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SYNCHRONIZATION & ANTI-SYNCHRONIZATION OF CHARGED PARTICLE IN THE FIELD OF THREE PLANE WAVES VIA ACTIVE CONTROL

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ABSTRACT

In this paper, we have investigated the synchronization and anti-synchronization behaviour of two identical dynamical model of charge particle in the field of three plane waves evolving from different initial conditions using the active control technique based on the Lyapunov stability theory and Routh-Hurwitz criteria. The designed controller, with our own choice of the coefficient matrix of the error dynamics that satisfy the Lyapunov stability theory and the Routh-Hurwitz criteria, are found to be effective in the stabilization of the error states at the origin, thereby, achieving synchronization and anti-synchronization between the states variables of two nonlinear dynamical systems under consideration. The results are validated by numerical simulations using mathematica.

Keywords: *Synchronization; Anti-synchronization (AS); Active control.*

1. INTRODUCTION

In 1990, after the introduction of PC method [1] to synchronize the two identical chaotic systems with different initial conditions, chaos synchronization has received an increasing attention and been investigated widely due to its potential application in almost all areas of engineering and sciences [2–4]. Many effective control methods [5–8] have been proposed to achieve chaos synchronization, such as linear and nonlinear feedback controls. However, most researches on chaos synchronization focused on certain chaotic systems without consideration of parameter variation uncertainty and external disturbance perturbation. But in practical situations, many chaotic systems are inevitably affected by parameter variations and external disturbances. Moreover, some or all of the system parameters and external disturbance uncertainties are unknown or variable from time to time. Therefore, investigation of system parameter variations and external disturbance perturbations in synchronization between drive and response chaotic systems has become an interesting and important research topic in recent years. In the work [9], an adaptive control law with single-state variable feedback was derived and applied to achieve the state synchronization of two identical Lorenz systems. An active sliding mode control was proposed by Zhang and Ma [10] to synchronize chaotic systems with parameter perturbation. Zhang et al. [11] also presented a sliding mode control to resolve the conquer synchronization problem in noise-perturbed chaotic systems. In the work [12], an intermittent parametric adaptive control method was studied to synchronize two logistic maps, and the corresponding sufficient conditions for synchronization are drawn. Based on Lyapunov stabilization theory, Huang et al. [13] proposed an adaptive controller with parameters identification for synchronizing a class of chaotic systems with unknown parameters. Park [14] developed a nonfragile controller using the Lyapunov functional technique combined with LMI technique to achieve synchronization problem of a class of chaotic systems with controller gain variations. Shahzad [15-17] has studied the synchronization and AS behavior for the two identical dynamical models of satellites motion using active control technique and found a robust synchronization as well as AS.

Keeping in mind the above studies, in this article, we have applied the active control technique based on the Lyapunov stability theory and the Routh-Hurwitz criteria to study the synchronization and AS behavior of two identical dynamical models of charge particle in the field of three plane waves evolving from different initial conditions. The system under consideration is chaotic for some values of parameter involved in the system. In synchronization, the two systems (master & slave) are synchronized that starts with different initial conditions. The same problem may be treated as the design of control laws for full chaotic slave system using the known information of the master system so as to ensure that the controlled receiver synchronizes with the master system. Hence, the slave chaotic system completely traces the dynamics of the master system in the course of time. The aim of this study is to trace the chaotic dynamics of the master system under study based on synchronization and AS phenomenon. To the best of my knowledge nobody studied this before.

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2. DESCRIPTION OF THE MODEL

The interaction of charged particles with a wave packet is a basic and challenging problem appearing in astrophysics, plasma physics, and condensed matter physics that may yield undesirable effects in a number of technological devices such as the destruction of magnetic surfaces in tokamaks. The simplest mathematical model [18] to examine the problem of regularization of the chaotic dissipative dynamics of a charged particle in a wave packet by a small amplitude uncorrelated wave, which is added to the initial wave packet, is given by:

$$\ddot{x}+\gamma\dot{x}=-\frac{e}{m}\left[E_0\sin\left(k_0x-\omega_0t\right)+E_c\sin\left(k_cx-\omega_ct\right)+E_s\sin\left(k_sx-\omega_st-\Psi\right)\right]. \tag{2.1}$$

where the amplitudes E_0, E_c, E_s, wave numbers k_0, k_c, k_s, and frequencies ω_0, ω_c, ω_s correspond to the main, chaos-inducing, and chaos-suppressing waves, respectively, Ψ is an initial phase, *e* and *m* are the charge and mass of the particle, respectively, and where weak dissipation ($\gamma \ll 1$) and non-uniform amplitudes ($E_{c,s}/E_0 < 1$) are assumed.

3. SYNCHRONIZATION VIA ACTIVE CONTROL

For a system of two coupled chaotic dynamical systems:

$$\text{Master System: }\begin{cases}\dot{x}_i = x_{i+1} & 1\le i\le n-1\\ \dot{x}_n = f(x,t) & x=[x_1,x_2...x_n]\in\Re^n\end{cases} \tag{3.1}$$

$$\text{Slave System: }\begin{cases}\dot{y}_i = y_{i+1}+u_i(t) & 1\le i\le n-1\\ \dot{y}_n = g(y,t)+u_n(t) & y=[y_1,y_2...y_n]\in\Re^n \ \&\ u=[u_1,u_2...u_n]\in\Re\end{cases}, \tag{3.2}$$

where $x(t)\in\Re^n$ & $y(t)\in\Re^n$ are the phase space (state variables), $f(x,t)$ & $g(y,t)$ are the corresponding nonlinear functions and $u(t)$ are the control functions to be determined, synchronization in a direct sense implies $\lim_{t\to\infty}\left|x(t)-y(t)\right|\to 0$. When this occurs the coupled systems are said to be completely synchronized. Since chaos synchronization is related to the observer problem in control theory [19], the problem may be treated as the design of control laws for full chaotic slave system using the known information of the master system so as to ensure that the controlled receiver synchronizes with the master system and hence, the slave chaotic system completely traces the dynamics of the master in the course of time.

In order to formulate the active controllers, we write the system (2.1) in two first order differential equations as shown below:

Let $x = x_1$ and $\dot{x}_1 = x_2$, then we have

$$\begin{aligned}&\dot{x}_1 = x_2,\\ &\dot{x}_2=-\gamma x_2-\frac{e}{m}\left[E_0\sin\left(k_0x_1-\omega_0t\right)+E_c\sin\left(k_cx_1-\omega_ct\right)+E_s\sin\left(k_sx_1-\omega_st-\Psi\right)\right].\end{aligned} \tag{3.3}$$

Let us define another system

$$\begin{aligned}&\dot{y}_1 = y_2+u_1(t),\\ &\dot{y}_2=-\gamma y_2-\frac{e}{m}\left[E_0\sin\left(k_0y_1-\omega_0t\right)+E_c\sin\left(k_cy_1-\omega_ct\right)+E_s\sin\left(k_sy_1-\omega_st-\Psi\right)\right]+u_2(t).\end{aligned} \tag{3.4}$$

Where (3.3) and (3.4) are called the master and slave systems respectively and in slave system, $u_1(t)$ and $u_2(t)$ are control functions to be determined. Let $e_i(t)=y_i(t)-x_i(t)$ be the synchronization errors such that $\lim_{t\to\infty}e_i(t)\to 0$ for $i=1,2$. From (3.3) and (3.4), we have

$$\begin{aligned}&\dot{e}_1(t)=e_2(t)+u_1(t),\\ &\dot{e}_2(t)=-\gamma e_2-\frac{eE_0}{m}\left[\sin\left(k_0y_1-\omega_0t\right)-\sin\left(k_0x_1-\omega_0t\right)\right]-\frac{eE_c}{m}\left[\sin\left(k_cy_1-\omega_ct\right)-\sin\left(k_cx_1-\omega_ct\right)\right]\\ &\qquad-\frac{eE_s}{m}\left[\sin\left(k_sy_1-\omega_st-\Psi\right)-\sin\left(k_sx_1-\omega_st-\Psi\right)\right]+u_2(t).\end{aligned} \tag{3.5}$$

In order to express (3.5) as only linear terms in $e_1(t)$ and $e_2(t)$, we redefine the control functions as follows:

$$u_1(t) = v_1(t),$$
$$u_2(t) = \frac{eE_0}{m}\left[\sin\left(k_0 y_1 - \omega_0 t\right) - \sin\left(k_0 x_1 - \omega_0 t\right)\right] + \frac{eE_c}{m}\left[\sin\left(k_c y_1 - \omega_c t\right) - \sin\left(k_c x_1 - \omega_c t\right)\right]$$
$$+\frac{eE_s}{m}\left[\sin\left(k_s y_1 - \omega_s t - \Psi\right) - \sin\left(k_s x_1 - \omega_s t - \Psi\right)\right] + v_2(t). \tag{3.6}$$

From (3.5) and (3.6), we have

$$\dot{e}_1(t) = e_2(t) + v_1(t),$$
$$\dot{e}_2(t) = -\gamma e_2 + v_2(t). \tag{3.7}$$

Equation (3.7) is the error dynamics, which can be interpreted as a control problem where the system, to be controlled is a linear system with control inputs $v_i(t) = v_i\left(e_i(t), e_i(t)\right)$ for $i = 1, 2$. As long as these feedbacks stabilize the system, $\lim_{t\to\infty} e_i(t) \to 0$ for $i = 1, 2$. This simply implies that the two systems (3.3) and (3.4) evolving from different initial conditions are synchronized. As functions of $e_1(t)$ and $e_2(t)$, we choose $v_1(t)$ and $v_2(t)$ as follows:

$$\begin{pmatrix} v_1(t) \\ v_2(t) \end{pmatrix} = D \begin{pmatrix} e_1(t) \\ e_2(t) \end{pmatrix} \tag{3.8}$$

where $D = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, is a 2×2 constant feedback matrix to be determined. Hence the error system (3.7) can be written as:

$$\begin{pmatrix} \dot{e}_1(t) \\ \dot{e}_2(t) \end{pmatrix} = C \begin{pmatrix} e_1(t) \\ e_2(t) \end{pmatrix} \tag{3.9}$$

where $C = \begin{pmatrix} a & 1+b \\ c & d-\gamma \end{pmatrix}$, is the coefficient matrix.

According to the Lyapunov stability theory and the Routh-Hurwitz criteria, if

$$a + d - \gamma < 0,$$
$$c\left(1+b\right) - a\left(d-\gamma\right) < 0. \tag{3.10}$$

then the eigen values of the coefficient matrix of error system (3.7) must be real or complex with negative real parts and, hence, stable synchronized dynamics between systems (3.3) and (3.4) is guaranteed. Let

$$a + d - \gamma = c\left(1+b\right) - a\left(d-\gamma\right) = -E, \tag{3.11}$$

Where $E > 0$ is a real number which is usually set equal to 1. There are several ways of choosing the constant elements *a*, *b*, *c*, *d* of matrix *D* in order to satisfy the Lyapunov stability theory and the Routh-Hurwitz criteria (3.10).

4. ANTI-SYNCHRONIZATION VIA ACTIVE CONTROL

Anti-synchronization (AS) of two coupled systems given by (3.1) and (3.2) means $\lim_{t\to\infty}\left|x(t) + y(t)\right| \to 0$. This phenomenon has been investigated both experimentally and theoretically in many physical systems [15, 20-25]. In a recent study of Shahzad [15], it has been found that the AS phenomenon was working faster than synchronization.

In order to formulate the active controllers for AS, we need to redefine the error functions as $e_i(t) = y_i(t) + x_i(t)$, where $e_i(t)$ are called the AS errors such that $\lim_{t\to\infty} e_i(t) \to 0$ for $i = 1, 2$. From (3.3) and (3.4), error dynamics can be written as:

$$\dot{e}_1(t) = e_2(t) + u_1(t),$$

$$\dot{e}_2(t) = -\gamma e_2 - \frac{eE_0}{m}\left[\sin\left(k_0 y_1 - \omega_0 t\right) + \sin\left(k_0 x_1 - \omega_0 t\right)\right] - \frac{eE_c}{m}\left[\sin\left(k_c y_1 - \omega_c t\right) + \sin\left(k_c x_1 - \omega_c t\right)\right]$$
$$-\frac{eE_s}{m}\left[\sin\left(k_s y_1 - \omega_s t - \Psi\right) + \sin\left(k_s x_1 - \omega_s t - \Psi\right)\right] + u_2(t). \tag{4.1}$$

In order to express (4.1) as only linear terms in $e_1(t)$ and $e_2(t)$, we redefine the control functions as follows:

$$u_1(t) = v_1(t),$$
$$u_2(t) = \frac{eE_0}{m}\left[\sin\left(k_0 y_1 - \omega_0 t\right) + \sin\left(k_0 x_1 - \omega_0 t\right)\right] + \frac{eE_c}{m}\left[\sin\left(k_c y_1 - \omega_c t\right) + \sin\left(k_c x_1 - \omega_c t\right)\right]$$
$$+\frac{eE_s}{m}\left[\sin\left(k_s y_1 - \omega_s t - \Psi\right) + \sin\left(k_s x_1 - \omega_s t - \Psi\right)\right] + v_2(t). \tag{4.2}$$

From (4.1) and (4.2), we have

$$\dot{e}_1(t) = e_2(t) + v_1(t),$$
$$\dot{e}_2(t) = -\gamma e_2(t) + v_2(t). \tag{4.3}$$

Equation (4.3) is the error dynamics, which can be interpreted as a control problem where the system, to be controlled is a linear system with control inputs $v_i(t) = v_i\left(e_i(t), e_i(t)\right)$ for $i = 1, 2$. As long as these feedbacks stabilize the system, $\lim_{t\to\infty} e_i(t) \to 0$ for $i = 1, 2$. This simply implies that the two systems (3.3) and (3.4) evolving from different initial conditions are anti-synchronized. As functions of $e_1(t)$ and $e_2(t)$, we choose $v_1(t)$ and $v_2(t)$ as follows:

$$\begin{pmatrix} v_1(t) \\ v_2(t) \end{pmatrix} = D \begin{pmatrix} e_1(t) \\ e_2(t) \end{pmatrix} \tag{4.4}$$

where $D = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, is a 2×2 constant feedback matrix to be determined. Hence the error system (4.3) can be written as:

$$\begin{pmatrix} \dot{e}_1(t) \\ \dot{e}_2(t) \end{pmatrix} = C \begin{pmatrix} e_1(t) \\ e_2(t) \end{pmatrix} \tag{4.5}$$

where $C = \begin{pmatrix} a & 1+b \\ c & d-\gamma \end{pmatrix}$, is the coefficient matrix.

According to the Lyapunov stability theory and the Routh-Hurwitz criteria, if

$$a + d - \gamma < 0,$$
$$c\left(1+b\right) - a\left(d-\gamma\right) < 0. \tag{4.6}$$

then the eigen values of the coefficient matrix of error system (4.3) must be real or complex with negative real parts and, hence, stable anti-synchronized dynamics between systems (3.3) and (3.4) is guaranteed. Let

$$a + d - \gamma = c\left(1+b\right) - a\left(d-\gamma\right) = -E. \tag{4.7}$$

Where $E > 0$ is a real number which is usually set equal to 1. There are several ways of choosing the constant elements *a*, *b*, *c*, *d* of matrix *D* in order to satisfy the Lyapunov stability theory and the Routh-Hurwitz criteria (4.6).

5. NUMERICAL SIMULATION

For the constant elements of feedback matrix, choosing $a = d = -0.5$ and for the parameters involved in system under investigation, $\gamma = 0.1$, $\omega_0 = 1$, $\omega_c = 2.26$, $\omega_s = 0.5$, $k_0 = 1$, $k_c = 1.22$, $k_s = 0.75$ $E_0 = 0.3$, $E_c = 0.3$, $E_s = 0.3$, $e = 0.4$, $\Psi = \pi/3$, $m = 0.4$ together with the initial conditions for synchronization are $\left[x_1(0), y_1(0)\right] = \left[0, 0\right]$ and $\left[x_2(0), y_2(0)\right] = \left[0.1, 1.5\right]$, and for AS are $\left[x_1(0), y_1(0)\right] = \left[0, 0\right]$ and $\left[x_2(0), y_2(0)\right] = \left[0.1, 1\right]$, we have simulated the system under consideration using *mathematica* for both synchronization as well as AS phenomenon. The results obtained show that the system under consideration achieved synchronization & AS. Phase plots, time series analysis and error analysis diagrams are the witness of achieving robust synchronization as well as AS between master and slave system given by (3.3) & (3.4). Further, it also has been confirmed by the convergence of the synchronization and AS quality defined by

$$e(t) = \sqrt{e_1^2(t) + e_2^2(t)} \tag{5.1}$$

Figure (7) confirms that the convergence quality in both synchronization and AS phenomenon which is almost same and sure for the simulated dynamical models of charge particle in the field of three plane waves.

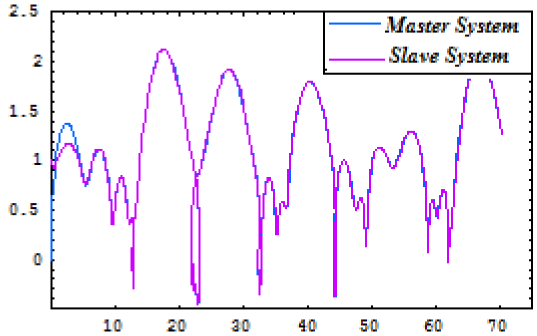

Figure 1: Phase Plot of Master & Slave System in Synch.

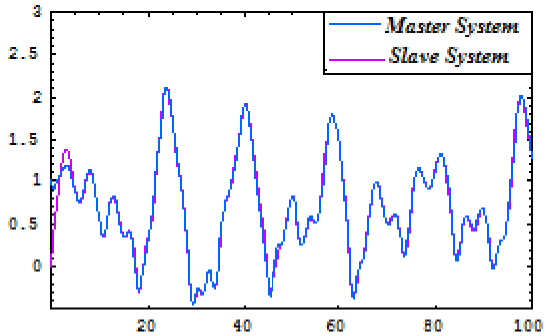

Figure 2: Time Series of Master & Slave System in Synch.

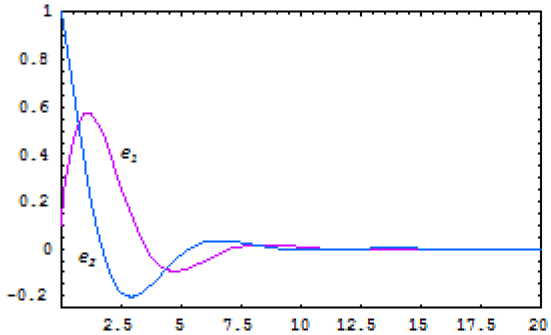

Figure 3: Time Series of $e_1(t)$ & $e_2(t)$ in Synch.

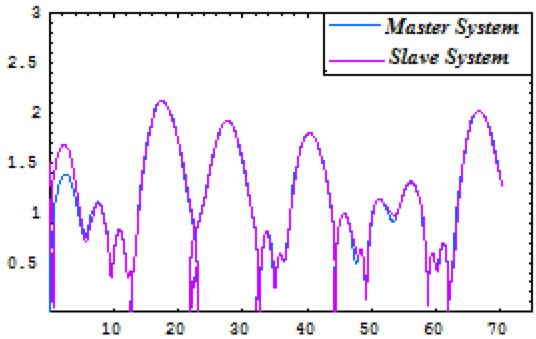

Figure 4: Phase Plot of Master & Slave System in AS

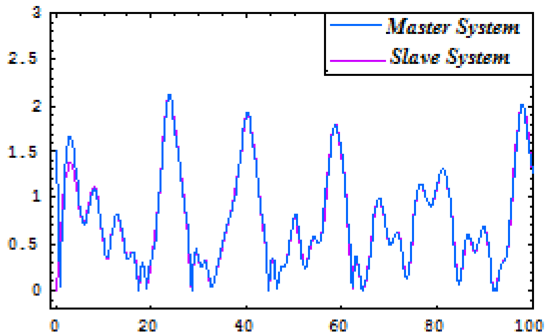

Figure 5: Time Series of Master & Slave System in AS

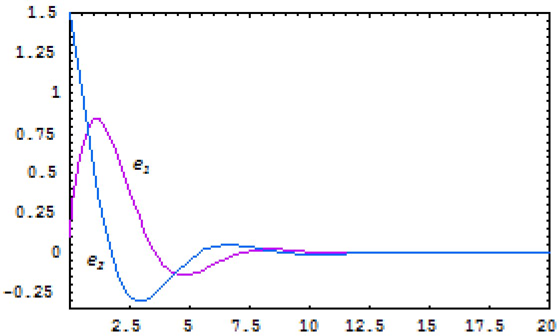

Figure 6: Time Series of $e_1(t)$ & $e_2(t)$ in AS

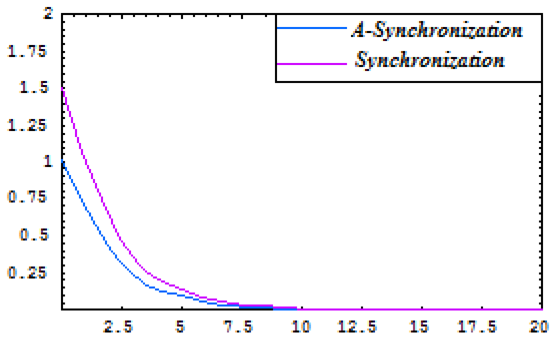

Figure 7: Convergence of Errors

6. CONCLUSION

In this paper, we have investigated the synchronization and AS behaviour of the two identical dynamical models of charge particle in the field of three plane waves evolving from different initial conditions via the active control technique based on the Lyapunov stability theory and the Routh-Hurwitz criteria. The results were validated by numerical simulations using *mathematica*. For the errors in synchronization and AS behavior of the system under study, we have observed that the rate of convergence of errors is almost same and sure in synchronization as well AS phenomenon that have been clearly seen in figure (7). We can conclude that the two identical dynamical models of charge particle in the field of three plane waves have achieved a robust synchronization as well as AS.

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