International Journal of Mathematical Archive-3(11), 2012, 3927-3934
IJMA Available online through www.ijma.info ISSN 2229 – 5046

COST-BENEFIT ANALYSIS OF A TWO-UNIT AUTOMATIC POWER FACTOR CONTROLLER SYSTEM WITH VARIATION IN POWER FACTOR AND OCCURRENCE OF NO OTHER EVENT DURING INSPECTION

GULSHAN TANEJA[1], ROOSEL JAIN[2,3*] and P. K. BHATIA[4]

[1]Department of Mathematics, Maharishi Dayanand University, Rohtak-124001, Haryana, India

[2]Department of Applied Mathematics, Deenbandhu Chhotu Ram University of Science &Technology, Murthal, (Sonepat)-131039 Haryana, India

[3]Department of Applied Mathematics, Amity Institute of Applied Sciences, Amity University, Noida-201301, U.P., India

[4]Department of Applied Mathematics, Deenbandhu Chhotu Ram University of Science &Technology, Murthal, (Sonepat)-131039 Haryana, India

(Received on: 31-10-12; Revised & Accepted on: 28-11-12)

ABSTRACT

Failures and financial constraints of complex power systems need the use of probabilistic methods to calculate reliability indices of system effectiveness. The present paper deals with probabilistic analysis of Automatic Power Factor Controller (APFC) working in industrial companies. The power factor correction of electrical loads and energy losses due to poor power factor are the problems common to all industrial companies. To control poor power factor, Automatic Power Factor Controller (APFC) systems are installed in such companies. Here, a system comprising of two identical cold standby units have been analyzed using semi-Markov processes and regenerative point technique. Initially, the system is operative with controlled power factor. Then it may transit to a state with uncontrolled power factor or it may undergo inspection on failure. The inspection is carried out to detect the type of failure which can be due to fuse blown off, transformer burnt, programming problem, output relay faulty. Out of above mentioned failures, first two are irreparable while last two are repairable. It is also assumed that during inspection, no other event takes place. The various reliability indices of system effectiveness are obtained and economic analysis is done to increase the profit of users of such systems graphically.

Key Words: *Automatic Power Factor Controller (APFC) system, controlled/uncontrolled power factor, reliability indices of system effectiveness, regenerative point technique, semi-Markov Processes.*

2000 Mathematics Subject Classification: *90B25 and 60K10.*

---

1. INTRODUCTION

The probabilistic analysis of the systems is done to calculate various reliability indices of system effectiveness which helps the users to increase the reliability of the system by minimizing failures, energy losses and maintainenances costs. Cold standby systems are analyzed by researchers like Gopalan and Muralidhar [1] including many others like [4], [5], [6] and [9]. In all these research papers, stochastic analysis of various situations is done by considering different failures and repair facilities. Parashar and Taneja [7] also analyzed a two unit PLC hot standby system with two types of repair facilities. Goyal et al [8] also developed a model for analyzing two-unit cold standby sulphated juice pump system working in sugar mill.

Here the system under consideration comprises two automatic power factor controllers wherein initially one unit is operative and the other is cold standby. The power factor correction of electrical loads is a problem common to all industrial companies. Most loads on an electrical distribution system are inductive in nature, therefore, take lagging currents. Some typical examples of such modern systems include transformers, fluorescent lighting, AC induction motors, Arc/induction furnaces etc. which draw not only active power (kW) from the supply but also inductive reactive power (kVAr). Also, apparent power (kVA) is combination of active and reactive power. Power factor is defined as the ratio of active power (kW) to apparent power (kVA) [2].

Corresponding author: *ROOSEL JAIN[*,2]Department of Applied Mathematics, Deenbandhu Chhotu Ram University of Science &Technology, Murthal,(Sonepat)-131039 Haryana, India*

The low power factor is highly undesirable as it causes an increase in current, resulting in additional losses of active power in all the elements of power systems. APFC is one of the systems which improves power factor and consists of number of capacitors that are switched on/off by means of contactors or relays. The capacitor draws a leading current and partly or completely neutralizes the lagging reactive component of load current. This helps in raising the power factor of the load. The APFC system is programmed to switch on/off capacitors automatically as and when needed to bring the power factor in the desired range (closer to one). The improvement in power factor is very important for consumers and generating stations as they have to pay electricity charges for their maximum demand in kVA plus the units consumed [2]. If the power factor is improved then there is reduction in maximum kVA demand and consequently there will be annual savings due to maximum demand charges. Hence, the reliability and profit aspects of APFC system are of utmost importance in the present scenario.

2. SYMBOLS AND NOTATIONS FOR STATES OF SYSTEM

λ	constant rate of failure
β_1	rate with which power factor changes from controlled mode to uncontrolled mode
β_2	rate with which power factor changes from uncontrolled mode to controlled mode
$i(t), I(t)$	p.d.f. and c.d.f. of inspection time
p_1	probability of failure of type I (Fuse blown off)
p_2	probability of failure of type II (Transform burnt)
p_3	probability of failure of type III (Programming Problem)
p_4	probability of failure of type IV (output relay faulty)
$g_1(t), G_1(t)$	p.d.f. and c.d.f. of failure of type I with controlled power factor
$g_2(t), G_2(t)$	p.d.f. and c.d.f. of failure of type II with controlled power factor
$g_3(t), G_3(t)$	p.d.f. and c.d.f. of failure of type III with controlled power factor
$g_4(t), G_4(t)$	p.d.f. and c.d.f. of failure of type IV with controlled power factor
© / Ⓢ	Laplace/ Stieltjes convolution
$h(t), H(t)$	p.d.f. and c.d.f. of the time of conversion of power factor from uncontrolled to controlled mode.
O	the unit is operative
CS	cold standby unit
C	power factor controlled
$\overline{C}$	power factor not controlled
F_i	unit is under inspection on failure
F_{r1}	the main unit is under repair in case of failure of type I (fuse blown off)
F_{r2}	the main unit is under repair in case of failure of type II (transformer burnt)
F_{r3}	the main unit is under repair in case of failure of type III (programming problem)
F_{r4}	the main unit is under repair in case of failure of type IV (output relay faulty)
F_{wi}	cold stand by unit waiting for inspection by the repairman
F_{R_1}	the continuation of repair of main unit from previous state in case of failure of type I
F_{R_2}	the continuation of repair of main unit from previous state in case of failure of type II
F_{R_3}	the continuation of repair of main unit from previous state in case of failure of type III
F_{R_4}	the continuation of repair of main unit from previous state in case of failure of type IV

3. STATE TRANSITION DIAGRAM AND ASSUMPTIONS

For the transition diagram given in Fig. 1 following assumptions are made :

1. Initially, one unit is operative while other unit is in cold standby mode. Both units are identical.
2. The repairman comes immediately as soon as the unit fails.
3. An inspection is carried out to detect the type of failure.
4. Failure times are assumed to have exponential distribution [3] whereas repair/replacement/inspection times have general distribution.
5. During inspection, no other event can take place.
6. On the arrival of the repairman, power factor is controlled first, if it is not controlled already.
7. All random variables are independent.

The system is analyzed by making use of semi-Markov processes and regenerative point technique. The various reliability indices of system effectiveness are obtained such as mean time to system failure (MTSF), availability when power factor is controlled, availability when power factor is not controlled, busy period of the repairman when fuse is blown off (Type I), busy period of the repairman when transformer is burnt (Type II), busy period of the repairman when there is programming problem (Type III), busy period of the repairman when output relay is faulty (Type IV), expected number of visits of the repairman, expected number of fuse replacement, expected number of transformer replacement. The profit incurred to the system is also evaluated and graphical study is also done. The data/information on failures, repairs, various costs involved etc. have been collected from a user of such systems. On the basis of the information gathered, estimated values of various rates/costs are obtained which are given as:

- Estimated value of failure rate (λ) = 0.001 per hour
- Estimated value of rate with which power factor changes from controlled mode to uncontrolled mode (β_1) = 0.02 per hour
- Estimated value of rate with which power factor changes from uncontrolled mode to controlled mode (β_2) = 0.2 per hour
- Probability of failure of type I (p_1) = 0.3
- Probability of failure of type II (p_2) = 0.2
- Probability of failure of type III (p_3) = 0.4
- Probability of failure of type IV (p_4) = 0.1
- Expected cost of fuse replacement (C_1) = 50 INR
- Expected cost of transformer replacement (C_2) = 150 INR
- Expected cost of visit of repairman (C_3) = 1000 INR

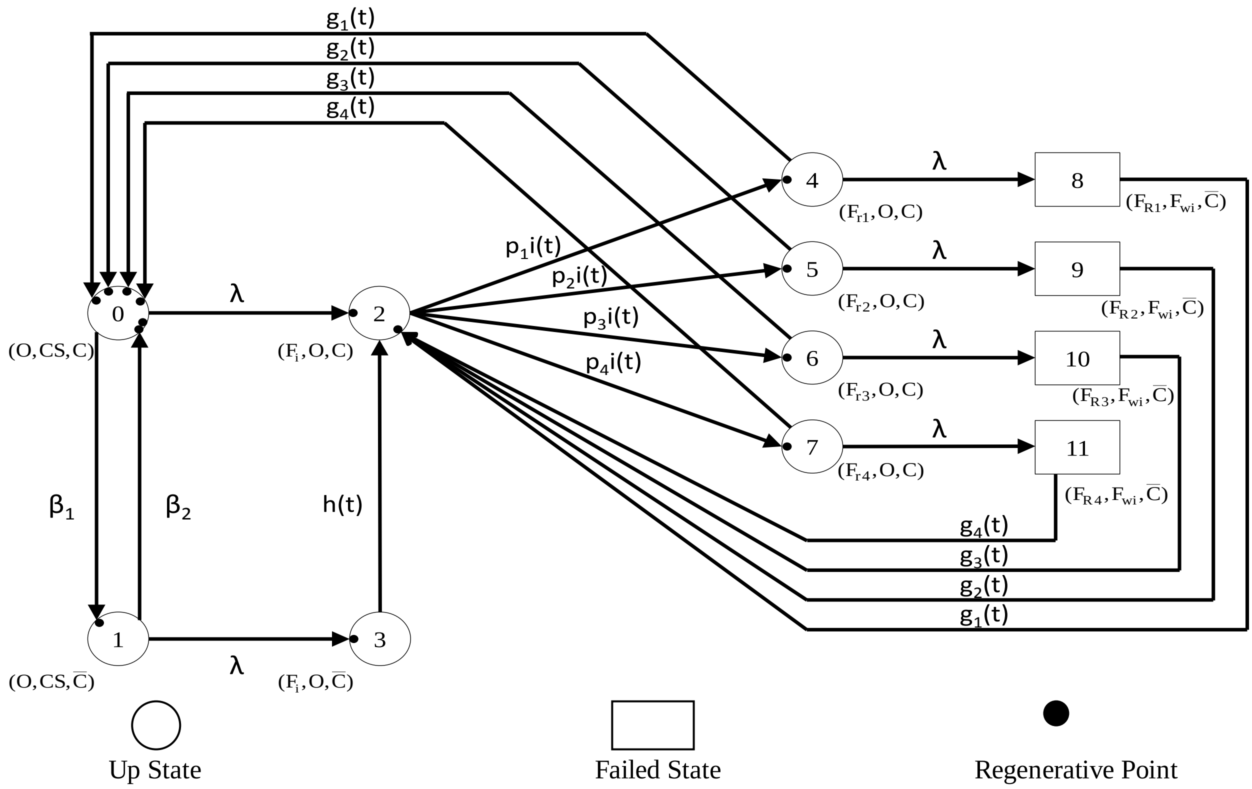

Fig.1: State Transition Diagram

4. TRANSITION PROBABILITES AND MEAN SOJOURN TIMES

A transition diagram showing the various states of system is shown in Fig. 1. The epochs of entry into states 0, 1,2,3,4,5,6,7 are regenerative points while 8, 9, 10, 11 are failed states. The transition probabilities from regenerative state to regenerative state are given below:

$dQ_{01}(t) = \beta_1 e^{-(\lambda+\beta_1)t}\,dt$, $dQ_{02}(t) = \lambda e^{-(\lambda+\beta_1)t}\,dt$, $dQ_{10}(t) = \beta_2 e^{-(\lambda+\beta_2)t}\,dt$, $dQ_{13}(t) = \lambda e^{-(\lambda+\beta_2)t}\,dt$, $dQ_{24}(t) = p_1\, i(t)dt$, $dQ_{25}(t) = p_2\, i(t)dt$, $dQ_{26}(t) = p_3\, i(t)dt$, $dQ_{27}(t) = p_4\, i(t)dt$, $dQ_{32}(t) = h(t)dt$, $dQ_{40}(t) = e^{-\lambda t}\, g_1(t)\,dt$, $dQ_{50}(t) = e^{-\lambda t}\, g_2(t)\,dt$, $dQ_{60}(t) = e^{-\lambda t}\, g_3(t)\,dt$, $dQ_{70}(t) = e^{-\lambda t}\, g_4(t)\,dt$, $dQ_{48}(t) = \lambda\, e^{-\lambda t}\, \overline{G_1}(t)\,dt$, $dQ_{59}(t) = \lambda\, e^{-\lambda t}\, \overline{G_2}(t)\,dt$, $dQ_{6,10}(t) = \lambda\, e^{-\lambda t}\, \overline{G_3}(t)\,dt$, $dQ_{7,11}(t) = \lambda\, e^{-\lambda t}\, \overline{G_4}(t)\,dt$, $dQ_{42}^{(8)}(t) = (\lambda\, e^{-\lambda t} ©1)\; g_1(t)\,dt$, $dQ_{52}^{(9)}(t) = (\lambda\, e^{-\lambda t} ©1)\; g_2(t)\,dt$, $dQ_{62}^{(10)}(t) = (\lambda\, e^{-\lambda t} ©1)\; g_3(t)\,dt$, $dQ_{72}^{(11)}(t) = (\lambda\, e^{-\lambda t} ©1)\; g_4(t)\,dt$

The non-zero element $p_{ij} = \lim_{s\to 0} q_{ij}^{*}(s)$ are given by

$$p_{01} = \frac{\beta_1}{\lambda+\beta_1},\; p_{02} = \frac{\lambda}{\lambda+\beta_1},\; p_{10} = \frac{\beta_2}{\lambda+\beta_2},\; p_{13} = \frac{\lambda}{\lambda+\beta_2},\; p_{24} = p_1,\; p_{25} = p_2,\; p_{26} = p_3,\; p_{27} = p_4,$$

$$p_{32} = 1,\; p_{40} = g_1^{*}(\lambda),\; p_{50} = g_2^{*}(\lambda),\; p_{60} = g_3^{*}(\lambda),\; p_{70} = g_4^{*}(\lambda),\; p_{48} = p_{42}^{(8)} = 1- g_1^{*}(\lambda),$$

$$p_{59} = p_{52}^{(9)} = 1- g_2^{*}(\lambda),\quad p_{6,10} = p_{62}^{(10)} = 1- g_3^{*}(\lambda),\quad p_{7,11} = p_{72}^{(11)} = 1- g_4^{*}(\lambda)$$

From above mentioned transition probabilities it can be verified that

$$p_{01} + p_{02} = 1,\; p_{10} + p_{13} = 1,\; p_{24} + p_{25} + p_{26} + p_{27} = 1,\; p_{32} = 1,\; p_{40} + p_{48} = p_{40} + p_{42}^{(8)} = 1$$

$$p_{50} + p_{59} = p_{50} + p_{52}^{(9)} = 1,\; p_{60} + p_{6,10} = p_{60} + p_{62}^{(10)} = 1,\; p_{70} + p_{7,11} = p_{70} + p_{72}^{(11)} = 1$$

The mean sojourn times (μ_i) in the regenerative state i is defined as the time of stay in that state before transition to any other state. If T denotes the sojourn time in the regenerative state i, then

$$\mu_i = E(T) = Pr(T>y)$$

$$\mu_0 = \frac{1}{\lambda+\beta_1},\quad \mu_1 = \frac{1}{\lambda+\beta_2},\quad \mu_2 = \int_0^\infty ti(t)dt,\quad \mu_3 = \int_0^\infty th(t)dt,\quad \mu_4 = \frac{1-g_1^{*}(\lambda)}{\lambda},\quad \mu_5 = \frac{1-g_2^{*}(\lambda)}{\lambda},$$

$$\mu_6 = \frac{1-g_3^{*}(\lambda)}{\lambda},\quad \mu_7 = \frac{1-g_4^{*}(\lambda)}{\lambda}$$

The unconditional mean time taken by the system to transit to any regenerative state j when time is counted from the epoch of entrance into state i is mathematically stated as

$$m_{ij} = \int_0^\infty t\,dQ_{ij}(t) = -q_{ij}^{*\prime}(0)$$

Also,

$$m_{01} + m_{02} = \mu_0,\; m_{10} + m_{12} = \mu_1,\; m_{40} + m_{4,12} = \mu_4,\; m_{40} + m_{42}^{(8)} = \int_0^\infty tg_1(t)dt = \kappa_1\,(\text{say}),\quad m_{50} + m_{59} = \mu_5$$

$$m_{50} + m_{52}^{(9)} = \int_0^\infty tg_2(t)dt = \kappa_2\,(\text{say}),\quad m_{60} + m_{6,10} = \mu_6,\quad m_{60} + m_{62}^{(10)} = \int_0^\infty tg_3(t)dt = \kappa_3\,(\text{say}),\quad m_{70} + m_{7,11} = \mu_7,$$

$$m_{70} + m_{72}^{(11)} = \int_0^\infty tg_4(t)dt = \kappa_4\,(\text{say})$$

5. MATHEMATICAL ANALYSIS OF VARIOUS RELIABILITY INDICES

For all the calculations given below failed states are considered as absorbing states and using the probabilistic arguments used for regenerative processes, the recursive relations are obtained for probabilistic analysis of reliability indices.

5.1 MEAN TIME TO SYSTEM FAILURE (MTSF)

This measure is defined as the expected time for which the system is in operation before it completely fails. $\varphi_i(t)$ is defined as the cumulative distribution function of first passage time from ith state to a failed state.

$$\phi_0(t) = Q_{01}(t) \circledS \phi_1(t) + Q_{02}(t) \circledS \phi_2(t)$$

$$\phi_1(t) = Q_{10}(t) \circledS \phi_0(t) + Q_{13}(t) \circledS \phi_3(t)$$

$$\phi_2(t) = Q_{24}(t) \circledS \phi_4(t) + Q_{25}(t) \circledS \phi_5(t) + Q_{26}(t) \circledS \phi_6(t) + Q_{27}(t) \circledS \phi_7(t)$$

$$\phi_3(t) = Q_{32}(t) \circledS \phi_2(t)$$

$$\phi_4(t) = Q_{40}(t) \circledS \phi_0(t) + Q_{48}(t)$$

$$\phi_5(t) = Q_{50}(t) \circledS \phi_0(t) + Q_{59}(t)$$

$$\phi_6(t) = Q_{60}(t) \circledS \phi_0(t) + Q_{6,10}(t)$$

$$\phi_7(t) = Q_{70}(t) \circledS \phi_0(t) + Q_{7,11}(t)$$

Taking Laplace Stieltjes Transform on both sides and solving equations for $\phi_0^{**}(s)$;

$$\text{MTSF} = \lim_{s\to 0} \frac{1-\phi_0^{**}(s)}{s} = \frac{D_0'(0)-N_0'(0)}{D_0(0)} = \frac{N_1}{D_1}$$

where $D_1 = (1-p_{01}\, p_{10})(1-p_{27}\, p_{70}-p_{26}\, p_{60}-p_{25}\, p_{50}-p_{24}\, p_{40})$

$N_1 = \mu_0 + p_{01}\,\mu_1 + p_{01}p_{13}\,\mu_3 + (\mu_2 + p_{24}\,\mu_4 + p_{25}\,\mu_5 + p_{26}\,\mu_6 + p_{27}\,\mu_7)(1-p_{01}\, p_{10})$

5.2 AVAILABILITY WHEN POWER FACTOR IS CONTROLLED

Using the probabilistic arguments and defining $AC_i(t)$ as the probability that the system is in up state when power factor is controlled at instant t, given that the system entered regenerative state i at t=0, we have the following recursive relations :

$AC_0(t) = M_0(t) + q_{01}(t) © AC_1(t) + q_{02}(t) © AC_2(t)$

$AC_1(t) = q_{10}(t) © AC_0(t) + q_{13}(t) © AC_3(t)$

$AC_2(t) = M_2(t) + q_{24}(t) © AC_4(t) + q_{25}(t) © AC_5(t) + q_{26}(t) © AC_6(t) + q_{27}(t) © AC_7(t)$

$AC_3(t) = q_{32}(t) © AC_2(t)$

$AC_4(t) = M_4(t) + q_{40}(t) © AC_0(t) + q_{42}^{(8)}(t) © AC_2(t)$

$AC_5(t) = M_5(t) + q_{50}(t) © AC_0(t) + q_{52}^{(9)} © AC_2(t)$

$AC_6(t) = M_6(t) + q_{60}(t) © AC_0(t) + q_{62}^{(10)} © AC_2(t)$

$AC_7(t) = M_7(t) + q_{70}(t) © AC_0(t) + q_{72}^{(11)} © AC_2(t)$

where $M_0(t) = e^{-(\lambda+\beta_1)t}$, $M_2(t)= e^{-\lambda t}\bar{I}(t)$, $M_4(t)= e^{-\lambda t}\overline{G_1}(t)$, $M_5(t)= e^{-\lambda t}\overline{G_2}(t)$, $M_6(t)= e^{-\lambda t}\overline{G_3}(t)$, $M_7(t)= e^{-\lambda t}\overline{G_4}(t)$

$$M_0^*(s) = \frac{1}{\lambda+\beta_1+s},\ M_2^*(s) = \frac{1-i^*(\lambda+s)}{\lambda+s},\ M_4^*(s) = \frac{1-g_1^*(\lambda+s)}{\lambda+s},\ M_5^*(s) = \frac{1-g_2^*(\lambda+s)}{\lambda+s},\ M_6^*(s) = \frac{1-g_3^*(\lambda+s)}{\lambda+s},$$

$$M_7^*(s) = \frac{1-g_4^*(\lambda+s)}{\lambda+s}$$

Taking Laplace transforms on both sides and solving for $AC_0^*(s)$;

$$AC_0^*(s) = \frac{N_2(s)}{D_2(s)}$$

For steady state, availability of the system is given by

$$AC_0 = \lim_{s\to 0} sAC_0^*(s) = \frac{N_2(0)}{D_2'(0)}$$

where $N_2(0) = (p_{27}p_{70}+p_{26}p_{60}+p_{25}p_{50}+p_{24}p_{40})\mu_0 + (1-p_{01}p_{10})\left[\left(\frac{1-i^*(\lambda)}{\lambda}\right)+\mu_4 p_{24}+\mu_5 p_{25}+\mu_6 p_{26}+\mu_7 p_{27}\right]$

$$D_2'(0) = (p_{27}p_{70}+p_{26}p_{60}+p_{25}p_{50}+p_{24}p_{40})(\mu_0+\mu_1 p_{01}+\mu_3 p_{01}p_{13}) + (1-p_{01}p_{10})$$
$$(\mu_2+p_{27}\kappa_4+p_{26}\kappa_3+p_{25}\kappa_2+p_{24}\kappa_1)$$

Similarly, Availability when power factor is not controlled can also be calculated using recursive relations as above.

For steady state, availability when power factor is not controlled of the system is given by

$$\overline{AC_0} = \lim_{s\to 0} s\overline{AC_0}^*(s) = \frac{N_3(0)}{D_2'(0)} \text{ where } N_3(0) = p_{01}(p_{27}p_{70}+p_{26}p_{60}+p_{25}p_{50}+p_{24}p_{40})\left[\mu_2+p_{13}\left(\frac{1-h^*(\lambda)}{\lambda}\right)\right]$$

5.3 BUSY PERIOD ANALYSIS OF TYPE I REPAIR

Using the probabilistic arguments and defining $BF_i(t)$ as the probability that the repairman is busy in the repair of Type I failure at instant t, given that the system entered regenerative state i at t=0, we have the following recursive relations :

$BF_0(t) = q_{01}(t) © BF_1(t) + q_{02}(t) © BF_2(t)$

$BF_1(t) = q_{10}(t) © BF_0(t) + q_{13}(t) © BF_3(t)$

$BF_2(t) = q_{24}(t) © BF_4(t) + q_{25}(t) © BF_5(t) + q_{26}(t) © BF_6(t) + q_{27}(t) © BF_7(t)$

$BF_3(t) = q_{32}(t) © BF_2(t)$

$BF_4(t) = W_4(t) + q_{40}(t) © BF_0(t) + q_{42}^{(8)}(t) © BF_2(t)$

$BF_5(t) = q_{50}(t) © BF_0(t) + q_{52}^{(9)} © BF_2(t)$

$BF_6(t) = q_{60}(t) © BF_0(t) + q_{62}^{(10)} © BF_2(t)$

$BF_7(t) = q_{70}(t) © BF_0(t) + q_{72}^{(11)} © BF_2(t)$

where $W_4(t) = \overline{G_1}(t)$

$$W_4^*(0) = \int_0^\infty \overline{G_1}(t)dt = \kappa_1 \text{(say)}$$

For steady state, $BF_0 = \dfrac{N_4(0)}{D_2'(0)}$ where $N_4(0) = p_{24}\kappa_1(1 - p_{01}p_{10})$

Similarly, busy period analysis of the repairman when transformer is burnt (Type II failure) (BT_0), busy period analysis of the repairman when there is programming problem (Type III failure) (BP_0), busy period analysis of the repairman when output relay is faulty (Type IV failure) (BO_0) can be evaluated for steady state.

5.4 EXPECTED NUMBER OF VISITS OF REPAIRMAN

Using the probabilistic arguments and defining $V_i(t)$ as the expected number of visits in (0,t], given that the system entered regenerative state i at t=0, we have the following recursive relations :

$V_0(t) = Q_{01}(t) Ⓢ V_1(t) + Q_{02}(t) Ⓢ (1 + V_2(t))$

$V_1(t) = Q_{10}(t) Ⓢ V_0(t) + Q_{13}(t) Ⓢ (1 + V_3(t))$

$V_2(t) = Q_{24}(t) Ⓢ V_4(t) + Q_{25}(t) Ⓢ V_5(t) + Q_{26}(t) Ⓢ V_6(t) + Q_{27}(t) Ⓢ V_7(t)$

$V_3(t) = Q_{32}(t) Ⓢ V_2(t)$

$V_4(t) = Q_{40}(t) Ⓢ V_0(t) + Q_{42}^{(8)}(t) Ⓢ V_2(t)$

$V_5(t) = Q_{50}(t) Ⓢ V_0(t) + Q_{52}^{(9)} Ⓢ V_2(t)$

$V_6(t) = Q_{60}(t) Ⓢ V_0(t) + Q_{62}^{(10)} Ⓢ V_2(t)$

$V_7(t) = Q_{70}(t) Ⓢ V_0(t) + Q_{72}^{(11)} Ⓢ V_2(t)$

Taking Laplace Stietjes Transform on both sides and solving for steady state,

$V_0 = \dfrac{N_8(0)}{D_2'(0)}$ where $N_8(0 \ni (p_{27}p_{70} + p_{26}p_{60} + p_{25}p_{50} + p_{24}p_{40})(1 - p_{01}p_{10})$

5.5 EXPECTED NUMBER OF FUSE REPLACEMENTS

Using the probabilistic arguments and defining $FR_i(t)$ as the expected number of replacements in (0,t], given that the system entered regenerative state i at t=0, we have the following recursive relations :

$FR_0(t) = Q_{01}(t) Ⓢ FR_1(t) + Q_{02}(t) Ⓢ FR_2(t)$

$FR_1(t) = Q_{10}(t) Ⓢ FR_0(t) + Q_{13}(t) Ⓢ FR_3(t)$

$FR_2(t) = Q_{24}(t) Ⓢ FR_4(t) + Q_{25}(t) Ⓢ FR_5(t) + Q_{26}(t) Ⓢ FR_6(t) + Q_{27}(t) Ⓢ FR_7(t)$

$FR_3(t) = Q_{32}(t) Ⓢ FR_2(t)$

$FR_4(t) = Q_{40}(t) Ⓢ (1 + FR_0(t)) + Q_{42}^{(8)}(t) Ⓢ (1 + FR_2(t))$

$FR_5(t) = Q_{50}(t) Ⓢ FR_0(t) + Q_{52}^{(9)} Ⓢ FR_2(t)$

$FR_6(t) = Q_{60}(t) Ⓢ FR_0(t) + Q_{62}^{(10)} Ⓢ FR_2(t)$

$FR_7(t) = Q_{70}(t) Ⓢ FR_0(t) + Q_{72}^{(11)} Ⓢ FR_2(t)$

Taking Laplace Stietjes Transform on both sides and solving for steady state,

$$FR_0 = \frac{N_9(0)}{D_2'(0)} \text{ where } N_9(0) = p_{24}(1 - p_{01}p_{10})$$

Also calculations are done to find expected number of transformer replacements (TR_0) and following result is obtained for steady state,

$$TR_0 = \frac{N_{10}(0)}{D_2'(0)} \text{ where } N_{10}(0) = p_{25}(1 - p_{01}p_{10})$$

6. COST-BENEFIT ANALYSIS

At steady state, the expected total profit P per unit time incurred to the system is given by:

$$P\ (\text{Profit}) = C_0(AC_0 + A\overline{C_0}) - C_{21}BF_0 - C_{22}BT_0 - C_{23}BP_0 - C_{24}BO_0 - C_1FR_0 - C_2TR_0 - C_3V_0 - A\overline{C_0}\ L\overline{C}$$

C_0	revenue per unit up time
C_{21}	cost per unit up time for which the repairman is busy for repairing the unit having failure of type I
C_{22}	cost per unit up time for which the repairman is busy for repairing the unit having failure of type II
C_{23}	cost per unit up time for which the repairman is busy for repairing the unit having failure of type III
C_{24}	cost per unit up time for which the repairman is busy for repairing the unit having failure of type IV
C_1	cost per fuse replacement
C_2	cost per transformer replacement
C_3	cost per visit of the repairman
$L\overline{C}$	loss per unit time when power factor is not controlled

7. DISCUSSION AND CONCLUSION

A particular case is discussed by assuming the repair/replacement is exponentially distributed as under:

$$i(t) = \beta e^{-\beta t},\quad g_1(t) = \alpha_1 e^{-\alpha_1 t},\quad g_2(t) = \alpha_2 e^{-\alpha_2 t},\quad g_3(t) = \alpha_3 e^{-\alpha_3 t},\quad g_4(t) = \alpha_4 e^{-\alpha_4 t},\quad h(t) = \gamma e^{-\gamma t}$$

Using the values estimated from the data/information collected i.e. ($p_1 = 0.3$, $p_2 = 0.2$, $p_3 = 0.4$, $p_4 = 0.1$, $\alpha_1 = 4$, $\alpha_2 = 2$, $\alpha_3 = 6$, $\alpha_4 = 10$, $\gamma = 2$, $\beta = 6$, $\beta_1 = 0.02$, $\beta_2 = 0.2$, $\lambda = 0.001$, $C_0 = 1000$, $C_1 = 50$, $C_2 = 250$, $C_3 = 1000$, $C_{21} = 100$, $C_{22} = 150$, $C_{23} = 50$, $C_{24} = 75$, $L\overline{C} = 500$) the values of various indices of reliability effectiveness can be obtained.

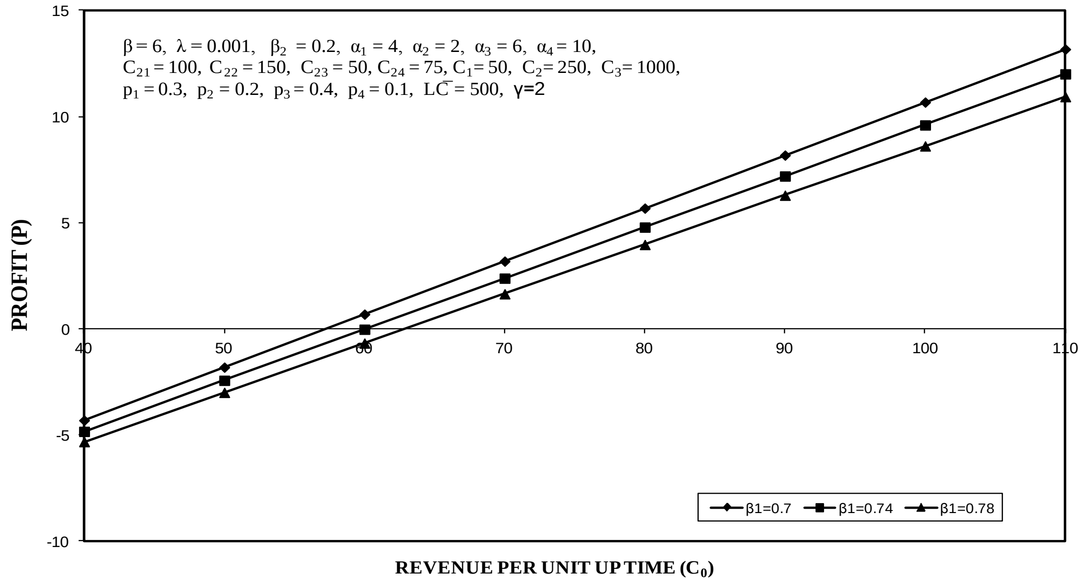

Fig. 2. Profit (P) versus revenue per unit up time with (C_0) for different values of the rate with which power factor changes from controlled mode to uncontrolled mode (β_1)

Fig. 2 shows the behavior of profit (P) with respect to revenue (C_0) per unit up time for different values of the rate with which power factor changes from controlled mode to uncontrolled mode (β_1). It can be concluded that the profit (P) increases with the increase in the value of C_0 and has higher values for lower rates of β_1. It can also be noticed that:

(i) For $\beta_1 = 0.7$ then $P >$ or $=$ or < 0 accordingly as $C_0 >$ or $=$ or < 57. So, for the model to be beneficial for $\beta_1 = 0.18$, the C_0 should be > 57.

(ii) Similarly, for $\beta_1 = 0.74$ and $\beta_1 = 0.78$, the values for C_0 should be greater than 60 and 63 respectively.

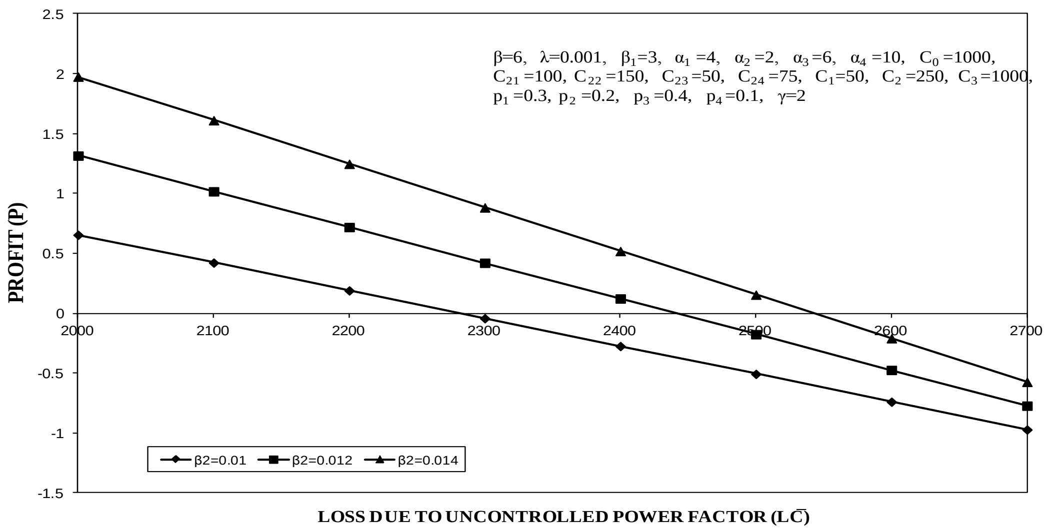

Fig. 3. Profit (P) versus loss due to uncontrolled power factor ($L\overline{C}$) for different values of the rate with which power factor changes from uncontrolled mode to controlled mode (β_2)

Fig. 3 reveals the behavior of profit (P) with respect to loss ($L\overline{C}$) due to uncontrolled power factor for different values of the rate with which power factor changes from uncontrolled mode to controlled mode (β_2). It can be concluded that the profit decreases with the increase in the value of $L\overline{C}$ and has higher values for higher rates of β_2. It can also be noticed that:

(i) For $\beta_2 = 0.01$ then P < or = or > 0 accordingly as $L\overline{C}$ > or = or < 2295. So, for the model to be beneficial for $\beta_2 = 0.01$, $L\overline{C}$ should be < 2295.

(ii) Similarly, for $\beta_2 = 0.012$ and $\beta_2 = 0.014$, the values of $L\overline{C}$ should be less than 2450 and 2550 respectively.

Many other graphs can be plotted by the users of such systems from the data/information given above to get the cut-off points so that profit can be increased to fix the cost of the product by the manufacturers.

REFERENCES

[1] Gopalan M. N.and Muralidhar M. N., "Cost analysis of one unit repairable system subject to online preventive maintenance and/or repair', Microelectron Relia., vol. 31(2/3), 1991, pp. 223-228.
[2] V. K. Mehta, "Principles of Power System" S. Chand & Company Ltd., 1994, pp. 123-127.
[3] Charles E. Ebeling, "Reliability and Maintainability Engineering" Tata McGraw-Hill, 2000, pp. 398-400.
[4] Taneja, G. and Nanda, J., "Probabilistic analysis of two-unit cold standby system with resume and repeat Repair policies.", Pure Applied Mathematika Sciences, LVII, 37-49. 2003.
[5] Tuteja, R. K, Parashar, Bhupender and Taneja, G. "Analysis of PLC hot standby system with single repair facility irrespective of type of failure." J. Indian Soc. Stat. Opers. Res. XXVI (1-4), 37-47, 2005.
[6] Khurana, Vipin, Rizwan, S.M. and Taneja, G.," Economic analysis of reliability model for two PLC cold standby system with four types of failures." Pure and Applied Mathematika Sciences, L XIII (1-2), 65-78, 2006
[7] Parashar, Bhupender and Taneja, G. , "Reliability and profit evaluation of a PLC hot standby system based on master slave concept and two types of repair facilities. " IEEE Transactions on Reliability, 56(3), 534-539, 2007.
[8] Goyal, Amit, Taneja, G. and Singh, D.V. "Economic comparative study between two models for sulphated juice pump systems working seasonally and having different configurations." International Journal of Engineering Science and Technology, 2(9), 4501-4510. 2010.
[9] Sharma, Upasana, Rekha and Taneja, G., "Analysis of a two standby oil delivering system with a provision of switching over to another system at need to increase the availability. Journal of Mathematics and Statistics, 7(1), 57-60. 2011.

Source of support: Nil, Conflict of interest: None Declared