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BIANCHI TYPE-I COSMOLOGICAL MODEL WITH CONSTANT DECELERATION PARAMETER IN SAEZ-BALLESTER THEORY OF GRAVITATION

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ABSTRACT

Bianchi type-I cosmological model in presence perfect fluid source in Saez-Ballester theory is investigated. Einstein's field equations are solvable with help of variation law of Hubble parameter, which gives a constant value of deceleration parameter. The law generates power-law and exponential form of average scale factor in terms of cosmic time. Kinematical properties of the models are discussed. It has been shown that the solutions are compatible with recent observations.

Keywords: *Cosmology, Bianchi type-I, Variation Hubble parameter, Saez-Ballester Theory.*

1. INTRODUCTION

In the last few decades there has been considerable interest in alternative theories of gravitation. The most popular among them are scalar-tensor theories proposed by Lyra (1951), Brans and Dicke (1961), Nordtvedt (1970), Ross (1972) and Dunn (1974), Wagoner (1970), Saez and Ballester (1985) have developed a theory in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields. In spite of the dimensionless character of the scalar field an antigravity regime appears. This theory suggests a possible way to solve the missing matter problem in non-flat FRW cosmologies. The scalar-tensor theories of gravitation play an important role to remove the graceful exit problem in the inflationary era (Pimental 1997).

The gravitational field equations with usual notation for Saez-Ballester theory (for $8\pi G = c = 1$) may be written as

$$R_{ij} - \frac{1}{2} g_{ij} R - \omega\phi^n \left(\phi,_i \ \phi,_j - \frac{1}{2} g_{ij} \phi,_l \ \phi,^l \right) = -T_{ij} \tag{1}$$

and the scalar field satisfies the equation

$$2\phi^n \phi,^i;_j + n\phi^{n-1} \phi,_l \ \phi,^l = 0 \tag{2}$$

Here n is an arbitrary constant and ω is dimensionless coupling constant. Comma and semi-colon respectively denote ordinary and covariant derivative with respect to cosmic time t. T_{ij} is the energy momentum tensor of matter.

The equation of motion

$$T^{ij}_{;j} = 0 \tag{3}$$

are consequences of the field equations (1) and (2).

The study of cosmological models in the frame work of scalar-tensor theories has been the active area of research for the last few decades. Singh and Rai (1983) gave a detailed discussion of Brans-Dicke cosmological models while Singh and Agarwal (1991), Shri Ram and Tiwari (1998), Reddy and Rao (2001) and Reddy et al. (2006) have studied Saez-Ballester cosmological models in four dimensions.

The astronomical observations have revealed that on large scale the universe is isotropic and homogeneous in its present state of evolution. But it might not be the same in the past. Therefore the models with anisotropic background that approach to isotropy at late times, are most suitable for describing the entire evolution of the universe. The spatially homogeneous and anisotropic Bianchi type-I space-time provides such a frame work. For studying the possible effects of anisotropy in the early universe on present day observations many researchers (Huang 1990; Chimento et al. 1997; Lima 1996; Lima and Carvalho 1994; Pradhan and Singh 2004; Saha 2005, 2006) have investigated Bianchi type-I models from different point of view.

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In this paper, we obtain new classes of Bianchi type-I cosmological models in Saez-Ballester theory with constant deceleration parameter. The plan of the paper is as follows. We present the metric and field equations in Sect. 2. In Sect. 3, we obtain exact solutions of the field equations by applying the law of variation for Hubble's parameter which yields a constant value of deceleration parameter. These solutions correspond to singular and non-singular models in two types of cosmologies. The physical and kinematical behaviors of the cosmological models are discussed. Finally,conclusions are summarized in the Sect. 4. It is shown that the models are compatible with the recent observations.

2. THE METRIC AND FIELD EQUATIONS

We consider the Bianchi type-I metric in the form

$$ds^2 = dt^2 - a^2(t)dx^2 - b^2(t)dy^2 - c^2(t)dz^2 \tag{4}$$

We assume that the cosmic matter is represented by the energy-momentum tensor of a perfect fluid

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij} \tag{5}$$

together with

$$g_{ij}u^i u^j = 1 \tag{6}$$

where u^i is the four velocity vector of the fluid and p and ρ are the proper pressure and energy density respectively . From (4)-(6) the components of T^i_j in commoving coordinates are

$$T^1_1 = T^2_2 = T^3_3 = -p, \qquad T^4_4 = \rho, \tag{7}$$

Now, the Saez-Ballester field equations (1) and (2) for the metric (4) with the help of equations (5)-(7) can be written as

$$\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{c}\dot{a}}{ca} = \rho - \frac{\omega}{2}\phi^n\dot{\phi}^2 \tag{8}$$

$$\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{b}\dot{c}}{bc} = -p + \frac{\omega}{2}\phi^n\dot{\phi}^2 \tag{9}$$

$$\frac{\ddot{c}}{c} + \frac{\ddot{a}}{a} + \frac{\dot{c}\dot{a}}{ca} = -p + \frac{\omega}{2}\phi^n\dot{\phi}^2 \tag{10}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} = -p + \frac{\omega}{2}\phi^n\dot{\phi}^2 \tag{11}$$

$$\ddot{\phi} + \left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c}\right)\dot{\phi} + \frac{n}{2}\frac{\dot{\phi}^2}{\phi} = 0 \tag{12}$$

and equation (3) which is a consequence of the field equations takes the form

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c}\right) = 0 \tag{13}$$

The average scale factor S for the metric (4) is defined as

$$S^3 = abc \tag{14}$$

The volume scale factor V is given by

$$V = S^3 = abc \tag{15}$$

We define the generalized mean Hubble's parameter H as

$$H = \frac{1}{3}\left(H_x + H_y + H_z\right) \tag{16}$$

where $H_x = \frac{\dot{a}}{a}, H_y = \frac{\dot{b}}{b}, H_z = \frac{\dot{c}}{c}$, are directional Hubble's parameters in the direction of x, y and z respectively.

From (14), (15) and (16) we obtain

$$H = \frac{\dot{S}}{S} = \frac{1}{3}\left(H_x + H_y + H_Z\right) \tag{17}$$

An important quantity q, the deceleration parameter in cosmology, is given by

$$q = -\frac{S\ddot{S}}{\dot{S}^2} \tag{18}$$

For the metric (4) in comoving coordinates the expansion scalar θ, shear scalar σ^2 are defined by

$$\theta = \left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c}\right) \tag{19}$$

$$\sigma^2 = \frac{1}{2}\sigma_{i\,j}\sigma^{i\,j} \tag{20}$$

$\sigma_{i\,j}$ being shear tensor.

3. SOLUTION OF THE FIELD EQUATIONS

Now we follow the approaches of Saha and Rikhvitsky (2006) and Singh and Chaubey (2007) to solved the field equations (8)-(11). Subtracting equation (9) from equation (10) we get

$$\frac{d}{dt}\left(\frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right) + \left(\frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right)\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c}\right) = 0 \tag{21}$$

From (15) and (21) we get

$$\frac{d}{dt}\left(\frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right) + \left(\frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right)\frac{\dot{V}}{V} = 0 \tag{22}$$

Integrating, which gives

$$\frac{a}{b} = d_1 \exp\left(x_1 \int \frac{dt}{V}\right) \tag{23}$$

Subtracting equation (9) from equation (11)

$$\frac{d}{dt}\left(\frac{\dot{a}}{a} - \frac{\dot{c}}{c}\right) + \left(\frac{\dot{a}}{a} - \frac{\dot{c}}{c}\right)\frac{\dot{V}}{V} = 0 \tag{24}$$

$$\frac{a}{c} = d_2 \exp\left(x_2 \int \frac{dt}{V}\right) \tag{25}$$

Subtracting equation (10) from equation (11)

$$\frac{d}{dt}\left(\frac{\dot{b}}{b} - \frac{\dot{c}}{c}\right) + \left(\frac{\dot{b}}{b} - \frac{\dot{c}}{c}\right)\frac{\dot{V}}{V} = 0 \tag{26}$$

Integrating, we get

$$\frac{b}{c} = d_3 \exp\left(x_3 \int \frac{dt}{V}\right) \tag{27}$$

where $d_1,\ d_2,\ d_3$ and $x_1,\ x_2,\ x_3$ are constants of integration. Using equations (23), (25) and (27) the values of $a(t),\ b(t)$ and $c(t)$ can be written explicitly as

$$a(t) = D_1 V^{\frac{1}{3}} \exp\left(X_1 \int \frac{dt}{V}\right) \tag{28}$$

$$b(t) = D_2 V^{\frac{1}{3}} \exp\left(X_2 \int \frac{dt}{V}\right) \tag{29}$$

$$c(t) = D_3 V^{\frac{1}{3}} \exp\left(X_3 \int \frac{dt}{V} \right) \tag{30}$$

where

$$D_1 = \sqrt[3]{d_1 d_2}, \quad D_2 = \sqrt[3]{d_1^{-1} d_3} \quad D_3 = \sqrt[3]{(d_2 d_3)^{-1}}$$
$$X_1 = x_1 + x_2, \ X_2 = x_3 - x_1, \ X_3 = -(x_2 + x_3)$$

the constants X_1, X_2, X_3 and D_1, D_2, D_3 satisfy the relations

$$X_1 + X_2 + X_3 = 0, \quad D_1 D_2 D_3 = 1 \tag{31}$$

We apply the special law of variation for generalized Hubble's parameter that yields a constant value of deceleration parameter. Since the line element (4) is completely characterized by Hubble parameter H. Therefore we assume a relation between Hubble parameter H and average scale factor S given by

$$H = lS^{-m} \tag{32}$$

where $l > 0$ and $m \geq 0$ are constants. This type of relation has been already discussed by Berman (1983) and Berman and Gomide (1988) in case of FRW models. Such relation gives a constant value of deceleration parameter. Later on, several authors (Sri Ram et al. 2009; Singh et al. 2012; Singh and Baghel 2009; Baghel and Singh 2012; Kumar 2011; Pradhan et al. 2011; Johri and Desikan 1994) have studied FRW and Bianchi type models by using the special law for Hubble parameter that yields constant value of deceleration parameter.

From (17) and (32), we get

$$\dot{S} = lS^{-m+1} \tag{33}$$

and

$$\ddot{S} = -l^2 (m-1) S^{-2m+1} \tag{34}$$

Substituting (33) and (34) into (18), we get

$$q = m - 1 \tag{35}$$

which is a constant. The sign of q indicates whether the model inflates or not. The positive sign of q i.e. $(m > 1)$ correspond to ''standard'' decelerating model whereas the negative sign of q i.e. $0 \leq m < 1$ indicates inflation. It may be noted that though the current observations of SN 1a and CMB favor accelerating models $(q < 0)$, they do not altogether rule out the decelerating ones which are also consistent with this observations (Vishwakarma, 2000).

Integrating equation (33) we obtain the law of average scale factor 'S' as

$$S = (lmt + c_1)^{\frac{1}{m}} \text{ for } m \neq 0 \tag{36}$$

and

$$S = c_2 e^{lt} \text{ for } m = 0 \tag{37}$$

where c_1 and c_2 are constants of integration. Thus, the law (32) provides two types of expansion in the universe i.e., (i) power law (36) and (ii) exponential law (37).

3.1 Case (i): When $n \neq 0$ i.e. model for power law expansion

Using equations (15) and (36) into (28)-(30), we obtain

$$a(t) = D_1 (lmt + c_1)^{\frac{1}{m}} \exp\left[\frac{X_1}{l(m-3)} (lmt + c_1)^{1-\frac{3}{m}} \right] \tag{38}$$

$$b(t) = D_1 (lmt + c_1)^{\frac{1}{m}} \exp\left[\frac{X_2}{l(m-3)} (lmt + c_1)^{1-\frac{3}{m}} \right] \tag{39}$$

$$c(t) = D_1 (lmt + c_1)^{\frac{1}{m}} \exp\left[\frac{X_3}{l(m-3)} (lmt + c_1)^{1-\frac{3}{m}} \right] \tag{40}$$

Hence the model (4) reduced to

$$ds^2 = dt^2 - D_1^2 (lmt + c_1)^{\frac{2}{m}} \exp\left[\frac{2X_1}{l(m-3)}(lmt + c_1)^{1-\frac{3}{m}}\right] dx^2$$

$$- D_2^2 (lmt + c_1)^{\frac{2}{m}} \exp\left[\frac{2X_2}{l(m-3)}(lmt + c_1)^{1-\frac{3}{m}}\right] dy^2 - D_3^2 (lmt + c_1)^{\frac{2}{m}} \exp\left[\frac{2X_3}{l(m-3)}(lmt + c_1)^{1-\frac{3}{m}}\right] dz^2 \quad (41)$$

The solution for the scalar function ϕ from equations (12), (14) and (36) we get

$$\phi = \left[\frac{k_1(n+2)}{2l(m-3)(lmt + c_1)^{\frac{3}{m}-1}} + \phi_0\right]^{\frac{2}{n+2}} \quad (42)$$

where k_1 is an integrating constant and $\phi_0 = \frac{k_2(n+2)}{2} = cons\tan t$. Using equations (42) and (38)-(40) into equations (8) and (9), the energy density and pressure of the fluid are given by

$$\rho = \frac{3l^2}{(lmt + c_1)^2} + \frac{\left(X_1X_2 + X_2X_3 + X_3X_1 + \omega/_2\, k_1^2\right)}{(lmt + c_1)^{\frac{6}{m}}} \quad (43)$$

$$p = \frac{1}{\left(lmt + c_1^2\right)^{\frac{6}{m}}}\left[\frac{\omega}{2}k_1^2 - X_1^2 - X_2^2 - X_1X_2\right] - \frac{l^2(3-2m)}{(lmt + c_1)^2} \quad (44)$$

The above solutions are valid for $m \neq 3$. The rate of expansion H_i in the direction of x, y and z read as

$$H_x = \frac{l}{(lmt + c_1)} + \frac{X_1}{(lmt + c_1)^{\frac{3}{n}}} \quad (45)$$

$$H_y = \frac{l}{(lmt + c_1)} + \frac{X_2}{(lmt + c_1)^{\frac{3}{n}}} \quad (46)$$

$$H_z = \frac{l}{(lmt + c_1)} + \frac{X_3}{(lmt + c_1)^{\frac{3}{n}}} \quad (47)$$

The Hubble parameter is given by

$$H = \frac{l}{(lmt + c_1)} \quad (48)$$

The expansion scalar $\theta = 3H$ is

$$\theta = \frac{3l}{(lmt + c_1)} \quad (49)$$

The components of shear tensor $(\sigma_i^{\ j})$ and conformal curvature tensor (C^i_{jij}) are given by

$$\sigma_1^{\ 1} = \frac{(2X_1 - X_2 - X_3)}{3(lmt + c_1)^{\frac{3}{n}}} \quad (50)$$

$$\sigma_2^{\ 2} = \frac{(2X_2 - X_3 - X_1)}{3(lmt + c_1)^{\frac{3}{n}}} \quad (51)$$

$$\sigma_3^{\ 3} = \frac{(2X_3 - X_1 - X_2)}{3(lmt + c_1)^{\frac{3}{n}}} \quad (52)$$

$$\sigma_4^{\ 4} = 0 \quad (53)$$

Thus

$$\sigma^2 = \frac{1}{2}\left[\left(\sigma_1{}^1\right)^2 + \left(\sigma_2{}^2\right)^2 + \left(\sigma_3{}^3\right)^2 + \left(\sigma_4{}^4\right)^2\right] \tag{54}$$

Therefore

$$\sigma = \frac{\left[\left(X_1{}^2 + X_2{}^2 + X_3{}^2\right) - \left(X_1X_2 + X_2X_3 + X_3X_1\right)\right]^{\frac{1}{2}}}{\sqrt{3}\left(lmt + c_1\right)^{\frac{3}{n}}} \tag{55}$$

$$C^1_{212} = \frac{l^2}{\left(lmt + c_1\right)^2} + \frac{l\left(X_1 + X_2\right)}{\left(lmt + c_1\right)^{\frac{3}{m}+1}} + \frac{X_1X_2}{\left(lmt + c_1\right)^{\frac{6}{m}}} \tag{56}$$

$$C^1_{313} = \frac{l^2}{\left(lmt + c_1\right)^2} + \frac{l\left(X_1 + X_3\right)}{\left(lmt + c_1\right)^{\frac{3}{m}+1}} + \frac{X_1X_3}{\left(lmt + c_1\right)^{\frac{6}{m}}} \tag{57}$$

$$C^2_{323} = \frac{l^2}{\left(lmt + c_1\right)^2} + \frac{l\left(X_2 + X_3\right)}{\left(lmt + c_1\right)^{\frac{3}{m}+1}} + \frac{X_2X_3}{\left(lmt + c_1\right)^{\frac{6}{m}}} \tag{58}$$

It is observed that the energy conservation equation (13) is identically satisfied. The spatial volume is zero and the expansion scalar is infinite at $t = t_1$ where $t_1 = -\frac{c_1}{lm}$, which shows that the universe starts evolving with zero volume at $t = t_1$ with an infinite rate of expansion. The directional scale factors vanish at $t = t_1$ and the parameters H_i, H, θ, σ and components of conformal curvature tensor $(C^i{}_{jij})$ diverge at the initial singularity. Therefore the model has a point singularity at the initial epoch (MacCallum 1971). The directional Hubble's parameters and the generalized Hubble's parameter are both infinite at this singularity point. The universe exhibits the power-law expansion after the big-bang impulse. As t increases the scale factors and the spatial volume increases but the expansion scalar and components of conformal curvature tensor decreases. Thus the rate of expansion slows down as the cosmic evolution progresses. For n>3, $\frac{\sigma}{\theta} \neq 0$ as $t \to \infty$, therefore, the model does not isotropize for large values of t. If n<3, the models are quasi-isotropic i.e. $\lim_{t\to\infty} \frac{\sigma}{\theta} = 0$.

The positive value of q indicates that the universe is decelerating. The scalar field ϕ, being infinite at $t = t_1$, decreases as time increases and ultimately becomes a constant for large time.

3.2 Case (ii): When $n = 0$ i.e. model for exponential law expansion

In this case we obtain an exponentially expanding non-singular cosmological model describing the accelerating phase of the present-day universe.

Using equations (15) and (37) into (28)-(30) we obtain

$$a(t) = D_1c_2e^{lt}\exp\left[-\frac{X_1}{3lc_2{}^3}e^{-3lt}\right] \tag{59}$$

$$b(t) = D_2c_2e^{lt}\exp\left[-\frac{X_2}{3lc_2{}^3}e^{-3lt}\right] \tag{60}$$

$$c(t) = D_3c_2e^{lt}\exp\left[-\frac{X_3}{3lc_2{}^3}e^{-3lt}\right] \tag{61}$$

Hence the model (4) is reduced to

$$ds^2 = dt^2 - D_1{}^2c_2{}^2e^{2lt}\exp\left[-\frac{2X_1}{3lc_2{}^3}e^{-3lt}\right]dx^2 - D_2{}^2c_2{}^2e^{2lt}\exp\left[-\frac{2X_2}{3lc_2{}^3}e^{-3lt}\right]dy^2$$

$$- D_3{}^2c_2{}^2e^{2lt}\exp\left[-\frac{2X_3}{3lc_2{}^3}e^{-3lt}\right]dz^2 \tag{62}$$

The solution for the scalar function ϕ from equations (12), (14) and (37) we get

$$\phi = \left[\phi_1 - \frac{(n+2)k_1}{6{c_2}^3 l} e^{-3lt} \right]^{\frac{2}{n+2}} \tag{63}$$

where k_3 is an integrating constant and $\phi_1 = \frac{k_3(n+2)}{2} = cons\tan t$. Using equations (63) and (59)-(61) into equations (8) and (9), the energy density and pressure of the fluid are given by

$$\rho = 3l^2 + \frac{e^{-6lt}}{{c_2}^6}\left[\frac{\omega}{2}{k_1}^2 + X_1X_2 + X_2X_3 + X_3X_1 \right] \tag{64}$$

$$p = -\left[3l^2 + \frac{e^{-6lt}}{{c_2}^6}\left\{ {X_1}^2 + {X_2}^2 + X_1X_2 - \frac{\omega}{2}{k_1}^2 \right\} \right] \tag{65}$$

The rate of expansion H_i in the direction of x, y and z are obtain as

$$H_x = l + \frac{X_1}{{c_2}^3} e^{-3lt} \tag{66}$$

$$H_y = l + \frac{X_2}{{c_2}^3} e^{-3lt} \tag{67}$$

$$H_z = l + \frac{X_3}{{c_2}^3} e^{-3lt} \tag{68}$$

Thus the Hubble's parameter H, expansion scalar θ, shear scalar σ and components of conformal curvature tensor $(C^i_{j\,i\,j})$ are given by are given by

$$H = l \tag{69}$$

$$\theta = 3l \tag{70}$$

$$\sigma = \frac{e^{-3lt}}{\sqrt{3}{c_2}^3}\left[\left({X_1}^2 + {X_2}^2 + {X_3}^3 \right) - \left(X_1X_2 + X_2X_3 + X_3X_1 \right) \right]^{\frac{1}{2}} \tag{71}$$

$$C^1_{212} = l^2 + \frac{l(X_1 + X_2)}{{c_2}^3} e^{-3lt} + \frac{X_1X_2}{{c_2}^6} e^{-6lt} \tag{72}$$

$$C^1_{313} = l^2 + \frac{l(X_1 + X_3)}{{c_2}^3} e^{-3lt} + \frac{X_1X_3}{{c_2}^6} e^{-6lt} \tag{73}$$

$$C^2_{323} = l^2 + \frac{l(X_2 + X_3)}{{c_2}^3} e^{-3lt} + \frac{X_2X_3}{{c_2}^6} e^{-6lt} \tag{74}$$

The conservation equation (13) is identically satisfied. From (37), it is observed that as $t \to -\infty$, $S \to 0$ which shows that the universe is infinitely old and has exponential inflationary phase. The directional scale factors and spatial volume of the universe increase exponentially with cosmic time and the shear scalar and components of conformal curvature tensor $(C^i_{j\,i\,j})$ are decreases with cosmic time, whereas the mean Hubble parameter and the expansion scalar are constant. Also for $t \to \infty$ the scale factors and volume of the universe becomes infinitely large, which shows that the universe is dominated by vacuum energy which drives the accelerated expansion of the universe. Since $\lim_{t\to\infty} \frac{\sigma}{\theta} = 0$, hence the model isotropize for large values of t. The derived model is non-singular. The scalar field ϕ is a decreasing function of time, which tends to constant for large time.

For n=0, we get $q=-1$; incidentally this value of deceleration parameter leads to $dH/dt = 0$, which implies the greatest value of Hubble's parameter and the fastest rate of expansion of the universe. Therefore, the derived model can be utilized to describe the dynamics of the late time evolution of the actual universe.

4. CONCLUSIONS

In this paper, we have investigated spatially homogeneous and an anisotropic Bianchi type-I cosmological models in presence of perfect fluid source in Saez-Ballester theory of gravity. In solving the field equations we have used the variation law of Hubble's parameter proposed by Berman (1983), which generates two types of cosmologies, (i) first form for $n \neq 0$ gives the solution for positive value of deceleration parameter for $n > 1$ and negative value of deceleration parameter for $0 \leq n < 1$, which shows the power-law expansion of the universe whereas (ii) second one (for n=0) gives the solution for negative value of deceleration parameter, which shows the exponential expansion of the universe. The power solution represents the singular model where the spatial scale factors and volume scalar vanish at $t = t_1$. All the physical parameters are infinite at the initial epoch and tends to zero as $t \to \infty$. There is a Point Type singularity (MacCallum 1971) at $t = t_1$ in the first model. The exponential solutions represents singularity free model of the universe. The directional scale factors are time dependent while the average Hubble parameter is constant. The expansion scalar is constant throughout the time of evolution, which shows that the constant expansion of the universe right from the beginning. In both the cases the scalar field ϕ is always decreasing function of cosmic time and for large time it is constant.

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