)=0.1$ and $\upsilon_A(1)=\upsilon_A(2)=0.7$. It is easy to check that A is an IFSC-ideal of X.

Now we give the characterization of IFSC-ideals of X.

Theorem 3.3: Let A be an intuitionistic fuzzy ideal of X. Then the following are equivalent:

(i) A is an IFSC-ideal of X,

(ii) $\mu_A(x*(x*y)) \geq \mu_A(y*(y*(x*(x*y))))$, $\upsilon_A(x*(x*y)) \leq \upsilon_A(y*(y*(x*(x*y))))$ for all $x, y \in X$.

(iii) $\mu_A(x*(x*y)) = \mu_A(y*(y*(x*(x*y))))$, $\upsilon_A(x*(x*y)) = \upsilon_A(y*(y*(x*(x*y))))$ for all $x, y \in X$.

(iv) if $x \leq y$, then $\mu_A(x) = \mu_A(y*(y*x))$
$\upsilon_A(x) = \upsilon_A(y*(y*x))$ for all $x, y \in X$.

(v) if $x \leq y$, then $\mu_A(x) \geq \mu_A(y*(y*x))$
$\upsilon_A(x) \leq \upsilon_A(y*(y*x))$ for all $x, y \in X$.

Proof. (i) Implies (ii) Suppose that A is an IFSC-ideal of X. by $(F1)$, $(F8)$ and $(F9)$ we have

$$\mu_A(x*(x*y)) \geq \min\{\mu_A((y*(y*(x*(x*y))))*0), \mu_A(0)\}$$
$$= \mu_A(y*(y*(x*(x*y))))$$

and $$\upsilon_A(x*(x*y)) \leq \max\{\upsilon_A((y*(y*(x*(x*y))))*0), \upsilon_A(0)\}$$
$$= \upsilon_A(y*(y*(x*(x*y)))).$$

(ii) implies (iii) Since $y*(y*(x*(x*y))) \leq x*(x*y)$, we have

$$\mu_A(y*(y*(x*(x*y)))) \geq \mu_A(x*(x*y))$$

and $$\upsilon_A(y*(y*(x*(x*y)))) \leq \upsilon_A(x*(x*y)).$$

Combining (ii) we obtain

$$\mu_A(x*(x*y)) = \mu_A(y*(y*(x*(x*y))))$$

and $$\upsilon_A(x*(x*y)) = \upsilon_A(y*(y*(x*(x*y)))).$$

(iii) implies (iv) If $x \leq y$, then $x*y=0$. By (iii) we have

$$\mu_A(x) = \mu_A(y*(y*x))$$

and $$\upsilon_A(x) = \upsilon_A(y*(y*x)).$$

(iv) implies (v) Trivial.

(v) implies (i) Since $x*(x*y) \leq y$, by (v) we have

$$\mu_A(x*(x*y)) \geq \mu_A(y*(y*(x*(x*y))))$$
$$\geq \min\{\mu_A((y*(y*(x*(x*y))))*z), \mu_A(z)\}$$

and $$\upsilon_A(x*(x*y)) \leq \upsilon_A(y*(y*(x*(x*y))))$$
$$\leq \max\{\upsilon_A((y*(y*(x*(x*y))))*z), \upsilon_A(z)\}.$$

Hence A is an IFSC-ideal of X, completing the proof.

Next we investigate the relation between IFSC-ideals and other intuitionistic fuzzy ideals.

Lemma 3.4: An intuitionistic fuzzy ideal A of X is an intuitionistic fuzzy p-ideal of X if and only if

$$\mu_A(x) \geq \mu_A(0*(0*x)),$$
$$\upsilon_A(x) \leq \upsilon_A(0*(0*x)), \text{ for all } x \in X.$$

Theorem 3.5: Any intuitionistic fuzzy p-ideal is an IFSC-ideal, but the converse is not true.

Proof. Let A be an intuitionistic fuzzy p-ideal of X. Then A is an intuitionistic fuzzy ideal. Because

$$[0*(0*(x*(x*y)))]*[y*(y*(x*(x*y)))] = [0*(y*(y*(x*(x*y))))]*[0*(x*(x*y))]$$
$$= [(0*y)*((0*y)*(0*(x*(x*y))))]*[0*(x*(x*y))]$$
$$\leq [0*(x*(x*y))]*[0*(x*(x*y))] = 0,$$

We have

$0*(0*(x*(x*y))) \leq y*(y*(x*(x*y)))$, and so $\mu_A(0*(0*(x*(x*y)))) \geq \mu_A(y*(y*(x*(x*y))))$ and $\upsilon_A(0*(0*(x*(x*y)))) \leq \upsilon_A(y*(y*(x*(x*y))))$.

By Lemma 3.4,

$\mu_A(x*(x*y)) \geq \mu_A(y*(y*(x*(x*y))))$ and $\upsilon_A(x*(x*y)) \leq \upsilon_A(y*(y*(x*(x*y))))$.

Hence A is an IFSC-ideal of X as Theorem 3.3 (ii).

To show the last half part, we see Example 3.2. It has known that A is an IFSC-ideal of X. But it is not an intuitionistic fuzzy p-ideal of X since

$$\mu_A(x) = 0.2 < 0.8 = \mu_A(0*(0*2))$$

and $$\upsilon_A(x) = 0.7 > 0.1 = \upsilon_A(0*(0*2)).$$

This completes the proof.

Definition 3.6: An intuitionistic fuzzy subset A of X is called an intuitionistic fuzzy sub implicative-ideal (briefly, IFSI-ideal) of X if it satisfies $(F1)$ and

$$(F10)\ \mu_A(y*(y*x)) \geq \min\{\mu_A(((x*(x*y))*(y*x))*z), \mu_A(z)\},$$
$$(F11)\ \upsilon_A(y*(y*x)) \leq \max\{\upsilon_A(((x*(x*y))*(y*x))*z), \upsilon_A(z)\},$$ for all $x, y, z \in X$.

Lemma 3.7: Let A be an intuitionistic fuzzy ideal of X. Then the following are equivalent:

(i) A is an IFSI-ideal of X,

(ii) $\mu_A(y*(y*x)) \geq \mu_A((x*(x*y))*(y*x))$,
$\upsilon_A(y*(y*x)) \leq \upsilon_A((x*(x*y))*(y*x))$, for all $x, y \in X$.

(iii) $\mu_A(y*(y*x)) = \mu_A((x*(x*y))*(y*x))$
$\upsilon_A(y*(y*x)) = \upsilon_A((x*(x*y))*(y*x))$, for all $x, y \in X$.

Theorem 3.8: Any IFSI-ideal is an intuitionistic fuzzy BCI-positive implicative ideal, but the converse is not true.

Theorem 3.9: Any IFSI-ideal is an IFSC-ideal, but the converse is not true.

Theorem 3.10: Any IFSC-ideal is an intuitionistic fuzzy ideal, but the converse does not hold.

Lemma 3.11: An intuitionistic fuzzy ideal A of X is an intuitionistic fuzzy BCI-positive implicative ideal of X if and only if

$$\mu_A(x*y) \geq \mu_A(((x*y)*y)*(0*y)),$$
$$\upsilon_A(x*y) \leq \upsilon_A(((x*y)*y)*(0*y)),$$ for all $x, y \in X$.

Now we give the characterization of intuitionistic fuzzy BCI-positive implicative ideals of X, which is needed in the sequel.

Theorem 3.12: An intuitionistic fuzzy ideal A of X is an intuitionistic fuzzy BCI-positive implicative ideal if and only if for all $x, y \in X$,

$$(F12)\ \mu_A(x*(x*(y*(y*x)))) \geq \mu_A((x*(x*y))*(y*x)),$$
$$(F13)\ \upsilon_A(x*(x*(y*(y*x)))) \leq \upsilon_A((x*(x*y))*(y*x)).$$

Proof. Let A be an intuitionistic fuzzy ideal satisfying $(F12)$ and $(F13)$. Since

$$((x*y)*((x*y)*x))*(x*(x*y)) = ((x*(x*(x*y)))*y)*((x*y)*x)$$
$$= ((x*y)*y)*(0*y),$$

We have

$$\mu_A[((x*y)*((x*y)*x))*(x*(x*y))] = \mu_A(((x*y)*y)*(0*y))$$

and $$\upsilon_A[((x*y)*((x*y)*x))*(x*(x*y))] = \upsilon_A(((x*y)*y)*(0*y)).$$

Substituting $x*y$ for x and x for y in $(F12)$ and $(F13)$, we have

$$\mu_A[(x*y)*((x*y)*(x*(x*(x*y))))] \geq \mu_A(((x*y)*y)*(0*y))$$

and $$\upsilon_A[(x*y)*((x*y)*(x*(x*(x*y))))] \leq \upsilon_A(((x*y)*y)*(0*y)).$$

Since

$$(x*y)*((x*y)*(x*(x*(x*y)))) = (x*y)*((x*y)*(x*y)) = x*y$$

We have

$$\mu_A(x*y) \geq \mu_A(((x*y)*y)*(0*y))$$

and $$\upsilon_A(x*y) \leq \upsilon_A(((x*y)*y)*(0*y)).$$

By Lemma 3.11, A is an intuitionistic fuzzy BCI-positive implicative ideal of X.

Conversely, let A be an intuitionistic fuzzy BCI-positive implicative ideal of X. Since

$$\begin{aligned}[((y*(y*x))*(y*x))*(x*y)]*[(x*(x*y))*(y*x)] &= [((y*(y*x))*(x*y))*(y*x)]*[(x*(x*y))*(y*x)] \\ &\leq [(y*(y*x))*(x*y)]*(x*(x*y)) \\ &\leq (y*(y*x))*x = 0,\end{aligned}$$

we have

$$\mu_A[((y*(y*x))*(y*x))*(x*y)] \geq \mu_A[(x*(x*y))*(y*x)]$$

and $$\upsilon_A[((y*(y*x))*(y*x))*(x*y)] \leq \upsilon_A[(x*(x*y))*(y*x)].$$

Let $s = y*x$ in $((y*(y*x))*(y*x))*(x*y)$. Then

(a_1) $\mu_A[((y*s)*s)*(x*y)] \geq \mu_A((x*(x*y))*(y*x))$

(a_2) $\upsilon_A[((y*s)*s)*(x*y)] \leq \upsilon_A((x*(x*y))*(y*x)).$

Let $t = x*(y*(y*x)) = x*(y*s)$. Because

$$\begin{aligned}[(((y*t)*s)*s)*(0*s)]*[((y*s)*s)*(x*y)] &= [(((y*s)*s)*(0*s))*(((y*s)*s)*(x*y))]*t \\ &\leq ((x*y)*(0*s))*t \\ &= ((x*t)*y)*(0*s) \\ &= ((x*(x*(y*s)))*y)*(0*s) \\ &\leq ((y*s)*y)*(0*s) \\ &= (0*s)*(0*s) = 0,\end{aligned}$$

we have

$$\mu_A[(((y*t)*s)*s)*(0*s)] \geq \mu_A[((y*s)*s)*(x*y)]$$

and $$\upsilon_A[(((y*t)*s)*s)*(0*s)] \leq \upsilon_A[((y*s)*s)*(x*y)].$$

By Lemma 3.11, we have

(b_1) $\mu_A((y*t)*s) \geq \mu_A[((y*s)*s)*(x*y)]$

(b_2) $\upsilon_A((y*t)*s) \leq \upsilon_A[((y*s)*s)*(x*y)].$

Since

$$\begin{aligned}[((x*t)*t)*(0*t)]*((y*t)*s) &= [((x*t)*((y*s)*t)]*(0*t) \\ &\leq ((x*t)*(y*s))*(0*t) \\ &= [(x*(x*(y*s)))*(y*s)]*(0*t) \\ &\leq ((y*s)*(y*s))*(0*t) \\ &= 0*(0*t),\end{aligned}$$

and

$$\begin{aligned}0*t &= 0*(x*(y*(y*x))) \\ &\leq 0*(x*x) = 0,\end{aligned}$$

We have $0*(0*t)=0,$ and so

$$\mu_A[((x*t)*t)*(0*t)] \geq \mu_A((y*t)*s)$$

and $$\upsilon_A[((x*t)*t)*(0*t)] \leq \upsilon_A((y*t)*s).$$

By Lemma 3.11 again, we have

(c_1) $\mu_A(x*t) \geq \mu_A((y*t)*s)$

(c_2) $\upsilon_A(x*t) \leq \upsilon_A((y*t)*s).$

Combining (a_1), (a_2), (b_1), (b_2) and (c_1), (c_2) we obtain

$$\mu_A(x*t) \geq \mu_A((x*(x*y))*(y*x))$$

and $$\upsilon_A(x*t) \leq \upsilon_A((x*(x*y))*(y*x)),$$

i.e., $$\mu_A(x*(x*(y*(y*x)))) \geq \mu_A((x*(x*y))*(y*x)),$$

$$\upsilon_A(x*(x*(y*(y*x)))) \leq \upsilon_A((x*(x*y))*(y*x)).$$

The proof is complete.

The following theorem shows the close relations among IFSI-ideals, IFSC-ideals and intuitionistic fuzzy BCI-positive implicative ideals.

Theorem 3.13: Let A be an intuitionistic fuzzy subset of X. Then A is an IFSI-ideal if and only if it is both an IFSC-ideal and an intuitionistic fuzzy BCI-positive implicative ideal.

Proof: If A is an IFSI-ideal, by Theorem 3.8 and 3.9, A is both an IFSC-ideal and an intuitionistic fuzzy BCI-positive implicative ideal. Conversely, if A is both an IFSC-ideal and an intuitionistic fuzzy BCI-positive implicative ideal, by Theorem 3.10, A is an intuitionistic fuzzy ideal. For any $x, y \in X$, by Theorem 3.3 (ii) and Theorem 3.12, we have

$$\mu_A(y*(y*x)) \geq \mu_A(x*(x*(y*(y*x)))) \geq \mu_A((x*(x*y))*(y*x))$$

and $$\upsilon_A(y*(y*x)) \leq \upsilon_A(x*(x*(y*(y*x)))) \leq \upsilon_A((x*(x*y))*(y*x)).$$

Hence A is an IFSI-ideal of X as Lemma 3.7(ii). The proof is complete.

Next we investigate the relation between IFSC-ideals and BCI-algebras.

Definition 3.14 [16]: A BCI-algebra is said to be commutative if and only if $x*(x*y)=(y*(y*(x*(x*y)))).$

If A is an intuitionistic fuzzy ideal of X, let $\mu_{A^*}=\mu_{\mu_A(0)}=\{x \in X / \mu_A(x)=\mu_A(0)\},$
$\upsilon_{A^*}=\upsilon_{\upsilon_A(0)}=\{x \in X / \upsilon_A(x)=\upsilon_A(0)\}$ and $B(X)=\{x \in X / 0 \leq x\}.$

Theorem 3.15: Let A be an intuitionistic fuzzy ideal of X. If $\langle X/\mu_A, X/\upsilon_A \rangle$ is an commutative BCI-algebra, then A is an IFSC - ideal of X. Conversely, if A is an IFSC - ideal with $\langle \mu_{A^*}, \upsilon_{A^*} \rangle \supseteq B(X)$, then $\langle X/\mu_A, X/\upsilon_A \rangle$ is a commutative BCI-algebra.

Proof. If $\langle X/\mu_A, X/\upsilon_A \rangle$ is an commutative BCI-algebra, then for any $x, y \in X$, we have

$$(\mu_y*(\mu_y*(\mu_x*(\mu_x*\mu_y)))) = (\mu_x*(\mu_x*\mu_y))$$

and $$(\upsilon_y*(\upsilon_y*(\upsilon_x*(\upsilon_x*\upsilon_y)))) = (\upsilon_x*(\upsilon_x*\upsilon_y)).$$

Namely $\mu_{(y*(y*(x*(x*y))))} = \mu_{(x*(x*y))}$ and $\upsilon_{(y*(y*(x*(x*y))))} = \upsilon_{(x*(x*y))}$.

Hence

$$\mu_A[(x*(x*y))*((y*(y*(x*(x*y))))] = \mu_A(0)$$

and $$\upsilon_A[(x*(x*y))*((y*(y*(x*(x*y))))] = \upsilon_A(0).$$

Thus $$\mu_A(x*(x*y)) \geq \min\{\mu_A((x*(x*y))*((y*(y*(x*(x*y)))), \mu_A((y*(y*(x*(x*y))))\} = \mu_A((y*(y*(x*(x*y)))),$$

and $$\upsilon_A(x*(x*y)) \leq \max\{\upsilon_A((x*(x*y))*((y*(y*(x*(x*y)))), \upsilon_A((y*(y*(x*(x*y))))\} = \upsilon_A((y*(y*(x*(x*y)))).$$

Therefore A is an IFSC-ideal of X.

Conversely, assume that A is an IFSC-ideal with $\langle \mu_{A^*}, \upsilon_{A^*} \rangle \supseteq B(X)$. For any $x, y \in X$, since

$$(x*(x*y))*((y*(y*(x*(x*y)))) \geq (x*(x*y))*(x*(x*y)) = 0,$$

we have $(x*(x*y))*((y*(y*(x*(x*y)))) \in B(X) \subseteq \langle \mu_{A^*}, \upsilon_{A^*} \rangle$, and so

$$\mu_A[(x*(x*y))*((y*(y*(x*(x*y))))] = \mu_A(0)$$

and $$\upsilon_A[(x*(x*y))*((y*(y*(x*(x*y))))] = \upsilon_A(0).$$

On the other hand,

$$(y*(y*(x*(x*y))))*(x*(x*y)) \leq (x*(x*y))*(x*(x*y)) = 0,$$

so

$$\mu_A[(y*(y*(x*(x*y))))*(x*(x*y))] = \mu_A(0)$$

and $$\upsilon_A[(y*(y*(x*(x*y))))*(x*(x*y))] = \upsilon_A(0).$$

Thus we obtain $\mu_{(y*(y*(x*(x*y))))} = \mu_{(x*(x*y))}$ and $\upsilon_{(y*(y*(x*(x*y))))} = \upsilon_{(x*(x*y))}$.

Namely $(\mu_y*(\mu_y*(\mu_x*(\mu_x*\mu_y)))) = (\mu_x*(\mu_x*\mu_y))$

and $$(\upsilon_y*(\upsilon_y*(\upsilon_x*(\upsilon_x*\upsilon_y)))) = (\upsilon_x*(\upsilon_x*\upsilon_y)).$$

It means that $\langle X/\mu_A, X/\upsilon_A \rangle$ is an commutative BCI-algebra. The proof is complete.

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