International Journal of Mathematical Archive-3(11), 2012, 3975-3981
Available online through www.ijma.info ISSN 2229 – 5046

PROFIT ANALYSIS OF A COLD STANDBY SYSTEM UNDER MAINTENANCE AND REPAIR SUBJECT TO RANDOM SHOCKS

S. C. Malik* & S. K. Chillar

Department of Statistics, M.D. University, Rohtak-124001, Haryana, India

(Received on: 10-10-12; Revised & Accepted on: 26-11-12)

ABSTRACT

A two-unit cold standby system is analyzed in which operative unit suffers a random shock with some probability. There is a single server who visits the system immediately to conduct maintenance and repair of the unit. The unit undergoes for maintenance if it is affected by impact of shocks. However, repair of the unit is done when it fails due to some other reasons. The unit works as new after maintenance and repair. The distributions of random shocks and failure times of the unit follow negative exponential while that of maintenance and repair times are taken as arbitrary with different probability density functions. The expressions for some performance measures of the system model are derived in steady state using semi-Markov process and regenerative point technique. The graphical study of the results obtained for mean time to system failure (MTSF), availability and profit has also been made giving some particular values to various costs and parameters.

Keywords: *Cold Standby System, Random Shocks, Maintenance, Repair and Profit Analysis.*

1. INTRODUCTION

The performance of most of the real world operating systems is significantly affected by varying environmental conditions. Shocks are the external environmental conditions which cause perturbation to the system, leading to its deterioration and consequent failure. The shocks may be caused by external factors such as fluctuation of unstable electric power, power failure, change in climate conditions, change of operator, etc. or due to internal factors such as stress and strain. Many systems like power generation and automotive industries are vulnerable to damage caused by shock attacking that may occur over the service life. Sometimes, a system may or may not be affected by the impact of shocks and the system may fail due to operation and / or due to random shocks. Shock models, which are one of the important models in the reliability theory have been extensively studied by the researchers including Murari and Al-Ali [1988] and Gupta and Chaudhary [1992].

The reliability and performance of the systems which are affected by the impact of shocks can be improved by various methods such as redundancy, maintenance and repair. Cold standby systems are one of the most important structures in reliability engineering and have been widely applied to industries. Therefore, a lot of work has been done by the researchers on reliability modeling of cold standby systems under different sets of assumptions on failure and repair policies. Murari and Goyal [1984] made a comparison of two-unit cold standby reliability models with three types of repair facilities. Recently, Malik and Anand [2010] probed reliability models of computer systems with the concept of redundancy in cold standby. Wu and Wu [2011] also discussed reliability of a two-unit cold standby repairable system under poison shocks.

Keeping the above observations and practical situations in mind, here we analyze a two-unit cold standby system subject to random shocks. The system may or may not be affected by the impact of shocks. There is a single server who visits the system immediately to conduct maintenance and repair of the unit. The unit undergoes for maintenance if it is affected by the impact of shocks. However, repair of the unit is done when it fails due to some other reasons. All random variables are uncorrelated to each other. The time to failure of the unit and time to occurrence of shock are exponentially distributed whereas the distributions of maintenance and repair times are taken as arbitrary with different probability density functions. The unit work as new after maintenance and repair. Various reliability indices such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server due to repair and maintenance, expected number of maintenance and repair and profit function are evaluated in steady state using semi-Markov process and regenerative point technique. The numerical results giving particular values to various costs and parameters are obtained for MTSF, availability and profit to depict their graphical behavior with respect to shock rate.

Corresponding author: *S. C. Malik**
Department of Statistics, M.D. University, Rohtak-124001, Haryana, India

2. NOTATIONS

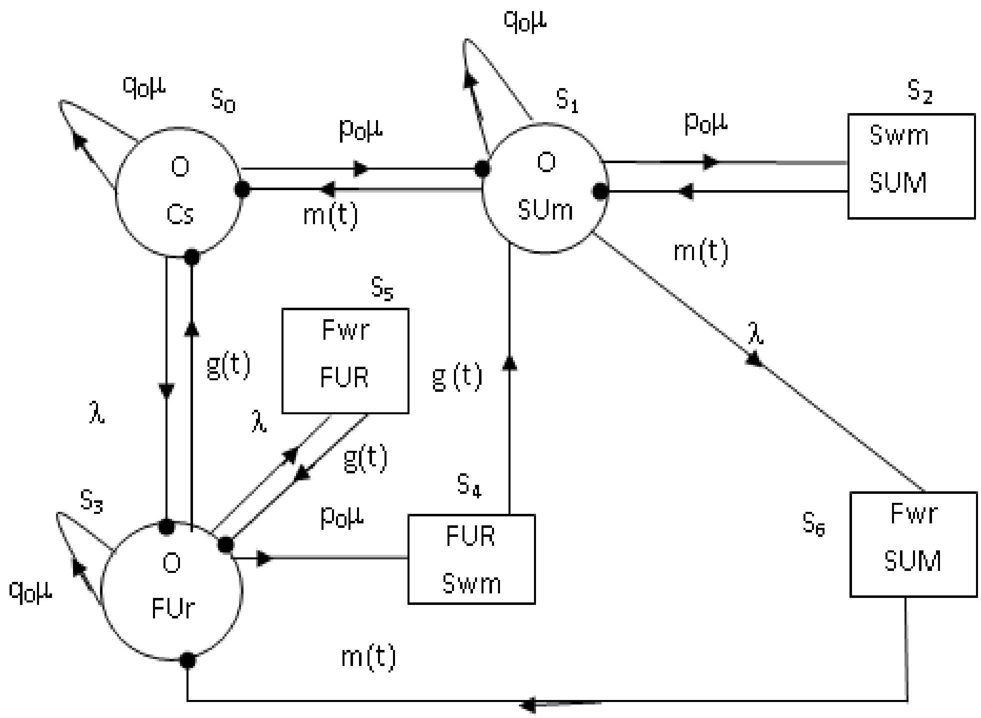
E : Set of regenerative states.
O : The unit is operative and in normal mode.
p_0 : The probability that shock is effective.
q_0 : The probability that shock is not effective.
μ : Constant rate of the occurrence of a shock.
λ : Constant failure rate of the unit.
m(t)/M(t) : pdf / cdf of maintenance time of the unit after the effect of a shock.
FUr / FWr /FUR: The Unit is completely failed and under repair / waiting for repair/ under continuous repair from previous state
SUm/SUM : Shocked unit under maintenance and under maintenance continuously from previous state
SWm : Shocked unit waiting for maintenance
g(t) / G(t) : pdf / cdf of repair time of the completely failed unit
$q_{ij}(t)$ / $Q_{ij}(t)$: pdf and cdf of direct transition time from a regenerative state i to a regenerative state jwithout visiting any other regenerative state
$q_{ij.k}(t)$ / $Q_{ij.k}(t)$: pdf and cdf of first passage time from a regenerative state i to a regenerative state j or to a failed state j visiting state k once in (0,t].
$M_i(t)$: Probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state.
$W_i(t)$: Probability that the server is busy in state S_i upto time t without making ransition to any other regenerative state or returning to the same via one or more non regenerative states.
m_{ij} :Contribution to mean sojourn time in state S_i when system transits directly to state S_j (S_i ,$S_j \in E$) so that $\mu_i = \sum_j m_{ij}$ where $m_{ij} = \int t\, dQ_{ij}(t) = -q_{ij}^{*/}(0)$ and μ_i is the mean sojourn time in state $S_i \in E$
(s) / © : Symbol for Stieltjes convolution / Laplace convolution.
~ / * : Symbol for Laplace Stieltjes Transform (LST) / Laplace Transform (LT).
/ (desh) : Symbol for derivative of the function.

The following are the possible transition states of the system

S_0 = (O, Cs), S_1 = (SUm, O), S_2 = (SUM, SWm), S_3= (O, FUr)

S_4 =(FUR, SWm),S_5=(FUR,FWr), S_6=(SUM,FWr),

The transition states S_0 , S_1 , S_3 , are regenerative and states S_2, S_4 ,S_5 , S_6 are non regenerative as shown in figure 1.



● Transition point ◯ Up-State ☐ Completely Failed

3. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt \quad \text{as}$$

$p_{00}=\dfrac{q_0\mu}{\lambda+\mu}$, $p_{01}=\dfrac{p_0\mu}{\lambda+\mu}$, $p_{03}=\dfrac{\lambda}{\lambda+\mu}$, $p_{10}=\dfrac{\theta}{\lambda+\mu+\theta}$, $p_{11}=\dfrac{q_0\mu}{\lambda+\mu+\theta}$, $p_{12}=\dfrac{p_0\mu}{\lambda+\mu+\theta}$,

$p_{16}=\dfrac{\lambda}{\lambda+\mu+\theta}$, $p_{30}=\dfrac{\gamma}{\lambda+\mu+\gamma}$, $p_{33}=\dfrac{q_0\mu}{\lambda+\mu+\gamma}$, $p_{34}=\dfrac{p_0\mu}{\lambda+\mu+\gamma}$, $p_{35}=\dfrac{\lambda}{\lambda+\mu+\gamma}$

$p_{21} = m^*(0)$, $p_{41}= g^*(0)$, $p_{53}= g^*(0)$, $p_{63} = m^*(0)$ (1)

For $m(t) = \theta e^{-\theta t}$ and $g(t) = \alpha e^{-\alpha t}$ we have

$p_{11.2}=\dfrac{p_0\mu}{\lambda+\mu+\theta}$, $p_{13.6}=\dfrac{\lambda}{\lambda+\mu+\theta}$, $p_{31.4}=\dfrac{p_0\mu}{\lambda+\mu+\gamma}$ (2)

It can be easily verified that

$p_{00}+p_{01}+p_{03}= p_{10}+p_{11}+p_{12}+p_{16}=p_{21}= p_{30}+p_{33}+p_{34}+p_{35} = p_{30}+p_{33}+p_{33.5}+p_{31.4} =$

$p_{41} = p_{53} =p_{63} = p_{10}+p_{11.2}+p_{11}+p_{13.6} =1$ (3)

The mean sojourn times μ_i for $m(t) = \theta e^{-\theta t}$, $g(t) = \alpha e^{-\alpha t}$ are

$\mu_0=\dfrac{1}{\lambda+\mu}$, $\mu_1 =\dfrac{1}{\lambda+\mu+\theta}$, $\mu_3 =\dfrac{1}{\lambda+\mu+\gamma}$, $\mu_1' =\dfrac{\lambda+p_0\mu+\theta}{\theta(\lambda+\mu+\theta)}$, $\mu_3' =\dfrac{\lambda+p_0\mu+\gamma}{\gamma(\lambda+\mu+\gamma)}$ (4)

where

$m_{00}+m_{01}+m_{03} = \mu_0$, $m_{10}+m_{11}+m_{12}+m_{16} = \mu_1$, $m_{30}+m_{33}+m_{34}+m_{35} = \mu_3$,

$m_{10}+m_{11}+m_{11.2}+m_{13.6} = \mu_1'$, $m_{30}+m_{33}+m_{33.5}+m_{31.4} = \mu_3'$ (5)

4. RELIABILITY AND MEAN TIME TO SYSTEM FALIURE (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:

$\phi_0(t) = Q_{00}(t)Ⓢ\phi_0(t) + Q_{01}(t)Ⓢ\phi_1(t) + Q_{03}(t)Ⓢ\phi_3(t)$

$\phi_1(t) = Q_{10}(t)Ⓢ\phi_0(t) + Q_{11}(t)Ⓢ\phi_1(t) + Q_{12}(t) + Q_{16}(t)$

$\phi_3(t) = Q_{30}(t)Ⓢ\phi_0(t) + Q_{34}(t) + Q_{35}(t) + Q_{33}(t)Ⓢ\phi_3(t)$ (6)

Taking LST of above relations (6) and solving for $\tilde{\phi}_0(s)$

We have $R^*(s) = \dfrac{1-\tilde{\phi}_0(s)}{s}$ (7)

The reliability of the system model can be obtained by taking Laplace inverse transform of (7).

The mean time to system failure (MTSF) is given by

$$\text{MTSF} = \lim_{s\to o}\frac{1-\tilde{\phi}_0(s)}{s} = \frac{N_1}{D_1} \quad (8)$$

where

$N_1=\mu_0(1-p_{11})(1-p_{33})+\mu_1 p_{01}(1-p_{33})+\mu_3 p_{03}(1-p_{11})$

and

$D_1=(1-p_{00})(1-p_{11})(1-p_{33})-p_{01}\,p_{10}(1-p_{33})-p_{03}\,p_{30}(1-p_{11})$

5. STEADY STATE AVAILABILITY

Let A_i (t) be the probability that the system is in up-state at instant'‘t’ given that the system entered regenerative state i at t = 0.

The recursive relations for A_i (t) are given as

$A_0(t) = M_0(t) + q_{00}(t) © A_0(t) + q_{01}(t) © A_1(t) + q_{03}(t) © A_3(t)$

$A_1(t) = M_1(t) + q_{10}(t) © A_0(t) + [q_{11}(t) + q_{11.2}(t)]© A_1(t) + q_{13.6}(t) © A_1(t)$

$$A_3(t) = M_3(t) + q_{30}(t) © A_0(t) + [q_{33.5}(t) + q_{33}(t)]© A_3(t) + q_{31.4}(t) © A_1(t) \quad (9)$$

where $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state, we have

$$M_0(t) = e^{-(\lambda+\mu)t},\ M_1(t) = e^{-(\lambda+\mu)t}\overline{M(t)},\ \ M_3(t) = e^{-(\lambda+\mu)t}\overline{G(t)} \quad (10)$$

Taking LT of above relations (9) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2}, \quad (11)$$

where

$N_2 = [(1-p_{11.2}-p_{11})(1-p_{33}-p_{33.5})-p_{31.4}p_{13.6}]\,\mu_0 + [p_{01}(1-p_{33}-p_{33.5})+p_{31.4}p_{03}]\mu_1+[p_{10}\,p_{13.6}+p_{03}(1-p_{11.2}-p_{11})]\mu_3$

and

$D_2 = [(1-p_{11.2}-p_{11})(1-p_{33}-p_{33.5})-p_{31.4}p_{13.6}]\,\mu_0 + [p_{01}(1-p_{33}-p_{33.5})+p_{31.4}p_{03}]\,\mu_1' + [p_{10}\,p_{13.6}+p_{03}(1-p_{11.2}-p_{11})]\,\mu_3'$

6. BUSY PERIOD ANALYSIS OF THE SERVER

(a)Due to repair

Let $B_i(t)$ be the probability that the server is busy at instant t given that the system entered regenerative state i at t = 0.The recursive relation for $B_i(t)$ are as follows:

$B^R_0(t) = q_{00}(t) © B^R_0(t) + q_{01}(t) © B^R_1(t) + q_{03}(t) © B^R_3(t)$

$B^R_1(t) = q_{10}(t) © B^R_0(t) + [q_{11}(t) + q_{11.2}(t)]© B^R_1(t) + q_{13.6}(t) © B^R_3(t)$

$$B^R_3(t) = W_3(t)+q_{30}(t) © B^R_0(t) + [q_{33.5}(t) + q_{33}(t)]© B^R_3(t) + q_{31.4}(t) © B^R_1(t) \quad (12)$$

where,

$W_3(t)= e^{-(\lambda+\mu)t}\overline{G(t)} + (\lambda e^{-(\lambda+\mu)t}©1)\overline{G(t)} + (p_0\mu e^{-(\lambda+\mu)t}©1)\overline{G(t)} + (q_0\mu e^{-(\lambda+\mu)t}©1)\overline{G(t)}$

Now taking L.T. of relations (12) and obtain the value of $B^{R}_0{}^*(s)$ and by using this, the time for which server is busy in steady state is given by

$$B^R_0 = N_3/D_2,$$

where

$N_3 = [p_{10}\,p_{13.6}+p_{03}(1-p_{11.2}-p_{11})]\,w_3^*(s)$ and D_2 is already defined.

(b) Due to Maintenance

$B^M_0(t) = q_{00}(t) © B^M_0(t) + q_{01}(t) © B^M_1(t) + q_{03}(t) © B^M_3(t)$

$B^M_1(t) = W_1(t)+ q_{10}(t) © B^M_0(t) + [q_{11}(t) + q_{11.2}(t)]© B^M_1(t) + q_{13.6}(t) © B^M_3(t)$

$$B^M_3(t) = q_{30}(t) © B^M_0(t) + [q_{33.5}(t) + q_{33}(t)]© B^M_3(t) + q_{31.4}(t) © B^M_1(t) \quad (13)$$

where,

$W_1(t)= e^{-(\lambda+\mu)t}\overline{M(t)} + (\lambda e^{-(\lambda+\mu)t} © 1)\overline{M(t)} + (p_0 \mu e^{-(\lambda+\mu)t} © 1)\overline{M(t)} + (q_0 \mu e^{-(\lambda+\mu)t} © 1)\overline{M(t)}$

Now taking L.T. of relations (13) and obtain the value of $B^M_0{}^*(s)$ and by using this, the time for which server is busy in steady state is given by

$$B^M_0 = N_4/D_2,$$

where

$N_4 = [p_{01}(1-p_{33}-p_{33.5}) + p_{31.4}p_{03}]\, w_1^*(s)$ and D_2 is already defined.

7. EXPECTED NUMBER OF MAINTENANCE

$N^M{}_0(t) = Q_{00}(t) Ⓢ N^M{}_0(t) + Q_{01}(t) Ⓢ N^M{}_1(t) + Q_{03}(t) Ⓢ N^M{}_3(t)$

$N^M{}_1(t) = Q_{10}(t) Ⓢ (1+N^M{}_0(t)) + Q_{11}(t) Ⓢ N^M{}_1(t) + Q_{11.2}(t) Ⓢ (1+N^M{}_1(t)) + Q_{13.6}(t) Ⓢ (1+N^M{}_3(t))$

$N^M{}_3(t) = Q_{30}(t) Ⓢ N^M{}_0(t) + [Q_{33.5}(t) + Q_{33}(t)] Ⓢ N^M{}_3(t) + Q_{31.4}(t) Ⓢ N^M{}_1(t)$ (14)

Now taking L.T. of relations (14) and obtain the value of $N^M_0{}^*(s)$ and by using this, the time for which server is busy in steady state is given by

$$N^M_0 = N_5/D_2$$

where

$N_5 = (p_{10}+p_{11.2}+p_{13.6})[p_{01}(1- p_{33}- p_{33.5})+p_{31.4}p_{03}]$ and D_2 is already defined.

8. EXPECTED NUMBER OF REPAIRS

$N^R{}_0(t) = Q_{00}(t) Ⓢ N^R{}_0(t) + Q_{01}(t) Ⓢ N^R{}_1(t) + Q_{03}(t) Ⓢ N^R{}_3(t)$

$N^R{}_1(t) = Q_{10}(t) Ⓢ N^R{}_0(t) + Q_{11}(t) Ⓢ N^R{}_1(t) + Q_{11.2}(t) Ⓢ N^R{}_1(t) + Q_{13.6}(t) Ⓢ N^R{}_3(t)$

$N^R{}_3(t) = Q_{30}(t) Ⓢ [1+N^R{}_0(t)] + Q_{33.5}(t) Ⓢ [1+N^R{}_3(t)] + Q_{33}(t) Ⓢ N^R{}_3(t) + Q_{31.4}(t) Ⓢ [1+N^R{}_1(t)]$ (15)

Now taking L.T. of relations (15) and obtain the value of $N^R_0{}^*(s)$ and by using this, the time for which server is busy in steady state is given by

$$N^R_0 = N_6/D_2$$

where

$N_6 = (p_{30}+p_{33.5}+p_{31.4})\,[p_{03}(1- p_{11}- p_{11.2}) +p_{01}p_{13.6}]$ and D_2 is already defined.

9. PROFIT ANALISIS

The profit incurred to the system model in steady state can be obtained as

$$P = K_0A_0 - K_1B_0^M - K_2B_0^R - K_3N_0^M - K_4N_0^R - K_5 \quad (16)$$

where

K_0 = Revenue per unit up-time of the system

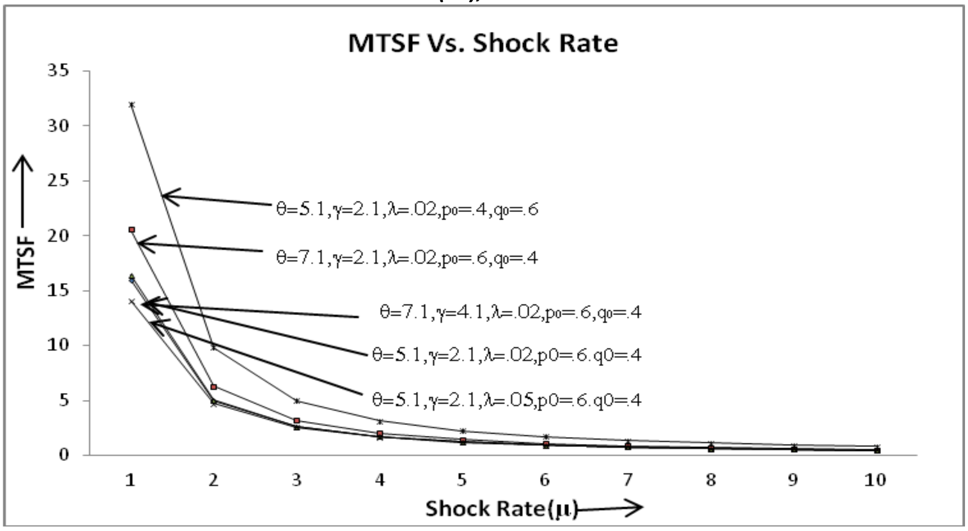
K_1 = Cost per unit time for which server is busy due to maintenance

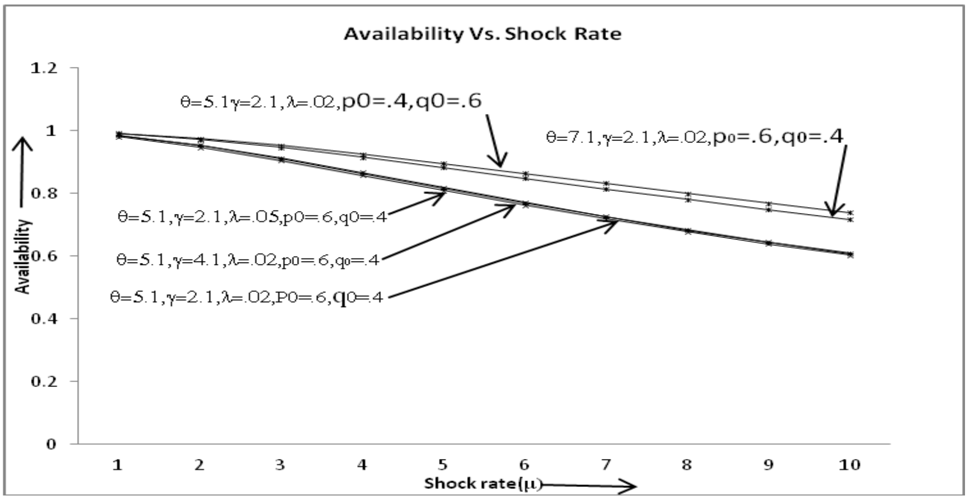
K_2 = Cost per unit time for which server is busy due to repair.

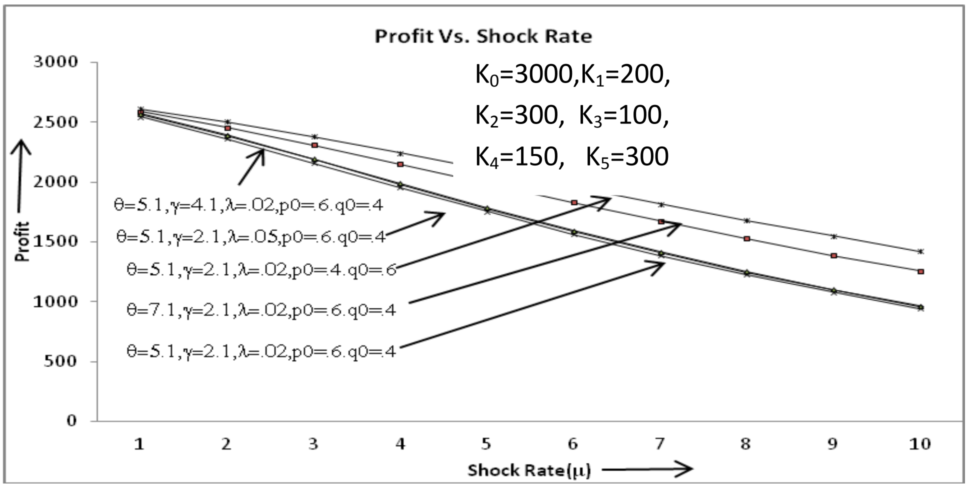
K_3 = Cost per unit maintenance of the shocked unit.

K_4 = Cost per unit repair of the failed unit.

K_5 = Total cost for the busy of the server and $A_0, B_0^M, B_0^R, N_0^M, N_0^R$ are already defined.







10. PARTICULAR CASE

Suppose $g(t) = \gamma e^{-\gamma t}$, $m(t) = \theta e^{-\theta t}$

We can obtain the following results

MTSF (T_0) = $\dfrac{N_1}{D_1}$, Availability (A_0) = $\dfrac{N_2}{D_2}$

Busy period due to repair $\left(B_0^R\right) = \dfrac{N_3}{D_2}$

Busy period due to maintenance $\left(B_0^M\right) = \dfrac{N_4}{D_2}$

Expected number of maintenance $\left(N_0^M\right) = \dfrac{N_5}{D_2}$

Expected number of repairs $\left(N_0^R\right) = \dfrac{N_6}{D_2}$ (18)

where

$N_1 = p_0\mu(p_0\mu+\lambda)(\lambda+p_0\mu+\gamma)+\lambda(\lambda+p_0\mu)(\lambda+p_0\mu+\theta)$

$D_1 = (\lambda+p_0\mu+\gamma)(\lambda+p_0\mu+\theta)+p_0\mu(\lambda+p_0\mu+\gamma)+\lambda(\lambda+p_0\mu+\theta)$

$$N_2 = \frac{[(\lambda+\mu+\gamma)(\lambda+\mu+\theta)-q_0\mu(\lambda+\mu+\theta)-q_0\mu(\lambda+\mu+\gamma)+(q_0\mu)^2]}{(\lambda+\mu)(\lambda+\mu+\gamma)(\lambda+\mu+\theta)}$$

$$D_2 = \frac{\{\theta\gamma(\lambda+\mu+\theta)-q_0\mu\theta\gamma+p_0\mu\gamma(\lambda+p_0\mu)\}(\lambda+\mu+\gamma)-q_0\mu\theta\gamma(\mu+\theta)+\theta\lambda(\lambda+p_0\mu)(\lambda+\mu+\theta)+(q_0\mu)^2\theta\gamma-p_0q_0\mu^2\gamma(\lambda+p_0\mu)-\lambda q_0\mu\theta(\lambda+p_0\mu+\gamma)}{\theta\gamma(\lambda+\mu)(\lambda+\mu+\gamma)(\lambda+\mu+\theta)}$$

$N_3 = \dfrac{\lambda(\lambda+p_0\mu+\theta)}{(\lambda+\mu)(\lambda+\mu+\theta)\gamma}$, $N_4 = \dfrac{p_0\mu(\lambda+p_0\mu+\gamma)}{\theta(\lambda+\mu)(\lambda+\mu+\gamma)}$, $N_5 = \dfrac{p_0\mu(\lambda+p_0\mu+\gamma)}{(\lambda+\mu)(\lambda+\mu+\gamma)}\dfrac{(\lambda+p_0\mu+\theta)}{(\lambda+\mu+\theta)}$,

$N_6 = \dfrac{\lambda(\lambda+p_0\mu+\gamma)}{(\lambda+\mu)(\lambda+\mu+\gamma)}\dfrac{(\lambda+p_0\mu+\theta)}{(\lambda+\mu+\theta)}$

11. CONCLUSION

For a particular case, numerical results are obtained to depict the graphical behavior of MTSF, availability and profit with respect to shock rate (μ) for fixed values of other parameters including p_0 =.6 and q_0 =.4 as shown in figures 2, 3 and 4 respectively. From figure 2, it is observed that MTSF declines rapidly with the increase of shock rate (μ). Also, MTSF decreases with the increase of failure rate (λ). But, if we increase repair and maintenance rates, the value of MTSF becomes more. And, if values of p_0 and q_0 are interchanged, i.e. p_0=.4 and q_0=.6, then MTSF increases many fold. Figures 3 and 4 indicate that availability and profit go on decreasing with the increase of shock rate (μ) and failure rate (λ). However, their values become more with the increase of maintenance and repair rates as well as by interchanging the values of p_0 and q_0.

Hence the study reveals that a two-unit cold standby system in which operative unit is affected by the impact of shocks can be made more reliable and profitable by increasing the maintenance rate of the shocked unit.

REFERENCES

1. Murari, K. and Goyal, V. (1984): Comparison of two-unit cold standby reliability models with three types of repair facilities. Journal of Microelectronics and Reliability, Vol. 24(1), *pp. 35 – 49.*
2. Murari, K. and Al-Ali, A.A. (1988): One unit reliability system subject to random shocks and preventive maintenance. Journal of Microelectronics and Reliability, Vol. 28 (3), pp. *373 –377.*
3. Gupta, R. and Chaudhary, A. (1992): A two-unit priority standby system subject to random shocks and releigh failure time distribution. Journal of Microelectronics and Reliability, Vol. 32 (12), pp. 1713-1723.
4. Mailk, S.C and Anand, Jyoti (2010): Reliability and economic analysis of a computer system with independent hardware and software failures. Bulletin of Pure and Applied Sciences. Vol.29E (No.1), pp.141-153.
5. Wu, Q. and Wu, S. (2011): Reliability analysis of a two-unit cold standby reparable systems under Poison shocks. Applied Mathematics and Computation, Vol. 218 (1), pp. 171-182.

Source of support: Nil, Conflict of interest: None Declared