

**Effect of Couple stress fluid with variable viscosity and an Endoscope on Peristaltic Motion**

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*(Received on: 31-10-12; Revised & Accepted on: 28-11-12)*

**ABSTRACT**

*The Problem of peristaltic transport of a couple stress fluid with variable viscosity through the gap between coaxial tubes where the outer tube is non-uniform with sinusoidal wave traveling down its wall and the inner tube is rigid. The relation between the pressure gradient, friction force on the inner and outer tube are obtained in terms of couple stress parameter and viscosity. The numerical solution of pressure gradient, outer and inner friction force and flow rate are shown graphically.*

**Keywords:** *Peristaltic transport, couple stress parameter, and flow rate.*

**1. INTRODUCTION**

The purpose of this paper is an attempt to understand the fluid mechanics in a physiological situation with the presence of an endoscope is placed concentrically. The pressure rise, peristaltic pumping, augmented pumping and friction force on the inner tube (endoscope) and outer tube is discussed by Srivastava et.al [1], and Siddiqui and Schwarz [2]. Latham [3] investigated the fluid mechanics of peristaltic pump and since then, other work on the same subject has been followed by Burns and Parker [4]. Barton and Raynor [5] have studied the case of a vanishingly small Reynolds number. Abd El Naby and El Misery [6] studied the effect of an endoscope and generalized Newtonian fluid on peristaltic motion. Gupta and Seshadri [7] studied peristaltic transport of a Newtonian fluid in non- uniform geometries. Srivastava and Srivastava [8] have investigated the effect of power law fluid in uniform and non-uniform tube and channel under zero Reynolds number and long wavelength approximation. Bohme and Friedrich [9] have investigated peristaltic flow of viscoelastic liquids and assumed that the relevant Reynolds number is small enough to neglect inertia forces, and that the ratio of the wavelength and channel height is large, which implies that the pressure is constant over the cross-section. Elshehawey et .al [10] studied peristaltic motion of generalized Newtonian fluid in a non-uniform channel under zero Reynolds number with long wavelength approximation. Most of studies on peristaltic motion, that assume physiological fluids behave like a Newtonian fluid with constant viscosity fail to give a better understanding when peristaltic mechanics involved in small blood vessel, lymphatic vessel, intestine, ducts efferent of the male reproductive tracts, and in transport of spermatozoa in the cervical canal. According to Haynes [11], Bugliarillo and Sevilla [12] and Goldsmith and Skalak [13] it is clear that in pre mentioned body organs, viscosity of the fluid varies across the thickness of the duct. Cotton and Williams [14] study the practical gastrointestinal endoscope. Rathod and Asha [15] studied the peristaltic transport of a couple stress fluids in uniform and non-uniform annulus moving with a constant velocity. Rathod and Asha [16] studied the effect of couple stress fluid and endoscope on peristaltic motion.

In the view of above discussion the effect of couple stress fluid with variable viscosity through the gap between inner and outer tubes where the inner tube is an endoscope and the outer tube has a sinusoidal wave traveling down its wall is the aim of present investigation.

**2. FORMULATION AND ANALYSIS**

Consider the two-dimensional flow of an incompressible Newtonian fluid with variable viscosity through the gap between inner and outer tubes where the inner tube is an endoscope and the outer tube has a sinusoidal wave traveling down its wall. The geometry of the two wall surface is given by the equation:

$$\bar{r}_1 = a_1, \tag{2.1}$$

$$\bar{r}_2 = a_2 + b \sin \frac{2\pi}{\lambda} (\bar{z} - c\bar{t}) \tag{2.2}$$

where  $a_1$  is the radius of endoscope  $a_2$  is the radius of the small intestine at inlet, b is the amplitude of the wave,  $\lambda$  is the wavelength,  $\bar{t}$  is time and c is the wave speed.

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In the fixed coordinates  $(\bar{r}, \bar{z})$  the flow in the gap between inner and outer tubes is unsteady but if we choose moving coordinates  $(\bar{r}, \bar{z})$  which travel in the  $\bar{z}$  - direction with the same speed as the wave, then the flow can be treated as steady. The coordinate's frames are related through:

$$\bar{z} = \bar{Z} - ct, \quad \bar{r} = \bar{R}, \tag{2.3}$$

$$\bar{w} = \bar{W} - c, \quad \bar{u} = \bar{U}, \tag{2.4}$$

where  $\bar{U}, \bar{W}$  and  $\bar{u}, \bar{w}$  are the velocity components in the radial and axial direction in the fixed and moving coordinates respectively. Equations of boundary condition in the moving coordinates are: Continuity equation:

$$\frac{1}{\bar{r}} \frac{\delta(\bar{r}\bar{u})}{\delta\bar{r}} + \frac{\delta\bar{w}}{\delta\bar{z}} = 0, \tag{2.5}$$

and the Navier Stokes equation:

$$\rho(\bar{u} \frac{\partial\bar{u}}{\partial\bar{r}} + \bar{w} \frac{\partial\bar{u}}{\partial\bar{z}}) = -\frac{\partial\bar{p}}{\partial\bar{r}} + \frac{\partial}{\partial\bar{r}} [2\bar{\mu}(\bar{r}) \frac{\partial\bar{u}}{\partial\bar{r}}] + \frac{2\bar{\mu}(\bar{r})}{\bar{r}} (\frac{\partial\bar{u}}{\partial\bar{r}} - \frac{\bar{u}}{\bar{r}}) + \frac{\partial}{\partial\bar{z}} [\bar{\mu}(\bar{r})(\frac{\partial\bar{u}}{\partial\bar{z}} + \frac{\partial\bar{w}}{\partial\bar{r}})] - \eta \nabla^2 (\nabla^2(\bar{u})) \tag{2.6}$$

$$\rho(\bar{u} \frac{\partial\bar{w}}{\partial\bar{r}} + \bar{w} \frac{\partial\bar{w}}{\partial\bar{z}}) = -\frac{\partial\bar{p}}{\partial\bar{z}} + \frac{\partial}{\partial\bar{z}} [2\bar{\mu}(\bar{r}) \frac{\partial\bar{w}}{\partial\bar{z}}] + \frac{1}{\bar{r}} \frac{\partial}{\partial\bar{r}} [\bar{\mu}(\bar{r})\bar{r} (\frac{\partial\bar{u}}{\partial\bar{z}} + \frac{\partial\bar{w}}{\partial\bar{r}})] - \eta \nabla^2 (\nabla^2(\bar{w})) \tag{2.7}$$

where  $\nabla^2 = \{ \frac{\partial}{\partial\bar{r}} [ \frac{1}{\bar{r}} (\frac{\partial}{\partial\bar{r}}) ] \}$

$\bar{p}$  is the pressure,  $\bar{\mu}(\bar{r})$  is the viscosity function and  $\eta$  is the couple stress parameter. The boundary condition are written

$$\bar{w} = -c \quad \nabla^2(\bar{w}) \text{ finite, at } \bar{r} = \bar{r}_1 \quad \bar{r} = \bar{r}_2 \tag{2.8a}$$

$$\bar{u} = 0 \quad \nabla^2(\bar{u}) = 0, \text{ at } \bar{r} = \bar{r}_1 \tag{2.8b}$$

Introducing the non dimensional variable and the Reynolds number (Re) and the wave number ( $\delta$ ) as follows:

$$r = \frac{\bar{r}}{a_{20}}, \quad R = \frac{\bar{R}}{a_{20}}, \quad r_1 = \frac{\bar{r}_1}{a_{20}} = \frac{a_{10}}{a_{20}} = \varepsilon < 1, \quad z = \frac{\bar{z}}{\lambda}, \quad Z = \frac{\bar{Z}}{\lambda}, \quad \mu(r) = \frac{\bar{\mu}(\bar{r})}{\mu_0},$$

$$u = \frac{\lambda\bar{u}}{a_{20}c}, \quad U = \frac{\lambda\bar{U}}{a_{20}c}, \quad w = \frac{\bar{w}}{c}, \quad W = \frac{\bar{W}}{c}, \quad \delta = \frac{a_{20}}{\lambda} \ll 1,$$

$$Re = \frac{ca_{20}\rho}{\mu_0}, \quad p = \frac{a_{20}^2\bar{p}}{c\lambda\mu_0}, \quad t = \frac{c\bar{t}}{\lambda}, \quad \phi = \frac{b}{a_{20}} < 1,$$

$$r_2 = \frac{\bar{r}_2}{a_{20}} = 1 + \phi \sin 2\pi z \tag{2.9}$$

where  $\varepsilon$  is the radius ratio,  $\phi$  is the amplitude ratio and  $\mu_0$  is the viscosity on the endoscope. Equation of motion and boundary conditions in the dimensionless form become:

$$\frac{1}{r} \frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0 \tag{2.10}$$

$$Re \delta^3 (u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z}) = -\frac{\partial p}{\partial r} + \delta^2 \frac{\partial}{\partial r} (2\mu(r) \frac{\partial u}{\partial r}) + \delta^2 \frac{\partial}{\partial z} [\mu(r)(\delta^2 \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r})] + \frac{2\delta^2 \mu(r)}{r} (\frac{\partial u}{\partial r} - \frac{u}{r}) - \frac{1}{\gamma^2} \nabla^2 (\nabla^2(u)) \tag{2.11}$$

$$Re \delta (u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z}) = -\frac{\partial p}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} [\mu(r)r(\delta^2 \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r})] + \delta^2 \frac{\partial}{\partial z} (2\mu(r) \frac{\partial w}{\partial z}) - \frac{1}{\gamma^2} \nabla^2 (\nabla^2(w)) \tag{2.12}$$

$\gamma = \sqrt{\frac{\eta}{\mu a_{20}^2}}$  is the couple stress parameter With the dimensionless boundary condition

$$w = -1 \quad \nabla^2(w) \text{ finite at } r = r_1, \quad r = r_2 \tag{2.13a}$$

$$u = 0 \quad \nabla^2(u) = 0 \quad \text{at } r = r_1. \tag{2.13b}$$

Using the long wavelength approximation and neglecting the wave number ( $\delta = 0$ ), one can reduce the equation:

$$\frac{\partial p}{\partial r} = 0 \tag{2.14}$$

$$\frac{\partial p}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} (\mu(r)r \frac{\partial w}{\partial r}) - \frac{1}{\gamma^2} \nabla^2(\nabla^2(w)) \tag{2.15}$$

The instantaneous volume flow rate in the fixed coordinate system is given by:

$$\bar{Q} = 2\pi \int_{\bar{r}_1}^{\bar{r}_2} \bar{W} \bar{R} d\bar{R} \tag{2.16}$$

where  $\bar{r}_1$  is a constant and  $\bar{r}_2$  is a function of  $\bar{Z}$  and  $\bar{t}$ . On substituting eq. (2.3) and (2.4) into eq. (2.16) and then integrating, one obtains:

$$\bar{Q} = \bar{q} + \pi c (\bar{r}_2^2 - \bar{r}_1^2) \tag{2.17}$$

$$\text{where } \bar{q} = 2\pi \int_{\bar{r}_1}^{\bar{r}_2} \bar{w} \bar{r} d\bar{r} \tag{2.18}$$

is the volume flow rate in the moving coordinate system and is independent of time. Here,  $\bar{r}_2$  is a function of  $\bar{z}$  alone and is defined through eq. (2.2). Using the dimensionless variable, eq. (2.18) becomes:

$$F = \frac{q}{2\pi a_2^2 c} = \int_{r_1}^{r_2} w r dr \tag{2.19}$$

The time- mean flow over a period  $T = \frac{\lambda}{c}$  at a fixed Z position is defined as:

$$Q = \frac{1}{T} \int_0^T \bar{Q} d\bar{t} \tag{2.20}$$

Using eqs. (2.17) and (2.18) in eq. (2.20) and integrating we get:

$$\bar{Q} = \bar{q} + \pi c (a_2^2 - a_1^2 + \frac{b^2}{2})$$

Which may be written as:

$$\frac{\bar{Q}}{2\pi a_{20}^2 c} = \frac{\bar{q}}{2\pi a_{20}^2 c} + \frac{1}{2} (1 - \varepsilon^2 + \frac{\phi^2}{2}) \tag{2.21}$$

On defining the dimensionless time-mean flow as:

$$\Theta = \frac{\bar{Q}}{2\pi a_{20}^2 c}$$

rewriting eq. (2.21) as:

$$\Theta = F + \frac{1}{2} (1 - \varepsilon^2 + \frac{\phi^2}{2}) \tag{2.22}$$

Solving eqs. (2.13) - (2.15), we obtain:

$$W = \frac{1}{2} \frac{dp}{dz} \left[ I_1(r) - \frac{I_2(r) \{I_1(r_2) - I_1(r_1)\} - I_1(r_1)I_2(r_2) - I_2(r_1)I_1(r_2)}{I_2(r_2) - I_2(r_1)} \right] - D - 1 \tag{2.23}$$

where  $D = \frac{1}{4\gamma^2} \left[ \left( \frac{\partial p}{\partial z} \right) \left\{ (r_2^2 - r_1^2) \frac{\log(r/r_1)}{\log(r_2/r_1)} - (r_2^2 - r_1^2) \right\} \left\{ \left\{ (r_2^2 - r_1^2) \frac{\log(r/r_1)}{\log(r_2/r_1)} - (r_2^2 - r_1^2) \right\} - 1 \right\} - \left[ \left\{ (r_2^2 - r_1^2) \frac{\log(r/r_1)}{\log(r_2/r_1)} - (r_2^2 - r_1^2) \right\} - 1 \right] \right]$

$$I_1(r) = \int \frac{r}{\mu(r)} dr \tag{2.24}$$

$$I_2(r) = \int \frac{dr}{r\mu(r)} \tag{2.25}$$

Using eq. (2.19) we obtain the relationship between  $\frac{dp}{dz}$  and F as follows:

$$F = \frac{1}{4} \frac{dp}{dz} \left[ \frac{(I_1(r_2) - I_1(r_1))^2}{I_2(r_2) - I_2(r_1)} - I_3 \right] - \frac{1}{2} (r_2^2 - r_1^2) - X \tag{2.26}$$

where X=

$$\frac{1}{8\gamma^2} \left[ \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2/r_1)} - (r_2^4 - r_1^4) \right\} + \frac{1}{2} (r_2^2 - r_1^2) \right] / \frac{1}{8\gamma^2} \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2/r_1)} - (r_2^4 - r_1^4) \right\} \left[ \frac{1}{8\gamma^2} \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2/r_1)} - (r_2^4 - r_1^4) - \frac{1}{2} (r_2^2 - r_1^2) \right\} \right]$$

$$I_3 = \int_{r_1}^{r_2} \frac{r^3}{\mu(r)} dr \tag{2.27}$$

Solving eq. (2.26) for  $\frac{dp}{dz}$ , we obtain:

$$\frac{dp}{dz} = \frac{4F + 2(r_2^2 - r_1^2)}{\left[ \frac{(I_1(r_2) - I_1(r_1))^2}{I_2(r_2) - I_2(r_1)} - I_3 - X \right]} \tag{2.28}$$

where X=

$$\frac{1}{8\gamma^2} \left[ \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2/r_1)} - (r_2^4 - r_1^4) \right\} + \frac{1}{2} (r_2^2 - r_1^2) \right] / \frac{1}{8\gamma^2} \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2/r_1)} - (r_2^4 - r_1^4) \right\} \left[ \frac{1}{8\gamma^2} \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2/r_1)} - (r_2^4 - r_1^4) - \frac{1}{2} (r_2^2 - r_1^2) \right\} \right]$$

The pressure rise  $\Delta P_\lambda$  and friction force on inner and outer tubes  $F_\lambda^{(i)}$  and  $F_\lambda^{(o)}$ , in their non-dimensional forms, are given by:

$$\Delta P_\lambda = \int_0^1 \left( \frac{dp}{dz} \right) dz \tag{2.29}$$

$$F_\lambda^{(i)} = \int_0^1 r_1^2 \left( - \frac{dp}{dz} \right) dz \tag{2.30}$$

$$F_{\lambda}^{(o)} = \int_0^1 r_2^2 \left(-\frac{dp}{dz}\right) dz \tag{2.31}$$

The effect of viscosity variation on peristaltic transport can be investigated through eqs. (2.29) - (2.31) for any given viscosity function  $\mu(r)$ .

For the present instigation, assume that the viscosity variation in the dimensionless form following Srivastava et al.[1], as follows:

$$\mu(r) = e^{-\alpha r} \tag{2.32}$$

Or

$$\mu(r) = 1 - \alpha r \quad \text{for } \alpha \ll 1 \tag{2.33}$$

where  $\alpha$  is viscosity parameter. The assumption is reasonable for the following physiological reason. Since normal person or animal or similar size takes 1 to 2L of fluid every day. On top of that, another 6 to 7L of fluid received by the small intestine daily as secretion from salivary glands, stomach, pancreas, liver and the small intestine itself, this implies that concentration of fluid is dependent on the radial distance.

Therefore, the above choice of  $\mu(r) = e^{-\alpha r}$  is justified.

Substituting eq.(2.33) into eqs.(2.24) ,(2.25) and (2.27), and using eq.(2.28), we obtain:

$$\begin{aligned} \frac{dp}{dz} = & \left[ \left( \{16\Theta - 8(1 - \varepsilon^2 + \frac{\phi^2}{2}) + 8(r_2^2 - r_1^2)\} \right) / \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} \right] \\ & \times \left( 1 - 4\alpha \left\{ \frac{(r_2^2 - r_1^2)(r_2^3 - r_1^3)}{3\log(r_2 / r_1)} - \frac{(r_2^2 - r_1^2)^2(r_2 - r_1)}{4(\ln(r_2 / r_1))^2} - \frac{(r_2^5 - r_1^5)}{5} \right\} \right) \\ & / \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} - \frac{1}{8\gamma^2} \left[ \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} + \frac{1}{2}(r_2^2 - r_1^2) \right] \\ & / \left[ \frac{1}{8\gamma^2} \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} \left[ \frac{1}{8\gamma^2} \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) - \frac{1}{2}(r_2^2 - r_1^2) \right\} \right] \right] d \quad z \tag{2.34} \end{aligned}$$

By taking the limit for eq.(2.34) when  $r_1$  asymptotes to zero and  $\alpha$  equals zero we get the same result obtained by Shapiro et al.[17] and Shukla et al. [18]. Substituting eq.(2.34) in eqs.(2.29)-(2.31) yield:

$$\begin{aligned} \Delta P_{\lambda} = & \int_0^1 \left[ \left( \{16\Theta - 8(1 - \varepsilon^2 + \frac{\phi^2}{2}) + 8(r_2^2 - r_1^2)\} \right) / \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} \right] \\ & \times \left( 1 - 4\alpha \left\{ \frac{(r_2^2 - r_1^2)(r_2^3 - r_1^3)}{3\log(r_2 / r_1)} - \frac{(r_2^2 - r_1^2)^2(r_2 - r_1)}{4(\log(r_2 / r_1))^2} - \frac{(r_2^5 - r_1^5)}{5} \right\} \right) \\ & / \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} - \frac{1}{8\gamma^2} \left[ \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} + \frac{1}{2}(r_2^2 - r_1^2) \right] \\ & / \left[ \frac{1}{8\gamma^2} \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} \left[ \frac{1}{8\gamma^2} \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) - \frac{1}{2}(r_2^2 - r_1^2) \right\} \right] \right] d \tag{2.35} \end{aligned}$$

$$\begin{aligned} F_{\lambda}^{(i)} = & \int_0^1 -r_1^2 \left[ \left( \{16\Theta - 8(1 - \varepsilon^2 + \frac{\phi^2}{2}) + 8(r_2^2 - r_1^2)\} \right) / \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} \right] \\ & \times \left( 1 - 4\alpha \left\{ \frac{(r_2^2 - r_1^2)(r_2^3 - r_1^3)}{3\log(r_2 / r_1)} - \frac{(r_2^2 - r_1^2)^2(r_2 - r_1)}{4(\log(r_2 / r_1))^2} - \frac{(r_2^5 - r_1^5)}{5} \right\} \right) \\ & / \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} - \frac{1}{8\gamma^2} \left[ \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} + \frac{1}{2}(r_2^2 - r_1^2) \right] \end{aligned}$$

$$\left/ \frac{1}{8\gamma^2} \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} \left[ \frac{1}{8\gamma^2} \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) - \frac{1}{2}(r_2^2 - r_1^2) \right\} \right] d \quad z \quad (2.36)$$

$$F_\lambda^{(\phi)} = \int_0^1 -r_2^2 \left[ \left( 16\Theta - 8(1 - \varepsilon^2 + \frac{\phi^2}{2}) + 8(r_2^2 - r_1^2) \right) / \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} \right. \\ \times \left( 1 - 4\alpha \left\{ \frac{(r_2^2 - r_1^2)(r_2^3 - r_1^3)}{3\log(r_2 / r_1)} - \frac{(r_2^2 - r_1^2)^2(r_2 - r_1)}{4(\log(r_2 / r_1))^2} - \frac{(r_2^5 - r_1^5)}{5} \right\} \right. \\ \left. / \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} \right] - \frac{1}{8\gamma^2} \left[ \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} + \frac{1}{2}(r_2^2 - r_1^2) \right] \\ \left. / \frac{1}{8\gamma^2} \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) \right\} \left[ \frac{1}{8\gamma^2} \left\{ \frac{(r_2^2 - r_1^2)^2}{\log(r_2 / r_1)} - (r_2^4 - r_1^4) - \frac{1}{2}(r_2^2 - r_1^2) \right\} \right] \right] d \quad (2.37)$$

### 3. RESULTS AND DISCUSSIONS

The dimensionless pressure rise ( $P_\lambda$ ) and the friction forces on the inner and outer tube for different given values of the dimensionless flow rate  $\Theta$ , amplitude ratio  $\phi$ , radius ratio  $\varepsilon$ , couple stress parameter  $\gamma$  and viscosity parameter  $\alpha$  are computed. As the integrals in equation. (2.35) to (2.37) are not integrable in the closed form so they are evaluated using  $a_{20}=1.25\text{cm}$   $a/\lambda=0.156$

The values of viscosity parameter  $\alpha$  as reported by Srivastava et.al [1] are  $\alpha=0.0$ ,  $\alpha=0.1$  Furthermore, since most routine upper gastrointestinal endoscopes are between 8-11 mm in diameter as reported by Cotton and Williams [20] and radius ratio 1.25cm reported by Srivastava and Srivastava [12].It may be noted that the theory of a long wavelength and zero Reynolds number of the of the present investigation remains applicable here as the radius is  $a_{20}=1.25\text{cm}$ , is small compared with the wavelength  $\lambda=8.01\text{cm}$ .

Fig. (1) Shows the pressure rise against the flow rate here it is observed that the pressure decreases with the increase of flow rate for different values of radius ratio  $\varepsilon = 0.32, \varepsilon = 0.38$  and  $\varepsilon = 0.44$  and for viscosity  $\alpha=0.0$  and  $\alpha=0.1$ . Fig (2) shows that as the viscosity increases the pressure rise decreases and as the amplitude ratio increases the pressure also increases.

Fig (3) and (4) shows the friction force on the outer tube for different values of radius ratio and viscosity, here it is observed that as radius ratio increases the friction force also increases and they are independent of radius ratio at certain values of the flow rate [for the values  $\phi=0.4$  and  $\alpha=0.0$  and  $\alpha=0.1$ ].

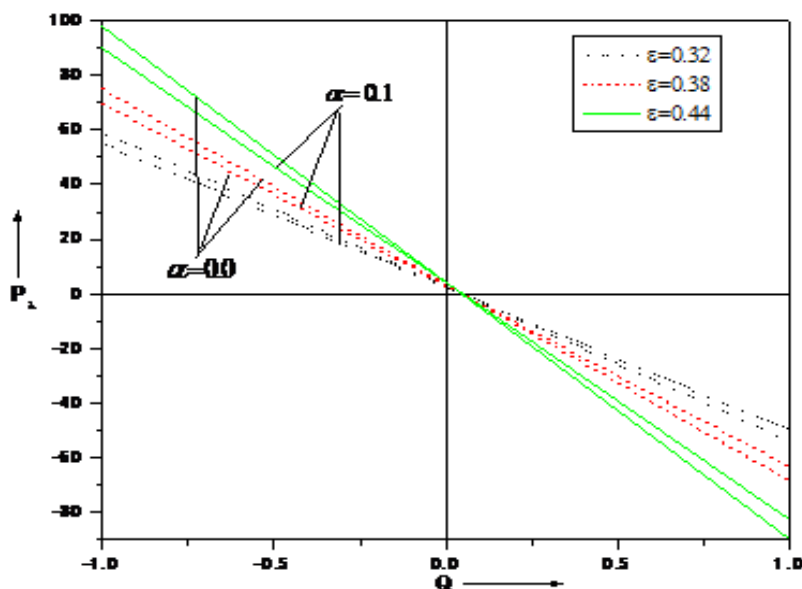
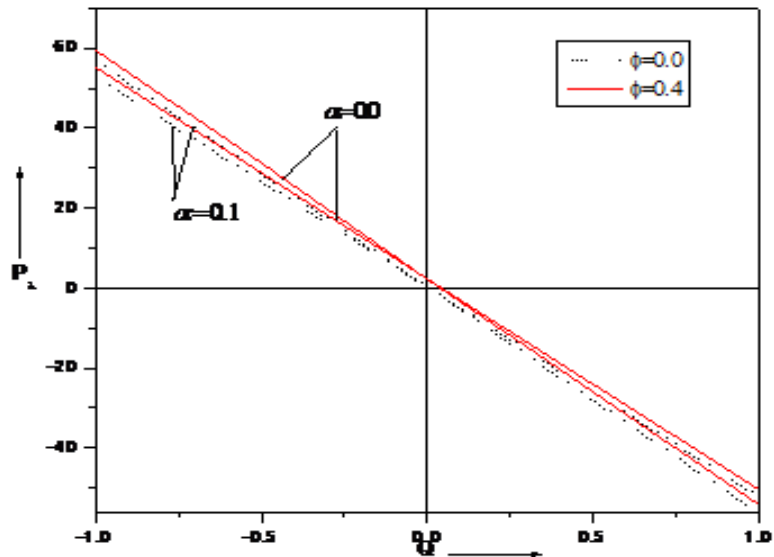
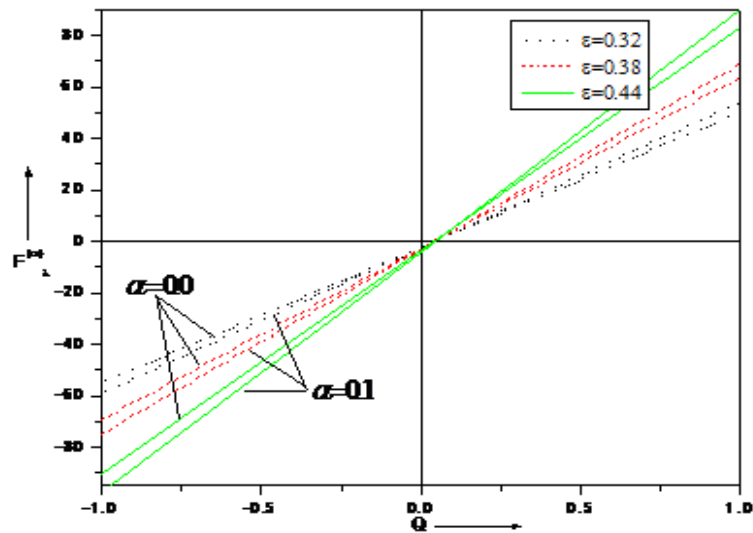


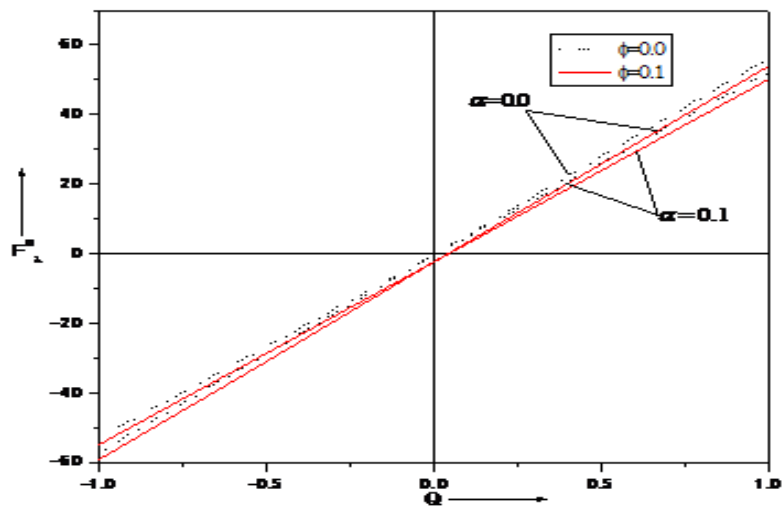
Fig (1)The pressure rise versus flow rate for  $\phi=0.4, \alpha=0.0$   $\alpha=0.1$  and  $\gamma=0.1$



Fig(2) The pressure rise versus flowrate for  $\alpha=0.32$ ,  $\alpha=0.0$ ,  $\alpha=0.1$ , and  $\gamma=0.1$



Fig(3) The friction force on the outertube versus flowrate for  $\beta=0.4$ , and  $\alpha=0.0$ , and  $\alpha=0.1$  and  $\gamma=0.1$



Fig(4) The friction force on the outertube versus flowrate for  $\alpha=0.32$ ,  $\alpha=0.0$ ,  $\alpha=0.1$ , and  $\gamma=0.1$

**REFERENCE:**

1. Srivastava L.M., Srivastava V.P. and Sinha S.N. *BioRheology* 20 (1983)153
2. Sinddiqui A.M. and Schwarz W.H. *J.Non Newtonian fluid Mech.* 53 (1994) 257
3. Latham T.W: M.S.Theis,MIT, Cambridge ,MA,U.S.A., 1966
4. Burns J.C and Parkes, T. *J.Fluid Mech.* 29 (1967) 731.
5. Barton C. and Raynor S.: *J.Bull. Math.Biophys.*30 (1968) 663
6. Abel El Naby A.H. and Abel El Naby A.E.Mx: *Appl.Math.Comput.* 128 (2001) 19
7. Gupta B.B and Sheshadri V : *J.Biomech* 9 (1976) 105.
8. Srivastava L.M.and Srivastava V.P. *Ann.Biomed.Engg.* 13 (1985) 137.
9. Elshehawey E.F. and Hakeem A.A. *J.Phys.Soc.Jpn.* 65 (1996) 3524
10. Elshehawey E.F, El.Misery E.F. and Hakeem A.A. 67 (1998) 434
11. Haynes:H.R. *Am.J.Physiol.*198 (1960) 1193.
12. Bugliarillo G. and Servilla J. : *BioRheology* 7 (1970) 85
13. Goldsmith H.L. and Skalak R.: *annu. Rev. Inc. Palo Alto publ.* 7 (1975) 231
14. Cotton P.B. and Williams C.B “Practical Gasstrointestinal Endoscopy” (Oxford, London, (1990) 3<sup>rd</sup> ed., Chap.2. p. 7
15. Rathod V.P and Asha.S.K “Peristaltic transport of a couple stress fluid in a uniform and Non uniform annulus”, *International journal of mathematical modeling , simulation and applications* Vol.2 No. 4,pp 414-426
16. Rathod V.P and Asha.S.K “Effect of couple stress fluid and an endoscope in peristaltic Motion” (2009), *Ultra Scinece* Vol.21 (1) M,83-90
17. Shapiro A.H., Jaffrin M.Y. and Weinberg S.L., *J.Fluid Mech.*37 (1969) 799.
18. Shukla J.B, Parihar.R.S. Rao P. and Rao S.P, *J.Fluid Mech.* 97 (1980) 225.

**Source of support: Nil, Conflict of interest: None Declared**