

On $\pi g\theta$ -Closed sets in Topological spaces

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ABSTRACT

In this paper a new class of sets called $\pi g\theta$ -closed is introduced and its properties are studied. Further the notion of $\pi g\theta$ - $T_{1/2}$ space and $\pi g\theta$ -continuity are introduced.

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Key words: $\pi g\theta$ -closed set, $\pi g\theta$ -open set, $\pi g\theta$ -continuity and $\pi g\theta$ - $T_{1/2}$ space.

1. INTRODUCTION

Velicko[23] introduced the notions of θ -open subsets, θ -closed subsets and θ -closure, for the sake of studying the important class of H-closed spaces in terms of arbitrary filterbases. Dontchev and Maki [2] alone have explored the concept of θ -generalized closed sets. The finite union of regular open set is π -open and subsequently the complement of π -open set is said to be π -closed, which has been highlighted by Zaitsev [23]. Dontchev and Noiri have explored the concept of quasi normal spaces and πg -closed sets. The studies of $\pi g\alpha$ -closed set [11], πgp -closed set [19], πgb -closed set[20], πgs -closed set[4] were introduced later.

The prime objective of this study is to explore the idea of $\pi g\theta$ -closed sets, its properties, characterization and its functions.

2. PRELIMINARIES

Throughout this paper (X, τ) and (Y, σ) represents non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ) , $cl(A)$ and $int(A)$ denote the closure of A and the interior of A respectively. (X, τ) will be replaced by X if there is no chance of confusion.

Let us recall the following definitions which we shall require later.

Definition 2.1: A subset A of space (X, τ) is called

- (1) a pre open set[16] if $A \subset int(cl(A))$ and a pre closed set if $cl(int(A)) \subset A$;
- (2) a semi-open set[3] if $A \subset cl(int(A))$ and semi-closed if $int(cl(A)) \subset A$;
- (3) a α -open set [17] if $A \subset int(cl(int(A)))$ and α -closed set if $cl(int(cl(A))) \subset A$;
- (4) a semi- pre open set[2] if $A \subset cl(int(cl(A)))$ and semi-pre closed set if $int(cl(int(A))) \subset A$;
- (5) a regular open set [21] if $A = int(cl(A))$ and a regular closed set if $A = cl(int(A))$.

Definition 2.2: A subset A of a space (X, τ) is called

- (1) a generalized closed (briefly g-closed) set [13] if $cl(A) \subset U$ whenever $A \subset U$ and U is open.
 - (2) a semi-generalized closed (briefly sg-closed) set [5] if $scl(A) \subset U$ whenever $A \subset U$ and U is semi-open.
 - (3) a generalized semi-closed (briefly gs-closed) set [3] if $scl(A) \subset U$ whenever $A \subset U$ and U is open.
 - (4) a α -generalized closed (briefly αg -closed) set [15] if $\alpha cl(A) \subset U$ whenever $A \subset U$ and U is open in (X, τ) .
 - (5) a generalized α -closed (briefly $g\alpha$ -closed) set[14] if $\alpha cl(A) \subset U$ whenever $A \subset U$ and U is α -open in (X, τ) .
 - (6) a θ -generalized closed (briefly θg -closed) set [7] if $cl_{\theta}(A) \subset U$ whenever $A \subset U$ and U is open in (X, τ) .
 - (7) πg -closed set [8] if $cl(A) \subset U$, whenever $A \subset U$ and U is π -open.
 - (8) $\pi g\alpha$ -closed set [11] if $\alpha cl(A) \subset U$, whenever $A \subset U$ and U is π -open.
 - (9) πgs -closed set [4] if $scl(A) \subset U$, whenever $A \subset U$ and U is π -open.
 - (10) πgb -closed set [20] if $bcl(A) \subset U$, whenever $A \subset U$ and U is π -open.
 - (11) πgp -closed set [19] if $pcl(A) \subset U$, whenever $A \subset U$ and U is π -open.
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The compliment of g -closed (resp. sg -closed, gs -closed, ag -closed, $g\alpha$ -closed, θg -closed, πg -closed, $\pi g\alpha$ -closed, $\pi g s$ -closed, $\pi g\theta$ -closed, $\pi g p$ -closed) is g -open (resp. sg - open, gs - open, ag - open, $g\alpha$ - open, θg - open, πg - open, $\pi g\alpha$ -open, $\pi g s$ - open, $\pi g\theta$ - open, $\pi g p$ - open)

Definition 2.3: Let (X, τ) be a topological space. A point $x \in X$ is said to be in the θ -closure of a subset $A \subseteq X$ if for each open neighbourhood U of x we have $cl(U) \cap A \neq \emptyset$. We shall denote the θ -closure of A by $cl_\theta(A)$. A subset $A \subseteq X$ is called θ - closed if $A = cl_\theta(A)$. The compliment of θ - closed is θ - open.

Definition 2.4: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called

- (1) π -open map [23] if $f(F)$ is π -open in Y for every π -open in X .
- (2) π -irresolute [23] if $f^{-1}(V)$ is π -closed in (X, τ) for every π -closed of (Y, σ) ;
- (3) θ -irresolute [18] if for each θ -open set V in Y , $f^{-1}(V)$ is θ -open in X ;
- (4) θ -continuous [18] if for each open set V in Y , $f^{-1}(V)$ is θ -open in X .

3. $\pi g\theta$ -CLOSED SETS

We introduce the following definition.

Definition 3.1: A subset A of (X, τ) is called $\pi g\theta$ -Closed set (briefly $\pi g\theta$ -closed) if $cl_\theta(A) \subset U$ whenever $A \subset U$ and U is π -open. By $\pi G\theta C(\tau)$ we mean the family of all $\pi g\theta$ -closed subsets of the space (X, τ) . The compliment of $\pi g\theta$ -closed is $\pi g\theta$ -open.

Theorem 3.2:

1. Every θ - closed set is $\pi g\theta$ -closed.
2. Every θg -closed set is $\pi g\theta$ -closed.
3. Every $\pi g\theta$ -closed set is πg -closed.
4. Every $\pi g\theta$ -closed set is $\pi g\alpha$ -closed.
5. Every $\pi g\theta$ -closed set is $\pi g s$ -closed.
6. Every $\pi g\theta$ -closed set is $\pi g b$ -closed.
7. Every $\pi g\theta$ -closed set is $\pi g p$ -closed.

Proof: Straight forward.

Converse of the above need not be true as seen in the following examples.

Example 3.3: Let $X = \{a, b, c\}, \tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \{b, c\}, X\}$. Let $A = \{b\}$. Then A is $\pi g\theta$ -closed but not θ -closed

Example 3.4: Let $X = \{a, b, c, d\}, \tau = \{\emptyset, \{b\}, \{d\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}, X\}$. Let $A = \{a, d\}$. Then A is $\pi g\theta$ -closed but not θg -closed.

Example 3.5: Let $X = \{a, b, c, d, e\}, \tau = \{\emptyset, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$. Let $A = \{a, e\}$. Then A is πg -closed but not $\pi g\theta$ -closed.

Example 3.6: Let $X = \{a, b, c, d\}, \tau = \{\emptyset, \{a\}, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, X\}$, Let $A = \{c\}$. Then A is $\pi g\alpha$ -closed but not $\pi g\theta$ -closed.

Example 3.7: Let $X = \{a, b, c, d\}, \tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, d\}, X\}$, Let $A = \{a\}$. Then A is $\pi g s$ -closed but not $\pi g\theta$ -closed.

Example 3.8: Let $X = \{a, b, c, d\}, \tau = \{\emptyset, \{a\}, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, X\}$. Let $A = \{a, c\}$. Then A is $\pi g b$ -closed but not $\pi g\theta$ - closed.

Example 3.9: Let $X = \{a, b, c, d\}, \tau = \{\emptyset, \{a\}, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, X\}$. Let $A = \{c\}$. Then A is $\pi g p$ -closed but not $\pi g\theta$ -closed. $\pi g\theta$ -closed is independent of closedness, α - closedness, semi -closedness, sg -closedness, gs -closedness, g - closedness, ag - closedness and $g\alpha$ - closedness, as seen in the following examples.

Example 3.10: Let $X = \{a, b, c, d\}, \tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, d\}, X\}$.(i) Let $A = \{d\}$. Then A is $\pi g\theta$ -closed but not g -closed.

Example 3.11: Let $X = \{a, b, c, d, e\}, \tau = \{\emptyset, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$. Let $A = \{e\}$. Then A is g -closed but not $\pi g\theta$ -closed.

Example 3.12: Let $X = \{a, b, c, \}$, $\tau = \{\Phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$. Let $A = \{b\}$. Then A is $\pi g\theta$ -closed but not closed, α -closed, semi-closed.

Example 3.13: Let $X = \{a, b, c, \}$, $\tau = \{\Phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$. Let $A = \{a\}$. Then A is closed, α -closed, semi-closed but not $\pi g\theta$ -closed.

Example 3.14: Let $X = \{a, b, c, d\}$, $\tau = \{\Phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$.

(i) Let $A = \{a, b, c\}$. Then A is $\pi g\theta$ -closed but neither sg -closed nor gs -closed.

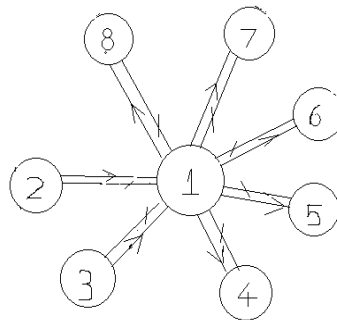
(ii) Let $A = \{a\}$. Then A is sg -closed, gs -closed but not $\pi g\theta$ -closed.

Example 3.15: Let $X = \{a, b, c, d\}$, $\tau = \{\Phi, \{a\}, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, X\}$.

(i) Let $A = \{b, d\}$. Then A is $\pi g\theta$ -closed but neither αg -closed nor $g\alpha$ -closed.

(ii) Let $A = \{c\}$. Then A is αg -closed, $g\alpha$ -closed but not $\pi g\theta$ -closed.

Remark 3.16: The above discussion are summarized in the following diagram.



1 = $\pi g\theta$ -closed, 2 = θ -closed, 3 = θg -closed, 4 = πg -closed, 5 = $\pi g\alpha$ -closed, 6 = $\pi g s$ -closed, 7 = $\pi g b$ -closed, 8 = $\pi g r$ -closed.

Theorem 3.17: If A is regular open and $\pi g\theta$ -closed, then A is θ -closed.

Proof: Let A be regular open and $\pi g\theta$ -closed. Since every regular open is π -open and since A is $\pi g\theta$ -closed, $cl_\theta(A) \subset A$. Then $A = cl_\theta(A)$. Hence A is θ -closed.

Theorem 3.18: Let A be $\pi g\theta$ -closed in (X, τ) . Then $cl_\theta(A) - A$ does not contain any non-empty π -closed set.

Proof: Let F be a non-empty π -closed set such that $F \subset cl_\theta(A) - A$. Since A is $\pi g\theta$ -closed, $A \subset X - F$ where $X - F$ is π -open implies $cl_\theta(A) \subset X - F$. Hence $F \subset X - cl_\theta(A)$. Now, $F \subset cl_\theta(A) \cap X - cl_\theta(A)$ implies $F = \emptyset$ which is a contradiction. Therefore $cl_\theta(A) - A$ does not contain any non-empty π -closed set.

Corollary 3.19: Let A be $\pi g\theta$ -closed in (X, τ) . Then A is θ -closed if and only if $cl_\theta(A) - A$ is π -closed.

Proof: Let A be θ -closed. Then $cl_\theta(A) = A$ implies $cl_\theta(A) - A = \emptyset$ which is π -closed. Assume $cl_\theta(A) - A = \emptyset$ is π -closed. Then $cl_\theta(A) - A = \emptyset$. Hence $cl_\theta(A) = A$.

Theorem 3.20: If A is $\pi g\theta$ -closed set and B is any set such that $A \subset B \subset cl_\theta(A)$, then B is $\pi g\theta$ -closed set.

Proof: Let $B \subset U$ and U be π -open. Given $A \subset B$. Then $A \subset U$. Since A is $\pi g\theta$ -closed, $A \subset U$ implies $cl_\theta(A) \subset U$. By assumption it follows that $cl_\theta(B) \subset cl_\theta(A) \subset U$. Then B is $\pi g\theta$ -closed.

Theorem 3.21: A finite union of $\pi g\theta$ -closed sets is always a $\pi g\theta$ -closed.

Proof: Let $A, B \in \pi G\theta C(X)$. Let U be any π -open set such that $(A \cup B) \subseteq U$. Since $cl_\theta(A \cup B) = cl_\theta(A) \cup cl_\theta(B) \subseteq U \cup U = U$. This implies $cl_\theta(A \cup B) \subseteq U$. Hence $A \cup B$ is also a $\pi g\theta$ -closed set.

Remark 3.22: Finite intersection of $\pi g\theta$ -closed sets need not be $\pi g\theta$ -closed.

Example 3.23: Let $X = \{a, b, c, d\}$ and $\tau = \{\Phi, \{a\}, \{b\}, \{a, b\}, \{a, b, d\}, X\}$. Let $A = \{a, b, c\}$ and $B = \{a, b, d\}$. Clearly A and B are $\pi g\theta$ -closed sets. But $A \cap B = \{a, b\}$ is not a $\pi g\theta$ -closed set.

4. $\pi g\theta$ -OPEN SETS

Definition 4.1: A set $A \subset X$ is called $\pi g\theta$ -open if and only if its complement is $\pi g\theta$ -closed.

Remark 4.2: $cl_0(X-A) = X - int_0(A)$. By $\pi G\theta O(\tau)$ we mean the family of all $\pi g\theta$ -open subsets of the space (X, τ) .

Theorem 4.3: If $A \subset X$ is $\pi g\theta$ -open if and only if $F \subset int_0(A)$ whenever F is π -closed and $F \subset A$.

Proof: Necessity: Let A be $\pi g\theta$ -open. Let F be π -closed and $F \subset A$. Then $X-A \subset X-F$ where $X-F$ is π -open. By assumption, $cl_0(X-A) \subset X-F$. By remark 4.2, $X - int_0(A) \subset (X-F)$. Thus $F \subset int_0(A)$.

Sufficiency: Suppose F is π -closed and $F \subset A$ such that $F \subset int_0(A)$. Let $X-A \subset U$ where U is π -open. Then $X-U \subset A$ where $X-U$ is π -closed. By hypothesis, $X-U \subset int_0(A)$ which implies $cl_0(X-A) \subset X - int_0(A) \subset U$. Thus $X-A$ is $\pi g\theta$ -closed and A is $\pi g\theta$ -open.

Theorem 4.4: If $int_0(A) \subset B \subset A$ and A is $\pi g\theta$ -open, then B is also $\pi g\theta$ -open.

Proof: Let $int_0(A) \subset B \subset A$. Thus $X-A \subset X-B \subset cl_0(X-A)$. Since $X - A$ is $\pi g\theta$ -closed, by theorem 3.20, $X-A \subset X-B \subset cl_0(X-A)$ implies $X-B$ is $\pi g\theta$ -closed.

Remark 4.5: For any $A \subset X$, $int_0(cl_0(A)-A) = \Phi$.

Theorem 4.6: If $A \subset X$ is $\pi g\theta$ -closed, then $cl_0(A) - A$ is $\pi g\theta$ -open.

Proof: Let A be $\pi g\theta$ -closed. Let F be π -closed. $F \subset cl_0(A) - A$. By theorem 3.18, $F = \Phi$. By remark 4.5, $int_0(cl_0(A)-A) = \Phi$. Thus $F \subset int_0(cl_0(A)-A) = \Phi$. Thus $cl_0(A) - A$ is $\pi g\theta$ -open.

Definition 4.7: A space (X, τ) is called a $\pi g\theta$ - $T_{1/2}$ space if every $\pi g\theta$ -closed set is θ -closed.

Theorem 4.8:

(i) $\theta O(\tau) \subset \pi G\theta O(\tau)$. (ii) A space (X, τ) is $\pi g\theta$ - $T_{1/2}$ space iff $\theta O(\tau) = \pi G\theta O(\tau)$.

Proof: Let A be θ -open, then $X-A$ is θ -closed. So $X-A$ is of $\pi g\theta$ -closed. Then A is of $\pi g\theta$ -open. Hence $\theta O(\tau) \subset \pi G\theta O(\tau)$.

(ii) **Necessity:** Let (X, τ) be $\pi g\theta$ - $T_{1/2}$ space. Let $A \in \pi G\theta O(\tau)$. Then $X-A$ is $\pi g\theta$ -closed. By hypothesis, $X - A$ is θ -closed, thus $A \in \theta O(\tau)$. Thus $\pi G\theta O(\tau) = \theta O(\tau)$.

Sufficiency: Let $\theta O(\tau) = \pi G\theta O(\tau)$. Let A be of $\pi g\theta$ -closed. Then $X-A$ is $\pi g\theta$ -open. $X-A \in \pi G\theta O(\tau)$. This implies $X-A \in \theta O(\tau)$. Hence A is θ -closed. This implies (X, τ) is a $\pi g\theta$ - $T_{1/2}$ space.

Theorem 4.9: For a topological space (X, τ) the following are equivalent.

(i) X is a $\pi g\theta$ - $T_{1/2}$ space. (ii) Every singleton set is π -closed or θ -open.

Proof: (i) \Rightarrow (ii): Let X be a $\pi g\theta$ - $T_{1/2}$ space. Let $x \in X$ and assuming that $\{x\}$ is not π -closed. Then clearly $X - \{x\}$ is trivially $\pi g\theta$ -closed in X . Since X is $\pi g\theta$ - $T_{1/2}$ space, $X - \{x\}$ is θ -closed. Therefore $\{x\}$ is θ -open.

(ii) \Rightarrow (i) Assume every singleton of X is either π -closed or θ -open. Let A be $\pi g\theta$ -closed set. Let $\{x\} \in cl_0(A)$.

Case (i): Let $\{x\}$ be π -closed. Suppose $\{x\} \notin cl_0(A)$. Then $\{x\} \in cl_0(A) - A$. By theorem.3.18, $\{x\} \in A$. Hence $cl_0(A) \subset A$.

Case (ii) : Let $\{x\}$ be θ -open. Since $\{x\} \in cl_0(A)$, we have $\{x\} \cap A \neq \Phi$ implies $\{x\} \in A$. Therefore $cl_0(A) \subset A$. Hence A is θ -closed.

5. $\pi g\theta$ -continuous and $\pi g\theta$ -irresolute functions

Definition 5.1.: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called $\pi g\theta$ -continuous if every $f^{-1}(V)$ is $\pi g\theta$ -closed in (X, τ) for every closed set V of (Y, σ) .

Definition 5.2: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called $\pi g\theta$ -irresolute if $f^{-1}(V)$ is $\pi g\theta$ -closed in (X, τ) for every $\pi g\theta$ -closed set V in (Y, σ) .

Remark 5.3: 1. $\pi g\theta$ -irresolute function is independent of θ -irresoluteness, as seen in the following examples.

Example 5.4:(a). Let $X=Y= \{a, b, c, d\}$, $\tau = \{\Phi, \{a\}, \{d\}, \{a, d\}, \{a, b, d\}, \{a, c, d\}, X\}$, $\sigma = \{\Phi, \{a\}, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, X\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an identity function. Then f is θ -irresolute but not $\pi g\theta$ -irresolute, since $f^{-1}\{b, c, d\} = \{b, c, d\}$ is not $\pi g\theta$ -closed in (X, τ) .

Example 5.4: (b). Let $X=Y=\{a, b, c, d\}$, $\tau = \{\Phi, \{a\}, \{d\}, \{a, d\}, \{a, b, d\}, \{a, c, d\}, X\}$, $\sigma = \{\Phi, \{a\}, \{c\}, \{d\}, \{a, d\}, \{a, c\}, \{c, d\}, \{a, c, d\}, \{a, b, d\}, X\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an identity function. Then f is $\pi g\theta$ -irresolute but not θ -irresolute, since $f^{-1}\{a, b, d\} = \{a, b, d\}$ is not θ -closed in (X, τ) .

Remark 5.3:2. Every θ -continuous is $\pi g\theta$ -continuous The converse of the above need not be true as seen in the following examples.

Example 5.5: Let $X=Y=\{a, b, c, d, e\}$, $\tau = \{\Phi, \{a, b\}, \{c\}, \{a, b, c\}, X\}$, $\sigma = \{\Phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an identity function. Then f is $\pi g\theta$ -continuous but not θ -continuous, since $f^{-1}\{c, d, e\} = \{c, d, e\}$ is not θ -closed in (X, τ) .

Remark 5.6: Composition of two $\pi g\theta$ -continuous function need not be $\pi g\theta$ -continuous.

Example 5.7: Let $X=Y=Z =\{a, b, c, d, e\}$, $\tau = \{\Phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$, $\sigma = \{\Phi, \{a, b\}, \{c\}, \{a, b, c\}, X\}$, $\eta = \{\Phi, \{e\}, \{a, b, c, d\}, X\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an identity function. Let $g: (Y, \sigma) \rightarrow (Z, \eta)$ be an identity function. Both f and g are $\pi g\theta$ -continuous but $g \circ f$ is not $\pi g\theta$ -continuous, since $(g \circ f)^{-1}\{a, b, c, d\} = \{a, b, c, d\}$ is not $\pi g\theta$ -closed in (X, τ) .

Definition 5.8: A function $f: X \rightarrow Y$ is said to be pre- θ -closed if $f(U)$ is θ -closed in Y for each θ -closed set in X .

Proposition 5.9: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be π -irresolute and pre- θ -closed map. Then $f(A)$ is $\pi g\theta$ -closed in Y for every $\pi g\theta$ -closed set A of X .

Proof: Let A be any $\pi g\theta$ -closed set of X and U be any π -open set of Y containing $f(A)$. Since f is π -irresolute, $f^{-1}(U)$ is π -open in X and $A \subseteq f^{-1}(U)$. Therefore we have $cl_{\theta}(A) \subseteq f^{-1}(U)$ and hence $f(cl_{\theta}(A)) \subseteq U$.

Since f is pre- θ -closed, $cl_{\theta}(f(A)) \subseteq cl_{\theta}(f(cl_{\theta}(A))) = f(cl_{\theta}(A)) \subseteq U$. Hence $f(A)$ is $\pi g\theta$ -closed in Y .

Theorem 5.10: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function.

- (i) If f is $\pi g\theta$ -irresolute and X is $\pi g\theta$ - $T_{1/2}$ space, then f is θ -irresolute.
- (ii) If f is $\pi g\theta$ -continuous and X is $\pi g\theta$ - $T_{1/2}$ space then f is θ -continuous.

Proof:

(i) Let V be θ -closed in Y . Since f is $\pi g\theta$ -irresolute, $f^{-1}(V)$ is $\pi g\theta$ -closed in X .

Since X is $\pi g\theta$ - $T_{1/2}$ space, $f^{-1}(V)$ is θ -closed in X . Hence f is θ -irresolute.

(ii) Let V be closed in Y . Since f is $\pi g\theta$ -continuous, $f^{-1}(V)$ is $\pi g\theta$ -closed in X .

By assumption, it is θ -closed. Therefore f is θ -continuous.

Theorem 5.11: If the bijective $f: (X, \tau) \rightarrow (Y, \sigma)$ is θ -irresolute and π -open map then f is $\pi g\theta$ -irresolute.

Proof: Let V be $\pi g\theta$ -closed in Y . Let $f^{-1}(V) \subset U$ where U is π -open in X . Then $V \subset f(U)$ and $f(U)$ is π -open implies $cl_{\theta}(V) \subset f(U)$. This implies $f^{-1}(cl_{\theta}(V)) \subset U$. Since f is θ -irresolute, $f^{-1}(cl_{\theta}(V))$ is θ -closed. Hence $cl_{\theta}(f^{-1}(cl_{\theta}(V))) \subset f^{-1}(cl_{\theta}(V)) \subset U$. Therefore f is $\pi g\theta$ -irresolute.

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