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DESINGNING GROUP ACCEPTANCE SAMPLING PLANS FOR THE GENERALISED RAYLEIGH DISTRIBUTION USING MINIMUM ANGLE METHOD

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ABSTRACT

In this paper, Designing Minimum angle Group acceptance sampling plans under the time truncated life test in which the proposed group acceptance sampling plan follows a generalized Rayleigh distribution is presented. Tables are constructed for the proposed plan according to group size, test termination ratio and true mean. Examples are provided.

Keywords: *Generalized Rayleigh distribution, Group Sampling plans and Producer's risk. Consumer's risk, Minimum angle method.*

INTRODUCTION

An acceptance sampling plan is concerned with accepting or rejecting a submitted lot of products on the basis of the quality of the products inspected in a sample taken from the lot. An acceptance sampling plan is a scheme that establishes the minimum sample size to be used for testing. This becomes particularly important if the quality of product is defined by its lifetime. Often, it is implicity assumed when designing a sampling plan that only a single item is put in a tester. The items in a tester can be regarded as a group and the number of items in a group is called the group size. An acceptance sampling plan based on such groups of items is called a group acceptance sampling plan (GASP).

The ordinary acceptance sampling plan under a truncated life test is the single sampling plan since it is customary to install one item to a single tester during the experiment. The ordinary acceptance sampling plans for different distributions have been developed by many researchers. The ordinary acceptance sampling plan is usually expensive to be implemented. In order to reduce the expensive, group sampling plans are used. The great advantage of group sampling plan is that it gives more strict inspection of the product since sample is distributed over more than one group. Comparing with ordinary acceptance plan group acceptance plan (GASP) is attracted by the researchers under a truncated life test. Aslam and Jun *et. al.* [1] Proposed the GASP's for the Weibull distribution and Aslam et al. [2] proposed the plans for the gamma distribution. More recently, Aslam et al.[3] proposed a modified group plan for the Weibull distribution. They showed that the modified plan was better than the plan in Aslam and Jun [1] in terms of sample size or the number of groups. Fixing the acceptance number in a sampling plan is very difficult. By the minimum angle criteria the optimum value of the acceptance number was designed.

The purpose of the paper is to design a group acceptance sampling plan under the time truncated life test for the generalized Rayleigh distribution using minimum angle method. An example is given for illustration. The tables 1–4 give Minimum angle method for GASP under Generalized Rayleigh distribution.

OPERATING CHARACTERISTICS FUNCTION

The probability of acceptance can be regarded as a function of the deviation of the specified value μ_0 of the mean from its true value μ. This function is called Operating Characteristic (OC) function of the sampling plan. When the sample size $n = rg$ is known, we can able to find the probability of acceptance of a lot when the quality of the product is sufficiently good. Using the probability of acceptance the corresponding to producer's risk and consumer's risk the values are tabulated to calculate the minimum angle method.

NOTATION

g	-	Number of groups
r	-	Number of items in a group
n	-	Sample size

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c	-	Acceptance number
t_0	-	Termination time
a	-	Test termination time multiplier
m	-	Shape parameters
β	-	Consumer's risk
P	-	Failure probability
L(p)	-	Probability of acceptance
μ	-	Mean life
μ_0	-	Specified life
θ	-	Minimum angle

GENERALIZED RAYLEIGH DISTRIBUTION

In Reliability analysis the Generalized Rayleigh distribution is used for lifetime modeling, life testing problems and acceptance sampling plans. Rayleigh distribution has many applications such as communications, engineering and life testing. The most important factor of Rayleigh distribution is that its failure is an increasing function of time. Aslam et al [2] developed the reliability plan using the Generalized Rayleigh Distribution. The cumulative distribution function (cdf) of the Generalized Rayleigh distribution is given by

$$F_k(t:\lambda) = 1 - \sum_{j=0}^{k} \frac{\left(\frac{t^2}{\lambda}\right)^j e^{-\frac{t2}{\lambda}}}{j!} \tag{1}$$

Where an unknown scale parameter and k is is a known non-negative integer shape parameter. The i-th moment of Generalized Rayleigh distribution is given by

$$E[T]^i = \frac{(k+\frac{i}{2}+1)^{\frac{1}{2}}}{(k+1)^{\frac{1}{2}}} \lambda^{\frac{1}{2}} \qquad \text{where I} = 1,2,3,\ldots\ldots \tag{2}$$

Where T is a random variable representing the lifetime. When i=1, from the Equation (2) we can get the mean value is given by

$$\mu = E[T] = m\lambda^{1/2} \tag{3}$$

Where $m = \sqrt{k+\frac{3}{2}}/\sqrt{k+1}$.

When k=0, the cdf of GR distribution reduces to the Rayleigh distribution.

DESIGN OF GROUP ACCEPTANCE SAMPLING PLAN FOR GENERALIZED RAYLEIGH DISTRIBUTION

The procedure for Design of GASP for generalised Rayleigh distribution which is based on the total number of failures from all groups is as follows according to (Aslam et al. [3])

(1) Randomly draw a sample of size n from a production lot, allocate r items to each of g groups (or testers) so that n=rg and put them on test until the pre-determined t_0 units of time.

(2) Accept the lot if the number of failures from all g groups is smaller than or equal to c. Truncate the test and reject the lot as soon as the number of failures from all g groups is larger than c before t_0, where g (or n) and c, are the parameters and r and t the pre-determined constants. It could be noticed that the proposed plan reduces to the ordinary acceptance sampling plan if the experimenter installs an item to the single tester (r=1) to find the probability of acceptance, we use binomial distribution to express the values in the OC curve for a sampling plan, when the lot size is large enough and the experimenter focuses only on two points, either to accept or reject the lot. The probability of rejecting a good lot is called the producer's risk which is denoted by α and the probability of accepting a bad lot is known as the consumer's risk which is labeled by β. The principle to develop a sampling plan is to find the sample size n and the acceptance number c to meet the required risks established from both producer and consumer.

The cdf given by Equation (1) with parameters μ and k. Let μ be the true population mean and μ_0 be the targeted population mean. It would be convenient to express the experiment time duration as $t_0 = a\mu_0$ where a is called the termination ratio. The probability of a failure before t_0 is given by

$$p = 1 - \sum_{j=0}^{k} \frac{\left(am\left(\frac{\mu_0}{\mu}\right)\right)^{2j} e^{-\left(am\left(\frac{\mu_0}{\mu}\right)\right)^2}}{j!} \tag{4}$$

Where m = $\mu/\sqrt{\lambda}$.

Aslam and Jun [2], studied the ratio of true mean life to the targeted mean life d= μ/μ_0 will be used as a measure of quality level for the products. Let the mean ratio, $d_1 = \mu_1/\mu_0$, be the acceptable reliability level (ARL) at the producer's risk and the mean ratio, $d_2 = \mu_2//\mu_0$ which is equal to 1, be the lot tolerance reliability level (LTRL) at the consumer's risk. The design parameters are, g and c, such that the following inequalities must be satisfied simultaneously,

$$L(p_1) = \sum_{i=0}^{c} \binom{rg}{i} \mathrm{p}_1^{\mathrm{i}} (1 - \mathrm{p}_1)^{\mathrm{rg}-\mathrm{i}} \geq 1 - \alpha$$

$$L(p_2) = \sum_{i=0}^{c} \binom{rg}{i} \mathrm{p}_2^{\mathrm{i}} (1 - \mathrm{p}_2)^{\mathrm{rg}-\mathrm{i}} \leq \beta \tag{5}$$

Where p_1 and p_2 are the respective failure probabilities at the quality levels d = d_1 and d = d_2, respectively,, and are given by

$$p_1 = 1 - \sum_{j=0}^{k} \frac{(am/d_1)^{2j}\, e^{-(am/d_1)^2}}{j!}$$

$$p_2 = 1 - \sum_{j=0}^{k} \frac{(am/d_2)^{2j}\, e^{-(am/d2)^2}}{j!} \tag{6}$$

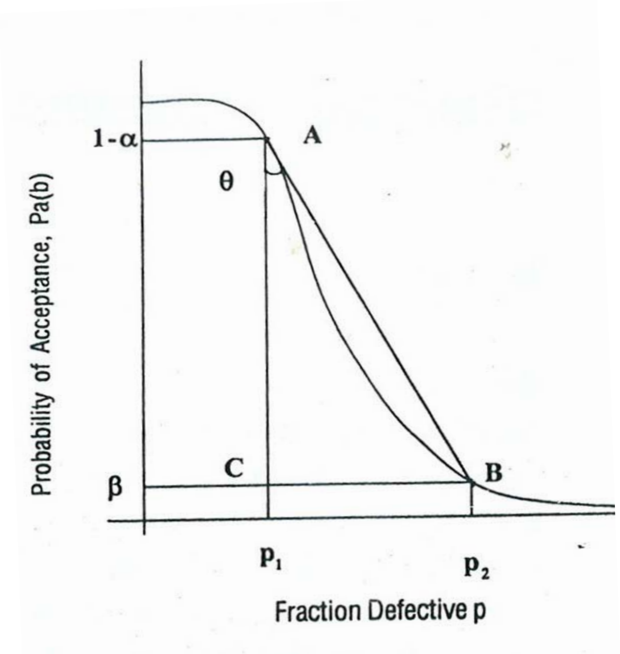
MINIMUM ANGLE METHOD

The practical performance of a sampling plan is revealed by it operating characteristic curve. Norman Bush et. al. [8] have used different techniques involving comparison of some portion of the OC curve to that of the ideal curve. The approach of minimum angle method by considering the tangent of the angle between the lines joining the points (AQL, 1- α) (LQL β) is shown in Figure where p_1 = AQL, p_2= LQL. By employing this method one can get a better discriminating plan with the minimum angle. Tangent of angle made by lines AB and AC is

tanθ = BC/AC

tanθ = $(p_2-p_1)/(Pa(p_1)-Pa(p_2))$ (7)

The smaller the value of this tanθ, closer is the angle θ approaching zero and the chord AB approaching AC, the ideal condition through (AQL, 1-α). This criterion minimizes simultaneously the consumer's and producer's risks. Thus both the producer and consumer favour the plans evolved by the criterion.



Minimum angle for given p_1 and p_2

In this paper the design parameters of the group acceptance sampling plan for Generalized Rayleigh distribution, using minimum angle method is introduced. The minimum angle method of the group sampling plan under generalized Rayleigh distribution for truncated life test is given below. The shape parameter (k= 0, 1, 2), termination ratio α (=.5, 1.0), mean ratio $d_1 = \mu/\mu_1$ (2, 4, 6, 8, 10), and the consumer's risk $\beta \leq .10$ and the number of testers r = 5, 10 and α = 0.05 are specified. The design parameters (c,g) of the proposed sampling plan and the probability of acceptance $L(p_1)$ and $L(p_2)$ is placed in Table 1 for k = 0, and in Table 4.

From the Tables 1, 2, 3 and 4 it can be noted that from the given values for mean ratio k and α, as r increases the design parameters, c and g, usually decrease and for given values for the remaining parameters, as the shape parameter, k, increases from 0 to 2 then the number of groups g generally increases.

DESIGNING GASP FOR THE GENERALIZED RAYLEIGH DISTRIBUTION USING MINIMUM ANGLE METHOD

- First calculate the mean ratio μ/μ_0 corresponding to d_1 and d_2, Where the mean ratio, $d_1 = \mu_1/\mu_0$, be the acceptable reliability level (ARL) at the producer's risk and the mean ratio, $d_2 = \mu_2//\mu_0$ which is equal to 1, be the lot tolerance reliability level (LTRL) at the consumer's risk.
- First calculate the mean ratio μ/μ_0 corresponding to d_1 and d_2.
- Select the values for termination ratio a, r for given shape parameter k = o, 1, 2.
- Locate the value of mean ratio corresponding to the probability of acceptance of GASP along with producer's and consumer's risk.
- Find tanθ from the table.
- Calculate the value $\theta = \tan^{-1}(\tan\theta)$
- Select the parameter of the sampling plan corresponding to smallest value of θ.

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CONSTRUCTION OF TABLES

The Tables are constructed using OC function for GASP plan, the probability of failure under Rayleigh distribution is given by the equation (3). Using the above values the minimum angle tanθ is calculated using the equation (4) Tables 1 and 2 give the proposed values of tanθ for various values c and g corresponding to the mean ratio and α below 5% and β below 10% for the given p_1 and p_2. Numerical value in these tables reveals the following facts.

For given mean ratio and various values c and g values Tables 1 and 2 can be used to select minimum angle of Group sampling plan under Rayleigh distribution for certain specified values of AQL and LQL. The parameter n=rg and θ can be obtained from the selected table corresponding to μ/μ_0 a, r and g along with producer's risk and consumer's risk.

Example 1: Suppose one want to design GASP under generalized Rayleigh distribution for given α = .05, β = .01, $\mu/\mu_0 = 2$, and a = .5 k = 0 r = 5 among the various values of θ the Minimum angle corresponds to c = 8 and g =9 the value $\theta = 12.919^0$ Thus, the desired sampling plan has parameters as (2,.5, 8, 9) as mean ratio, Number of items, Acceptance number, Number of groups respectively.

Example 2: For given $\mu/\mu_0 = 6$ and a = .5, k = 0, r = 5 Minimum angle corresponds to c =4 and g =8 the value $\theta = 16.334^0$

Example 3: For given $\mu/\mu_0 = 2$ and a = 1, k = 0, r = 5 Minimum angle corresponds to c =8 and g = 3 the value $\theta =26.1^0$

Example 4: For given $\mu/\mu_0 = 4$ and a = 1, k = 0, r = 5 Minimum angle corresponds to c = 8 and g = 4 the value $\theta = 34.113^0$

From the above values we come to know that minimum angle plan of GASP under Generalized Rayleigh distribution is given by (2, 5, 8, 9) corresponding to (μ/μ_0, r, c, g). Thus we design the Group sampling plan with Rayleigh distribution for the given values termination ratio a and the number of testers r corresponding to the minimum number of groups using minimum angle method.

TABLE: 1 MINIMUM ANGLE GASP FOR RAYLEIGH DISTRIBUTION k = 0 & r = 5.

	μ/μ_0	c	g	$L(p_1)$	$L(p_2)$	ntanθ	θ
a = .5	2	6	7	0.972522	0.061415	0.237354	13.3523
		7	8	0.980957	0.051919	0.232773	13.10356
		8	9	0.986724	0.043984	0.22939	12.91955
	4	5	6	0.999955	0.072819	0.302086	16.80885
		1	3	0.957105	0.033874	0.303364	16.87589
		5	7	0.999887	0.025178	0.287342	16.03158
	6	1	3	0.990462	0.033874	0.305642	16.99536
		5	7	0.999999	0.025178	0.299925	16.69532
		4	8	0.999952	0.002336	0.293072	16.33439

	8	2	4	0.999813	0.033853	0.307171	17.07542
		3	6	0.999976	0.008691	0.299323	16.66366
		4	7	0.999998	0.008452	0.299244	16.65951
	10	2	4	0.999949	0.033853	0.309214	17.18232
		3	7	0.999992	0.002227	0.299399	16.66766
		3	8	0.999986	0.00054	0.298896	16.64118
a = 1	2	6	2	0.988911	0.194423	0.579932	30.1108
		8	3	0.983932	0.043453	0.489908	26.10061
		9	4	0.949607	0.002501	0.48648	25.942
	4	2	2	0.951714	0.000279	0.71156	35.43414
		5	3	0.998974	0.000501	0.678039	34.13879
		8	4	0.999982	0.000555	0.677392	34.11338
	6	4	2	0.99998	0.015108	0.73498	36.31514
		5	3	0.999987	0.000501	0.724234	35.91334
		2	1	0.999432	0.090212	0.796134	38.52451
	8	1	1	0.995267	0.012769	0.754021	37.01706
		3	2	0.999954	0.002513	0.742724	36.60218
		5	3	0.999999	0.000501	0.741195	36.54566
	10	2	1	0.999971	0.090212	0.823047	39.456
		2	2	0.999675	0.000279	0.749228	36.84157

TABLE:2 MINIMUM ANGLE GASP FOR RAYLEIGH DISTRIBUTION k = 0 & r = 10.

	μ/μ_0	c	g	$L(p_1)$	$L(p_2)$	ntanθ	θ
a = .5	2	5	3	0.960048	0.072819	0.243742	13.69827
		7	4	0.980957	0.051919	0.232773	13.10356
		8	5	0.974484	0.016777	0.225805	12.72427
	4	3	4	0.988163	0.00054	0.283585	15.83253
		4	5	0.994971	0.000152	0.281534	15.72369
		6	6	0.999629	0.000181	0.28023	15.65445
	6	1	2	0.983298	0.007217	0.299538	16.67496
		2	3	0.996728	0.001952	0.293909	16.37852
		5	5	0.999989	0.000645	0.292565	16.30764
	8	2	2	0.999813	0.033853	0.307171	17.07542
		1	3	0.987668	0.000286	0.300506	16.72584
		4	4	0.999997	0.002336	0.29741	16.56301
	10	3	4	0.999986	0.00054	0.298896	16.64118
		4	5	0.999999	0.000152	0.298776	16.63487
		2	2	0.999949	0.033853	0.309214	17.18232
a = 1	2	5	1	0.950921	0.063783	0.519365	27.44579
		10	2	0.981808	0.009394	0.473819	25.3525
		14	3	0.981798	0.000432	0.469497	25.1499
	4	2	1	0.951714	0.000279	0.71156	35.43414
		6	2	0.999011	1.53E-05	0.677684	34.12486
		8	2	0.999982	0.000555	0.677392	34.11338
	6	4	1	0.99998	0.015108	0.73498	36.31514
		6	2	0.999993	1.53E-05	0.723878	35.89995
		6	3	0.999869	1.24E-10	0.723956	35.90289

	8	3	1	0.999954	0.002513	0.742724	36.60218
		5	2	0.999996	1.84E-06	0.740828	36.53207
		5	3	0.999955	9.09E-12	0.740857	36.53316
	10	2	1	0.999675	0.000279	0.749228	36.84157
		9	2	0.9998	0.002501	0.750653	36.89382
		4	1	0.9998	0.015108	0.760261	37.24432

TABLE – 3 MINIMUM ANGLE GASP FOR RAYLEIGH DISTRIBUTION k = 1 & r = 5

	μ/μ_0	c	g	L (p_1)	L(p_2)	ntanθ	θ
a =0.5	2	1	19	0.968228	0.102539	0.042843	2.453242
		3	36	0.997962	0.067954	0.03988	2.283762
		2	26	0.993257	0.103917	0.041704	2.388079
		0	17	0.780102	0.031104	0.049518	2.834866
		5	47	0.999924	0.088914	0.040712	2.331337
	4	1	20	0.999824	0.087118	0.043625	2.497937
		2	27	0.999997	0.090199	0.043764	2.505912
		0	12	0.988688	0.086318	0.044125	2.526514
		1	20	0.999824	0.087118	0.043625	2.497937
		3	36	0.999971	0.067954	0.04272	2.446169
		1	22	0.999787	0.062586	0.042485	2.432729
	6	3	34	0.999991	0.088281	0.043839	2.510165
		1	21	0.999992	0.073895	0.043158	2.471242
		2	27	0.999981	0.090199	0.043931	2.51545
		0	17	0.996799	0.031104	0.041388	2.370028
	8	2	26	0.999782	0.103917	0.044632	2.555552
		0	19	0.998864	0.020677	0.040886	2.341302
		1	20	0.999999	0.087118	0.043811	2.508585
		2	28	0.999762	0.078147	0.043385	2.484202
	10	1	19	0.999661	0.102539	0.044572	2.552083
		0	18	0.999558	0.02536	0.041061	2.351291
		2	27	0.999521	0.090199	0.043967	2.517513
a =1	2	1	2	0.942494	0.080731	0.366211	20.11328
		2	4	0.956817	0.01072	0.333567	18.447
		3	7	0.950913	0.000275	0.331974	18.36482
	4	0	3	0.957415	0.00138	0.368648	20.2363
		4	4	0.999989	0.108797	0.395465	21.57708
		2	4	0.999973	0.01072	0.356269	19.60939
		3	7	0.999997	0.000275	0.352538	19.4195
		7	8	0.999621	0.010386	0.356139	19.60278
	6	0	2	0.994126	0.012397	0.36135	19.86734
		3	4	0.999932	0.040062	0.369553	20.28194
		2	5	0.999872	0.001813	0.355392	19.56483
	8	1	3	0.999996	0.012793	0.359752	19.78632
		4	5	0.999631	0.028386	0.365525	20.0786
		3	4	0.999841	0.040062	0.36997	20.30298
	10	0	2	0.999226	0.012397	0.360001	19.79895
		6	7	0.999654	0.014462	0.360473	19.82285
		5	5	0.999557	0.074645	0.383917	21.00265

TABLE– 4: MINIMUM ANGLE GASP FOR RAYLEIGH DISTRIBUTION k = 2 & r = 5

	μ/μ_0	c	g	$L(p_1)$	$L(p_2)$	$\tan\theta$	θ
a=0.5	2	0	140	0.958103	0.097094	0.003792	0.217259
		1	240	0.997439	0.091858	0.003605	0.206566
		2	320	0.999855	0.099668	0.003627	0.207803
		1	280	0.999999	0.05347	0.003513	0.20127
		0	160	0.999204	0.069583	0.003577	0.204931
	4	2	320	0.999897	0.099668	0.003693	0.211597
		2	340	0.999864	0.07896	0.003611	0.206896
		0	140	0.999938	0.097094	0.003684	0.211066
		1	260	0.999652	0.070221	0.003577	0.204952
		1	280	0.999562	0.05347	0.003514	0.201329
		2	320	0.999875	0.099668	0.003694	0.21166
	6	1	200	0.999742	0.154999	0.003936	0.22552
		0	160	0.999997	0.069583	0.003575	0.204817
		2	360	0.999654	0.062262	0.003547	0.203217
		1	7	0.993881	0.081325	0.120522	6.872249
	8	1	7	0.993881	0.081325	0.120522	6.872249
		2	10	0.999358	0.067029	0.117966	6.727846
		1	10	0.999995	0.018081	0.115334	6.579069
		0	4	0.998778	0.09025	0.12465	7.10526
	10	2	9	0.999582	0.102309	0.126155	7.190143
		2	12	0.999885	0.027618	0.116522	6.646223
		0	6	0.999834	0.027112	0.116481	6.643923
a = 1	2	1	7	0.9999775	0.081325	0.123334	7.030987
		2	9	0.999999	0.102309	0.126222	7.193922
		2	10	0.999876	0.067029	0.121449	6.92458
	4	1	7	0.999477	0.081325	0.123339	7.031311

CONCLUSION

In this paper minimum angle of group acceptance sampling plans under generalized Rayleigh distribution with different shape parameters are discussed. According to M. Aslam for given mean ratio $\mu/\mu_0 = 2$, a = .5 and r = 5 the producer's risk is .0098 and consumer's risk is .05 is obtained for c = 8 and g = 19. Whereas when we apply minimum angle method for the same parameters the producer's risk is .0133 and consumer's risk is .04 is obtained for c = 8 and g = 9 which reduces the number of groups. Therefore this plan reduces time cost and cost of the tester. This criterion minimizes simultaneously the consumer's and producer's risk. This minimum angle plan provides better discrimination of accepting good lots among minimum number of groups.

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