

A NEW NOTION OF OPEN SETS IN TOPOLOGICAL SPACES

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ABSTRACT

The determination of this paper is to introduce the new family of sets, namely S_g^* -open sets and S_g^* -closed sets. Further we define S_g^* -interior and S_g^* -closure and discuss its properties. Additionally we relate S_g^* -open sets and S_g^* -closed sets with some other sets.

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1. INTRODUCTION

Throughout this paper, we denote a topological space by (X, τ) . For a subset A of (X, τ) , the closure and interior of A with respect to τ are denoted by $Cl(A)$ and $Int(A)$ respectively. The concept of semi-open sets [3] and generalized closed (briefly g -closed) sets [4] were introduced by Norman Levine in 1963 and 1970 respectively. After the works of Levine, in the year 1987, P. Battacharyya and B.K. Lahiri [1] introduced the concept of semi-generalized closed sets with the help of semi-openness. Recently, A. Robert and S. Pious Missier [6] have introduced the concept of semi*-open sets and studied their properties.

In this direction, we shall introduce a new family of sets called S_g^* -open sets, using the semi-generalized closure operator. This definition enables us to obtain some more results. Further we establish the relationship between this S_g^* -open sets and some nearly open sets. Besides we prove the class of S_g^* -open sets are in between the class of semi-open sets due to Levine and the class of open sets. The notion of S_g^* -closed sets are also introduced. In addition to this, we define S_g^* -interior and S_g^* -closure of a subset and discuss some of its properties.

2. PRELIMINARIES

Definition 2.1: [3]

- (i) A subset A of a topological space (X, τ) is **semi-open** if there is an open set U in X such that $U \subseteq A \subseteq Cl(U)$ or equivalently if $A \subseteq Cl(Int(A))$.
- (ii) The complement of semi-open set is called **semi-closed set**.
- (iii) The family of all semi-open sets in (X, τ) is denoted by $SO(X, \tau)$.
- (iv) The **semi-interior** (briefly $sInt(A)$) of A is defined as the union of all semi-open sets of X contained in A .
- (v) The **semi-closure** (briefly $sCl(A)$) of A is defined as the intersection of all semi-open sets of X containing A .

Definition 2.2:

- (i) A subset A of a space X is **semi-generalized closed** [1] (briefly Sg -closed) if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X .
- (ii) A subset A of a space X is **semi-star generalized closed** [2] (briefly S^*g -closed) if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X .
- (iii) The complement of semi-generalized closed set (resp. semi-star generalized closed) is called **semi-generalized open** [1] (resp. semi star-generalized open [2]). It is denoted by Sg -open (resp. S^*g -open).
- (iv) The **semi-generalized interior** [2] (briefly $sInt^*(A)$) of A is defined as the union of all Sg -open sets of X contained in A .
- (iii) If A is a subset of a space X , the **semi-generalized closure** [5] (briefly $sCl^*(A)$) of A is defined as the intersection of all Sg -closed sets in X containing A .

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Results 2.3:[5] Let A and B be two subsets of a space (X, τ)

- (i) $A \subseteq sCl^*(A) \subseteq sCl(A) \subseteq Cl(A)$
- (ii) $sCl^*(\phi) = \phi$ and $sCl^*(X) = X$
- (iii) $sCl^*(A \cup B) \supseteq sCl^*(A) \cup sCl^*(B)$
- (iv) $sCl^*(sCl^*(A)) = sCl^*(A)$

Definition 2.4: [6] A subset A of a topological space (X, τ) is called a **semi*-open set** (briefly S^* -open) if there is an open set U in X such that $U \subseteq A \subseteq Cl^*(U)$.

3. S_g^* -OPEN SETS

Definition 3.1: A subset A of a topological space (X, τ) is called a **S_g^* -open set** if there is an open set U in X such that $U \subseteq A \subseteq sCl^*(U)$. The collection of all S_g^* -open sets in (X, τ) is denoted by $S_g^*O(X, \tau)$.

Theorem 3.2: A subset A of a topological space (X, τ) is S_g^* -open set iff $A \subseteq sCl^*(Int(A))$.

Proof: Necessity. If A is S_g^* -open, then there exists an open set $U \subseteq A \subseteq sCl^*(U)$. Let $U = Int U \subseteq Int(A)$. Then $sCl^*U = sCl^*(Int(U)) \subseteq sCl^*(Int(A))$. Hence $A \subseteq sCl^*(Int(A))$.

Sufficiency. Let $A \subseteq sCl^*(Int(A))$. Then for $U = Int(A)$, we get U is an open set in X such that $U \subseteq A \subseteq sCl^*(U)$.

Theorem 3.3: Let $\{A_\alpha\}$ be a collection of S_g^* -open sets in a topological space X. Then $\cup A_\alpha$ is S_g^* -open.

Proof: A_α is S_g^* -open implies for each α , there is an open set U_α in X such that $U_\alpha \subseteq A_\alpha \subseteq sCl^*(U_\alpha)$. Then $\cup U_\alpha \subseteq \cup A_\alpha \subseteq \cup sCl^*(U_\alpha) \subseteq sCl^*(\cup U_\alpha)$. Since $\cup U_\alpha$ is open, $\cup A_\alpha$ is S_g^* -open.

Remark 3.4: The problem of determining the intersection of S_g^* -open sets is S_g^* -open remains open.

Theorem 3.5: Every open set is S_g^* -open

Proof: Let A be an open set in X. Then $Int(A) = A$. Hence $A \subseteq sCl^*(A) = sCl^*(Int(A))$. Therefore by the necessary and sufficient condition of S_g^* -open, A is S_g^* -open.

Remark 3.6: The converse of Theorem 3.5 need not be true as can be seen from the following example.

Example 3.7: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}\}$. In the topological space (X, τ) , the subsets $\{a, b\}$ and $\{a, c\}$ are S_g^* -open but not open.

Theorem 3.8: If a subset A is S_g^* -open and B is open, then so is $A \cup B$.

Proof: Follows from Theorem 3.5 and Theorem 3.3.

Theorem 3.9: If A is S_g^* -open in X and B is open in X, then $A \cap B$ is S_g^* -open.

Proof: Since A is S_g^* -open in X, there exists an open set U in X such that $U \subseteq A \subseteq sCl^*(U)$. Now B is open implies $U \cap B \subseteq A \cap B \subseteq sCl^*(U) \cap B \subseteq sCl^*(U \cap B)$. Therefore $A \cap B$ is S_g^* -open.

Theorem 3.10: Let $A \subseteq B \subseteq sCl^*(A)$. If A is S_g^* -open in the topological space X, then B is S_g^* -open.

Proof: A is S_g^* -open in X \Rightarrow There exists an open set U in X such that $U \subseteq A \subseteq sCl^*(U)$. Therefore $U \subseteq B \Rightarrow sCl^*(U) \subseteq sCl^*(B)$.

Hence $U \subseteq B \subseteq sCl^*(U)$ and B is S_g^* -open.

Theorem 3.11: A subset A of X is S_g^* -open if and only if A contains a S_g^* -open set about each of its points.

Proof: Necessity: Obvious.

Sufficiency: Let $x \in A$. Then there is a S_g^* -open set U_x containing x such that $U_x \subseteq A$. Then we have $\cup \{U_x : x \in A\} = A$. By Theorem 3.3, A is S_g^* -open.

Theorem 3.12: Every S_g^* -open set is semi-open.

Proof: Let A be a S_g^* -open set. Then there exists an open set U in X such that $U \subseteq A \subseteq sCl^*(U)$. Note that $sCl^*(U) \subseteq sCl(U) \subseteq Cl(U)$. Therefore $U \subseteq A \subseteq Cl(U)$. Hence A is semi-open.

Remark 3.13: Converse of the above Theorem 3.12 is not true as can be seen from the following example.

Example 3.14: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{c\}, \{a,c\}\}$. In this space, $SO(X, \tau) = \{X, \phi, \{a\}, \{c\}, \{a,c\}, \{a,b\}, \{b,c\}\}$ and $S_g^*O(X, \tau) = \{X, \phi, \{a\}, \{c\}, \{a,c\}\}$. Here the subsets $\{a, b\}$ and $\{b, c\}$ are semi-open but not S_g^* -open.

Theorem 3.15: Every S_g^* -open set is semi-generalised open.

Proof: It follows from two facts that every S_g^* -open set is semi-open and every semi open set is semi-generalised open [1].

Remark 3.16: Converse of the above Theorem 3.15 is not true as can be seen from the following example.

Example 3.17: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a, b\}\}$. In this topological space (X, τ) , $SgO(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $S_g^*O(X, \tau) = \{X, \phi, \{a, b\}\}$. It shows that the subsets $\{a\}$ and $\{b\}$ are semi-generalised open but not S_g^* -open.

Theorem 3.18: For any topological space (X, τ) , $\tau \subseteq S_g^*O(X, \tau) \subseteq SO(X, \tau) \subseteq SgO(X, \tau)$

Proof: It follows from Theorem 3.5, Theorem 3.12 and Theorem 3.15.

Remark 3.19: S_g^* -open sets and S^*g -open sets are independent as shown by the following examples.

Example 3.20: Let $X = \{a, b, c, d, e\}$ and $\tau = \{X, \phi, \{a\}\}$. $S^*gO(X, \tau) = \{X, \phi, \{a\}\}$. $S_g^*O(X, \tau) = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, e\}, \{a, b, c\}, \{a, b, d\}, \{a, b, e\}, \{a, c, d\}, \{a, c, e\}, \{a, d, e\}, \{a, b, c, d\}, \{a, b, d, e\}, \{a, c, d, e\}, \{a, b, c, e\}\}$. Here the subsets $\{a, b\}, \{a, c\}, \{a, d\}, \{a, e\}, \{a, b, c\}, \{a, b, d\}, \{a, b, e\}, \{a, c, d\}, \{a, c, e\}, \{a, d, e\}, \{a, b, c, d\}, \{a, b, d, e\}, \{a, c, d, e\}$ and $\{a, b, c, e\}$ are S_g^* -open but not S^*g -open.

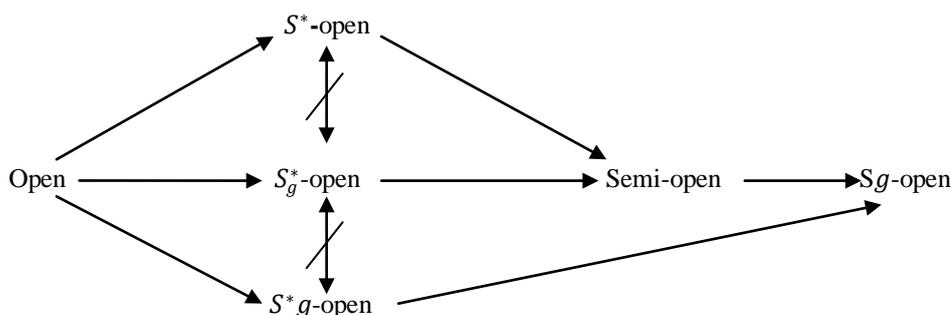
Example 3.21: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a, b\}\}$. $S^*gO(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. $S_g^*O(X, \tau) = \{X, \phi, \{a, b\}\}$. Here the subsets $\{a\}$ and $\{b\}$ are S^*g -open but not S_g^* -open.

Remark 3.22: S_g^* -open sets and S^* -open sets are independent as shown by the following examples.

Example 3.23: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}\} = S^*O(X, \tau)$. $S_g^*O(X, \tau) = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$. The subsets $\{a, b\}$ and $\{a, c\}$ are S_g^* -open but not S^* -open.

Example 3.24: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}\} = S_g^*O(X, \tau)$. $S^*O(X, \tau) = \{X, \phi, \{a\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. The subsets $\{a, b\}$ and $\{b, c\}$ are S^* -open but not S_g^* -open.

Remark 3.25: From the above Theorems and Remarks, we have the following diagram.



Definition 3.26: The S_g^* -interior of A is defined as the union of all S_g^* -open sets of X contained in A. It is denoted by $S_g^*Int(A)$.

Definition 3.27: Let A be a subset of X. A point $x \in X$ is called S_g^* -interior point of A if A contains a S_g^* -open set containing x.

Theorem 3.28: If A is any subset of X, $S_g^*Int(A)$ is S_g^* -open. In fact $s_g^*Int(A)$ is the largest S_g^* -open set contained in A.

Proof: Follows from Definition 3.26 and Theorem 3.3.

Theorem 3.29: A subset A of X is S_g^* -open if and only if $s_g^*Int(A)=A$.

Proof: If A is S_g^* -open then $s_g^*Int(A)=A$ is obvious. Conversely, let $s_g^*Int(A)=A$. By Theorem 3.28, $s_g^*Int(A)$ is S_g^* -open and hence A is S_g^* -open.

Theorem 3.30: If A is a subset of X, then $s_g^*Int(A)$ is the set of all S_g^* -interior points of A.

Proof: If $x \in s_g^*Int(A)$, then x belongs to some S_g^* -open subset U of A. That is, x is a S_g^* -interior point of A.

Corollary 3.31: A subset A of X is S_g^* -open if and only if every point of A is a S_g^* -interior point of A.

Proof: It follows from Theorem 3.29 and Theorem 3.30.

Theorem 3.32: If A and B are subsets of a topological space (X, τ) , then the following results hold:

- (i) $s_g^*Int(\phi)=\phi$.
- (ii) $s_g^*Int(X)=X$.
- (iii) $s_g^*Int(A) \subseteq A$.
- (iv) $A \subseteq B \implies s_g^*Int(A) \subseteq s_g^*Int(B)$.
- (v) $Int(A) \subseteq s_g^*Int(A) \subseteq sInt(A) \subseteq sInt^*(A) \subseteq A$.
- (vi) $s_g^*Int(A \cup B) \supseteq s_g^*Int(A) \cup s_g^*Int(B)$.
- (vii) $s_g^*Int(A \cap B) \subseteq s_g^*Int(A) \cap s_g^*Int(B)$.
- (viii) $s_g^*Int(s_g^*Int(A)) = s_g^*Int(A)$.
- (ix) $Int(s_g^*Int(A)) = Int(A)$.
- (x) $s_g^*Int(Int(A)) = Int(A)$.

Proof: (i), (ii), (iii) and (iv) follows from Definition 3.26.

(v) follows from Theorem 3.18.

(vi) and (vii) follow from (iv) above.

(viii) and (ix) follows from Theorem 3.28 and Theorem 3.29.

(x) $Int(A)$ is open which implies $Int(A)$ is S_g^* -open. Now by Theorem 3.29, $s_g^*Int(Int(A)) = Int(A)$.

4. S_g^* -CLOSED SETS

Definition 4.1: A subset A of a topological space (X, τ) is called a S_g^* -closed set if $X \setminus A$ is S_g^* -open. The collection of all S_g^* -closed sets in (X, τ) is denoted by $S_g^*C(X, \tau)$.

Theorem 4.2: A subset A of a topological space (X, τ) is S_g^* -closed set if and only if there exists a closed set F in X such that $sInt^*(F) \subseteq A \subseteq F$.

Proof: Necessity. A is S_g^* -closed $\implies X \setminus A$ is S_g^* -open. By Definition 3.1, there exists an open set U in X such that $U \subseteq X \setminus A \subseteq sCl^*(U)$. This implies $X \setminus U \supseteq A \supseteq X \setminus sCl^*(U)$. Since $X \setminus sCl^*(U) = sInt^*(X \setminus U)$, $sInt^*(X \setminus U) \subseteq A \subseteq X \setminus U$ where $X \setminus U$ is a closed set in X.

Sufficiency. Suppose there exists a closed set F in X such that $sInt^*(F) \subseteq A \subseteq F$. Then $X \setminus sInt^*(F) \supseteq X \setminus A \supseteq X \setminus F$. Note that $X \setminus sInt^*(F) = sCl^*(X \setminus F)$. Hence $X \setminus F \subseteq X \setminus A \subseteq sCl^*(X \setminus F)$ where $X \setminus F$ is an open set in X. This implies $X \setminus A$ is a S_g^* -open set and A is a S_g^* -closed set.

Theorem 4.3: A subset A of a topological space (X, τ) is S_g^* -closed set iff $sInt^*(Cl(A)) \subseteq A$.

Proof: Necessity. If A is a S_g^* -closed set, then by Theorem 4.2 there exists a closed set F such that $sInt^*(F) \subseteq A \subseteq F$. Take $F = Cl(A)$. Then $Cl(A) \subseteq Cl(F) \implies sInt^*(Cl(A)) \subseteq sInt^*(Cl(F)) = sInt^*(F)$. Hence $sInt^*(Cl(A)) \subseteq A$.

Sufficiency. Let $sInt^*(Cl(A)) \subseteq A$. Then for $F = Cl(A)$, we get F is a closed set in X such that $sInt^*(F) \subseteq A \subseteq F$. Hence by Theorem 4.2, A is S_g^* -closed in X.

Theorem 4.4: If $\{A_\alpha\}$ is a collection of S_g^* -closed sets in a topological space X, then $\bigcap A_\alpha$ is S_g^* -closed.

Proof: A_α is S_g^* -closed in $X \Rightarrow X \setminus A_\alpha$ is S_g^* -open in $X \Rightarrow$ By Theorem 3.3, $UX \setminus A_\alpha$ is S_g^* -open in $X \Rightarrow X \setminus \bigcap A_\alpha$ is S_g^* -open in $X \Rightarrow \bigcap A_\alpha$ is S_g^* -closed.

Remark 4.5: The problem of determining the union of S_g^* -closed sets is S_g^* -closed remains open.

Theorem 4.6: Suppose A is S_g^* -closed in X and $sInt^*(A) \subseteq B \subseteq A$. Then B is S_g^* -closed in X .

Proof: A is S_g^* -closed in X implies $X \setminus A$ is S_g^* -open in X . $sInt^*(A) \subseteq B \subseteq A \Rightarrow X \setminus sInt^*(A) \supseteq X \setminus B \supseteq X \setminus A \Rightarrow X \setminus A \subseteq X \setminus B \subseteq sCl^*(X \setminus A)$. Now by Theorem 3.10, $X \setminus B$ is S_g^* -open in X . Hence B is S_g^* -closed in X .

Theorem 4.7: Every closed set is S_g^* -closed.

Proof: Let A be a closed set in X . Then $X \setminus A$ is open in X . By Theorem 3.5, $X \setminus A$ is S_g^* -open. Hence A is S_g^* -closed.

Remark 4.8: The converse of Theorem 4.7 need not be true as shown in the following example.

Example 4.9: Let $X = \{a, b, c, d, e\}$ and $\mathcal{F} = \{X, \phi, \{c, d, e\}\}$. $S_g^*C(X, \tau) = \{X, \phi, \{c\}, \{d\}, \{e\}, \{c, d\}, \{c, e\}, \{d, e\}, \{c, d, e\}\}$. Here the subsets $\{c\}, \{d\}, \{e\}, \{c, d\}, \{c, e\}$ and $\{d, e\}$ are S_g^* -closed set but not closed.

Theorem 4.10: If a subset A is S_g^* -closed and B is closed, then $A \cap B$ is S_g^* -closed.

Proof: This follows from Theorem 4.7 and Theorem 4.4.

Theorem 4.11: If A is S_g^* -closed in X and B is closed in X , then $A \cup B$ is S_g^* -closed.

Proof: A is S_g^* -closed in X implies $X \setminus A$ is S_g^* -open in X . Also B is closed in X implies $X \setminus B$ is -open in X . Hence by Theorem 3.9, $X \setminus A \cap X \setminus B$ is S_g^* -open in X . Note $X \setminus A \cap X \setminus B = X \setminus (A \cup B)$. Therefore $X \setminus (A \cup B)$ is S_g^* -open and so $A \cup B$ is S_g^* -closed.

Theorem 4.12: Every S_g^* -closed set is semi-closed.

Proof: Let A be a S_g^* -closed set. Then $X \setminus A$ is a S_g^* -open set. Now by Theorem 3.12, $X \setminus A$ is semi-open which implies A is semi-closed.

Remark 4.13: Converse of the above Theorem 4.12 is not true as can be seen from the following example.

Example 4.14: Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$. In this space, $SC(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ and $S_g^*C(X, \tau) = \{X, \phi, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Here the subsets $\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}$ and $\{b, c\}$ are semi-closed but not S_g^* -closed.

Theorem 4.15: Every S_g^* -closed set is semi-generalised closed.

Proof: It follows from two facts that every S_g^* -closed set is semi-closed and every semi-closed set is semi-generalised closed [1].

Remark 4.16: Converse of the above Theorem 4.15 is false as shown in the following example.

Example 4.17: Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$. In this space, $S_gC(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}\}$ and $S_g^*C(X, \tau) = \{X, \phi, \{a\}, \{d\}, \{a, d\}, \{a, b, d\}, \{a, c, d\}\}$. It shows that the subsets $\{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}$ and $\{c, d\}$ are semi-generalized closed but not S_g^* -closed.

Remark 4.17: The concept of S_g^* -closed sets and S^*g -closed sets are independent as shown by the following examples.

Example 4.18: Let $X = \{a, b, c, d, e\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c, d, e\}\}$. $S^*gC(X, \tau) = \{X, \phi, \{a\}, \{c, d, e\}, \{b, c, d, e\}, \{a, c, d, e\}\}$. $S_g^*C(X, \tau) = \{X, \phi, \{a\}, \{c\}, \{d\}, \{e\}, \{a, c\}, \{a, d\}, \{a, e\}, \{c, d\}, \{c, e\}, \{d, e\}, \{a, c, d\}, \{a, c, e\}, \{a, d, e\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}\}$. Here the subsets $\{c\}, \{d\}, \{e\}, \{a, c\}, \{a, d\}, \{a, e\}, \{c, d\}, \{c, e\}, \{d, e\}, \{a, c, d\}, \{a, c, e\}$ and $\{a, d, e\}$ are S_g^* -closed but not S^*g -closed.

Example 4.19: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a, b\}\}$. $S^*gC(X, \tau) = \{X, \phi, \{c\}, \{b, c\}, \{a, c\}\}$. $S_g^*O(X, \tau) = \{X, \phi, \{c\}\}$. Here the subsets $\{b, c\}$ and $\{a, c\}$ are S^*g -closed but not S_g^* -closed.

Remark 4.20: S_g^* -closed sets and S^* -closed sets are independent as shown by the following examples.

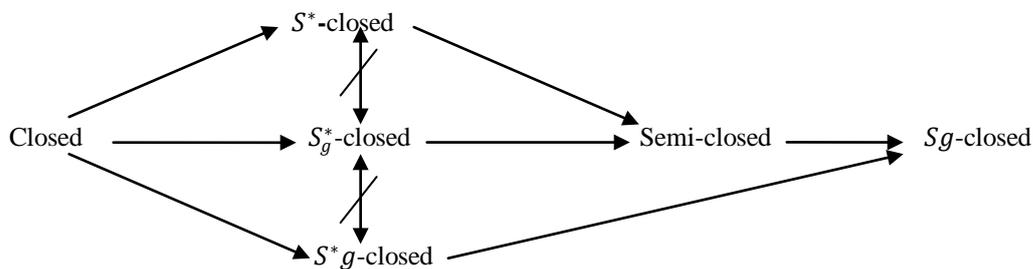
Example 4.21: Let $X = \{a, b, c, d, e\}$ and $\tau = \{X, \phi, \{a\}\}$. $S^*C(X, \tau) = \{X, \phi, \{b, c, d, e\}\}$. $S_g^*O(X, \tau) = \{X, \phi, \{b\}, \{c\}, \{d\}, \{e\}, \{b, c\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}, \{b, c, d\}, \{b, c, e\}, \{b, d, e\}, \{c, d, e\}, \{b, c, d, e\}\}$. Here the subsets $\{b\}, \{c\}, \{d\}, \{e\}, \{b, c\}, \{b, d\}, \{b, e\}, \{c, d\}, \{c, e\}, \{d, e\}, \{b, c, d\}, \{b, c, e\}, \{b, d, e\}$ and $\{c, d, e\}$ are S_g^* -closed but not S^* -closed.

Example 4.22: Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$. In this space, $S^*C(X, \tau) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ and $S_g^*C(X, \tau) = \{X, \phi, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Here the subsets $\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}$ and $\{b, c\}$ are S^* -closed but not S_g^* -closed.

Theorem 4.23: Let (X, τ) be any topological space and \mathcal{F} be the family of all closed sets, then $\mathcal{F} \subseteq S_g^*C(X, \tau) \subseteq SC(X, \tau) \subseteq SgC(X, \tau)$

Proof: It follows from Theorem 4.7, Theorem 4.12 and Theorem 4.15.

Remark 4.24: From the above discussions we have the following figure.



Definition 4.25: The S_g^* -closure of A is defined as the intersection of all S_g^* -closed sets of X containing A . It is denoted by $s_g^*Cl(A)$.

Theorem 4.26: If A is any subset of X , $s_g^*Cl(A)$ is S_g^* -closed. In fact $s_g^*Cl(A)$ is the smallest S_g^* -closed set in X containing A .

Proof: Follows from Definition 4.25 and Theorem 4.4.

Theorem 4.27: A subset A of X is S_g^* -closed iff $s_g^*Cl(A) = A$.

Proof: If A is S_g^* -closed then $s_g^*Cl(A) = A$ is obvious. Conversely, suppose $s_g^*Cl(A) = A$. By Theorem 4.26, $s_g^*Cl(A)$ is S_g^* -closed and hence A is S_g^* -closed.

Theorem 4.28: If A and B be subsets of a topological space (X, τ) , then the following results hold:

- (i) $s_g^*Cl(\phi) = \phi$.
- (ii) $s_g^*Cl(X) = X$.
- (iii) $A \subseteq s_g^*Cl(A)$.
- (iv) $A \subseteq B \implies s_g^*Cl(A) \subseteq s_g^*Cl(B)$.
- (v) $A \subseteq sCl^*(A) \subseteq sCl(A) \subseteq s_g^*Cl(A) \subseteq Cl(A)$.
- (vi) $s_g^*Cl(A \cup B) \supseteq s_g^*Cl(A) \cup s_g^*Cl(B)$.
- (vii) $s_g^*Cl(A \cap B) \subseteq s_g^*Cl(A) \cap s_g^*Cl(B)$.
- (viii) $s_g^*Cl(s_g^*Cl(A)) = s_g^*Cl(A)$.
- (ix) $Cl(s_g^*Cl(A)) = Cl(A)$.
- (x) $s_g^*Cl(Cl(A)) = Cl(A)$.

Proof: (i), (ii), (iii) and (iv) follows from Definition 4.25.

(v) Follows from Result 2.3, Theorem 4.7, Theorem 4.12 and Theorem 4.15.

(vi) and (vii) follow from (iv) above.

(viii) and (ix) follows from Theorem 4.26 and Theorem 4.27.

(x) Since $Cl(A)$ being closed, $Cl(A)$ is S_g^* -closed. Hence by Theorem 4.27, $s_g^*Cl(Cl(A))=Cl(A)$.

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