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ON PRESERVING (1, 2)*-g- CLOSED SETS IN BITOPOLOGY

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ABSTRACT

The purpose of this paper is to introduce new weak forms of continuity and closure (which we call (1, 2)-a-continuity and (1, 2)*-a-closure) and to use these forms to strengthen some of the bitopological results of Ravi et al. [6].*

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1. INTRODUCTION:

Generalized closed sets (or g-closed sets) were introduced by Levine in [4]. Levine established conditions under which functions and inverse functions preserve g-closed sets. In [3], Cueva improved one of Levine's results. These results have recently been generalized to bitopological spaces by Ravi *et al* [6]. In [2], Baker strengthened some of the topological results of Cueva and Levine. The purpose of this paper is to introduce new weak forms of continuity and closure namely (1,2)*-a-continuity and (1,2)*-a-closed functions in bitopological spaces and to use these forms to strengthen some of the bitopological results of Ravi *et al* [6]. We also characterize (1, 2)*-$T_{1/2}$ spaces in terms of (1,2)*-a-continuity and (1,2)*-a-closure. Finally some of the basic properties of (1, 2)*-a-continuous functions and (1,2)*-a-closed functions are investigated.

2. PRELIMINARIES:

Throughout the paper, X, Y and Z denote bitopological spaces.

Definition: 2.1[5]
A subset S of a space (X, τ_1, τ_2) is said to be $\tau_{1,2}$-open if $S = A \cup B$ where $A \in \tau_1$ and $B \in \tau_2$.
The complement of $\tau_{1,2}$-open set is $\tau_{1,2}$-closed.

The family of all $\tau_{1,2}$-open [resp. $\tau_{1,2}$-closed] subsets of X is denoted by (1,2)*-O(X) [resp. (1,2)*-C(X)].

Note: 2.2[5]
Notice that $\tau_{1,2}$-open subsets of X need not necessarily form a topology.

Definition: 2.3[5]
Let S be a subset of X. Then
(i) The $\tau_{1,2}$-interior of S, denoted by $\tau_{1,2}$-int(S), is defined as $\cup\{F|F\subseteq S$ and F is $\tau_{1,2}$-open$\}$;

(ii) The $\tau_{1,2}$-closure of S, denoted by $\tau_{1,2}$-cl(S), is defined as $\cap\{F|S\subseteq F$ and F is $\tau_{1,2}$-closed$\}$.

Definition: 2.4[6]
A subset S of a space (X, τ_1, τ_2) is said to be (1,2)*-g-closed if $\tau_{1,2}$-cl(S) $\subseteq$ U whenever S $\subseteq$ U and U is $\tau_{1,2}$-open in X.

The complement of (1,2)*-g-closed set is (1,2)*-g-open.

Definition: 2.5[6]
A function f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be (1,2)*-g-continuous if $f^{-1}(F)$ is (1,2)*-g-closed set in X for every $\sigma_{1,2}$-closed set F of Y.

Theorem: 2.6[6]
A subset S of X is (1,2)*-g-closed if and only if $\tau_{1,2}$-cl(S)–S contains no non-empty $\tau_{1,2}$-closed set.

Theorem: 2.7[6]
If S is a (1,2)*-g-closed subset in X and if f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (1,2)*-continuous and (1,2)*-closed, then f(S) is (1,2)*-g-closed set in Y.

Theorem: 2.8[6]
A subset A of X is (1,2)*-g-open in X if and only if F $\subseteq$ $\tau_{1,2}$-int(A) whenever F is $\tau_{1,2}$-closed set and F $\subseteq$ A.

Theorem:2.9[6]
If f: X $\to$ Y is (1,2)*-g-continuous and (1,2)*-closed and if G is a (1,2)*-g-open (or (1,2)*-g-closed) subset of Y, then $f^{-1}(G)$ is (1,2)*-g-open (or (1,2)*-g-closed) in X.

Theorem: 2.10[6]
Suppose that A $\subseteq$ B $\subseteq$ X, A is a (1,2)*-g-closed set relative to B and that B is a (1,2)*-g-closed subset of X. Then A is (1, 2)*-g-closed relative to X.

Definition: 2.11[6]

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A space X is said to be a $(1,2)^*$-$T_{1/2}$ space if every $(1,2)^*$-g-closed subset of X is $\tau_{1,2}$-closed in X.

3. (1, 2)*-a-CLOSED AND (1, 2)*-a-CONTINUOUS FUNCTIONS:

Definition: 3.1
A function f: X → Y is said to be approximately (1,2)*-closed (or (1,2)*-a-closed) if $f(F) \subseteq \sigma_{1,2}$-int(A) whenever F is a $\tau_{1,2}$-closed subset of X, A is a (1,2)*-g-open subset of Y and $f(F) \subseteq A$.

Definition: 3.2
A function f: X → Y is said to be approximately (1,2)*-continuous (or (1,2)*-a-continuous) if $\tau_{1,2}$-cl(A) $\subseteq f^{-1}(V)$ whenever V is an $\sigma_{1,2}$-open subset of Y, A is a (1,2)*-g-closed subset of X, and $A \subseteq f^{-1}(V)$.
Clearly, (1, 2)*-closed functions are (1, 2)*-a-closed and (1,2)*-continuous functions are (1,2)*-a-continuous. The following example shows the converse implications do not hold.

Example: 3.3
Consider the Sierpinski space X = {a,b} with the topologies $\tau_1 = \{\phi, X, \{a\}\}$ and $\tau_2 = \{\phi, X\}$. Define the function f: X → X by f(a) = b and f(b) = a. Clearly, f is neither (1,2)*-closed nor (1,2)*-continuous. However, since the image of every $\tau_{1,2}$-closed set is $\sigma_{1,2}$-open, f is (1,2)*-a-closed. Similarly, since the inverse image of every $\sigma_{1,2}$-open set is $\tau_{1,2}$-closed, f is (1,2)*-a-continuous.

Theorem: 3.4 If f: X → Y is a bijection, then f is (1,2)*-a-closed if and only if f^{-1} is (1,2)*-a-continuous**.**

Proof: The proof of this result is a straight forward argument using complements and is omitted.

4. PRESERVING (1, 2)*-g-CLOSED SETS:

The following theorem strengthens Theorem 2.9 by replacing the closure requirement with (1,2)*-a-closure.

Theorem: 4.1 If f: X → Y is (1,2)*-g-continuous and (1,2)*-a-closed, then $f^{-1}(A)$ is (1,2)*-g-closed ((1,2)*-g-open) whenever A is a (1,2)*-g-closed ((1,2)*-g-open) subset of Y.

Proof: Assume that A is a (1,2)*-g-closed subset of Y and let $f^{-1}(A) \subseteq U$, where U is an $\tau_{1,2}$-open subset of X. Taking complements we obtain $X–U \subseteq f^{-1}(Y–A)$ or $f(X–U) \subseteq Y–A$. Since f is (1, 2)*-a-closed, $f(X–U) \subseteq \sigma_{1,2}$-int(Y–A) = Y–$\sigma_{1,2}$-cl(A). It follows that $X–U \subseteq X - f^{-1}(\sigma_{1,2}\text{-cl}(A))$ and hence $f^{-1}(\sigma_{1,2}\text{-cl}(A)) \subseteq U$. Since f is (1, 2)*-g-continuous, $f^{-1}(\sigma_{1,2}\text{-cl}(A))$ is (1,2)*-g-closed. Thus we have $\tau_{1,2}\text{-cl}(f^{-1}(A)) \subseteq \tau_{1,2}\text{-cl}(f^{-1}(\sigma_{1,2}\text{-cl}(A))) \subseteq U$ which implies that $f^{-1}(A)$ is (1,2)*-g-closed.

It is shown that inverse images of (1, 2)*-g-open sets are (1, 2)*-g-open by applying complementation to the result just obtained and using the fact that f^{-1} preserves complements.

The following theorem strengthens Theorem 2.7 by replacing the continuity requirement with (1, 2)*-a-continuity.

Theorem: 4.2 If f: X → Y is (1,2)*-a-continuous and (1,2)*-closed, then f(A) is (1,2)*-g-closed in Y whenever A is a (1,2)*-g-closed subset of X.

Proof: Assume that $A \subseteq X$ is (1,2)*-g-closed and that $f(A) \subseteq V$, where V is an $\sigma_{1,2}$-open subset of Y. Then $A \subseteq f^{-1}(V)$ and, since f is (1,2)*-a-continuous, $\tau_{1,2}\text{-cl}(A) \subseteq f^{-1}(V)$. Then $f(\tau_{1,2}\text{-cl}(A)) \subseteq V$ and since f is (1,2)*-closed, we have $\sigma_{1,2}\text{-cl}(f(A)) \subseteq \sigma_{1,2}\text{-cl}(f(\tau_{1,2}\text{-cl}(A))) = f(\tau_{1,2}\text{-cl}(A)) \subseteq V$. Therefore $\tau_{1,2}\text{-cl}(f(A)) \subset V$ and hence f(A) is (1,2)*-g-closed.

5. PROPERTIES OF (1, 2)*-a-CLOSED AND (1, 2)*-a-CONTINUOUS FUNCTIONS:

In this section, we use (1, 2)*-a-closed and (1, 2)*-a-continuous functions to characterize $(1,2)^*$-$T_{1/2}$ spaces. Also we establish sufficient conditions for a function to be (1, 2)*-a-closed or (1, 2)*-a-continuous. Finally we investigate some of the properties of these functions involving restriction and composition.

Theorem: 5.1 A space X is a $(1, 2)^*$-$T_{1/2}$ space if and only if, for every space Y and every function f: X → Y, f is (1,2)*-a-continuous.

Proof: The necessity follows from the definition of (1,2)*-a-continuity. For the sufficiency, let A be a non-empty (1,2)*-g-closed subset of X and let Y be the set X with topologies $\sigma_1 = \{\phi, Y, A\}$, $\sigma_2 = \{\phi, Y\}$. Finally let f: X → Y be the identity mapping. By assumption f is (1,2)*-a-continuous. Since A is (1,2)*-g-closed in X and $\sigma_{1,2}$-open in Y, and $A \subseteq f^{-1}(A)$, it follows that $\tau_{1,2}\text{-cl}(A) \subseteq f^{-1}(A) = A$. Hence A is $\tau_{1,2}$-closed in X and therefore X is $(1,2)^*$-$T_{1/2}$ space.

An analogous argument proves the following result for (1, 2)*-a-closed functions.

Theorem: 5.2 A space Y is a $(1, 2)^*$-$T_{1/2}$-space if and only if, for every space X and every function f: X → Y, f is (1,2)*-a-closed.

The next two results follow easily from the definitions. These results were used in Example 3.3.

Theorem: 5.3. If f: X → Y is a function for which f(F) is $\sigma_{1,2}$-open in Y for every $\tau_{1,2}$-closed subset F of X, then f is (1,2)*-a-closed.

Theorem: 5.4 If f: X → Y is a function for which $f^{-1}(V)$ is $\tau_{1,2}$-closed in X for every $\sigma_{1,2}$-open subset V of Y, then f is (1,2)*-a-continuous.

Since the identity mapping on any space is both (1, 2)*-a-continuous and (1, 2)*-a-closed, it is clear the converses of Theorems 5.3 and 5.4 do not hold.

Theorem: 5.5 In a bitopological space X, (1, 2)*-O(X) = (1,2)*-C(X) if and only if every subset of X is a (1,2)*-g-closed set.

Proof: Suppose that (1, 2)*-O(X) = (1,2)*-C(X) and that $A \subseteq U \in$ (1,2)*-O(X). Then $\tau_{1,2}\text{-cl}(A) \subseteq \tau_{1,2}\text{-cl}(U) = U$ and A is (1,2)*-g-closed. Conversely, suppose that every subset of X is (1, 2)*-g-closed. Let $U \in$ (1, 2)*-O(X). Then since $U \subseteq U$ and

U is (1, 2)*-g-closed, we have $\tau_{1,2}$-cl(U) ⊆ U and U ∈ (1,2)*-C(X). Thus (1, 2)*-O(X) ⊆ (1, 2)*-C(X). If F ∈ (1, 2)*-**C(X),** then X–F ∈ (1, 2)*-O(X) ⊆ (1,2)*-C(X) and hence F ∈ (1,2)*-O(X). Finally, (1, 2)*-O(X) = (1, 2)*-C(X).

Theorem: 5.6 If the $\sigma_{1,2}$-open and $\sigma_{1,2}$-closed sets of Y coincide, then a function f: X → Y is (1,2)*-a-closed if and only if f(F) is $\sigma_{1,2}$-open for every $\tau_{1,2}$-closed subset F of X.

Proof: Assume f is (1,2)*-a-closed. By Theorem 5.5, all subsets of Y are (1, 2)*-g-closed (and hence all are (1,2)*-g-open). So, for any $\tau_{1,2}$-closed subset F of X, f(F) is (1,2)*-g-open in Y. Since f is (1, 2)*-a-closed, f(F) ⊆ $\sigma_{1,2}$-int(f(F)). Therefore f(F) = $\sigma_{1,2}$-int (f(F)), ie., f(F) is $\sigma_{1,2}$-open. The converse can be easily shown.

Corollary: 5.7

If the $\sigma_{1,2}$-open and $\sigma_{1,2}$-closed sets of Y coincide, then a function f: X → Y is (1,2)*-a-closed if and only if it is (1,2)*-closed.

The proofs of the following results for (1, 2)*-a-continuous functions are analogous and are omitted.

Theorem: 5.8

If the $\tau_{1,2}$-open and $\tau_{1,2}$-closed sets of X coincide, then a function f: X → Y is (1,2)*-a-continuous if and only if f^{-1}(V) is $\tau_{1,2}$-closed for every $\sigma_{1,2}$-open subset V of Y.

Corollary: 5.9

If the $\tau_{1,2}$-open and $\tau_{1,2}$-closed sets of X coincide, then a function f: X → Y is (1,2)*-a-continuous if and only if it is (1,2)*-continuous.

Compositions of (1, 2)*-a-continuous ((1, 2)*-a-closed) functions are not, in general, (1,2)*-a-continuous ((1,2)*-a-closed). However, the following results do hold.

Theorem: 5.10 If f: X → Y is (1, 2)*-closed and g: Y → Z is (1, 2)*-a-closed, then gof: X → Z is (1,2)*-a-closed.

Proof: Let F be a $\tau_{1,2}$-closed subset of X and A a (1,2)*-g-open subset of Z for which gof (F) ⊆ A. Since f is (1, 2)*-closed, f(F) is $\sigma_{1,2}$-closed in Y. Because g is (1,2)*-a-closed, g(f(F)) ⊆ $\eta_{1,2}$-int(A).

Theorem: 5.11 If f: X → Y is (1,2)*-a-closed and g: Y → Z is (1,2)*-open and inversely preserves (1,2)*-g-open sets, then gof: X → Z is (1,2)*-a-closed**.**

Proof: Let F be a $\tau_{1,2}$-closed subset of X and A a (1,2)*-g-open subset of Z for which gof (F) ⊆ A. Then f(F) ⊆ g^{-1}(A). Since g^{-1}(A) is (1,2)*-g-open and f is (1,2)*-a-closed, f(F) ⊆ $\sigma_{1,2}$-int (g^{-1}(A)). Thus gof (F) = g(f(F)) ⊆ g($\sigma_{1,2}$-int (g^{-1}(A))) ⊆ $\eta_{1,2}$-int (g(g^{-1}(A))) ⊆ $\eta_{1,2}$-int(A).

Theorem: 5.12 If f: X → Y is (1, 2)*-a-continuous and g: Y → Z is (1,2)*-continuous, then gof: X → Z is (1,2)*-a-continuous.

Proof: Assume A is a (1,2)*-g-closed subset of X and V is an $\eta_{1,2}$-open subset of Z for which A ⊆ $(gof)^{-1}$ (V). Since g is (1, 2)*-continuous, g^{-1}(V) is $\sigma_{1,2}$-open. Because f is (1, 2)*-a-continuous, $\tau_{1,2}$-cl(A) ⊆ $(gof)^{-1}$(V) = f^{-1}(g^{-1}(V)).

Corollary: 5.13

Let f_α: X → Y_α be a function for each α ∈ $\mathcal{A}$ and f: X → Π Y_α the product map given by f(x) = (f_α(x)). If f is (1, 2)*-a-continuous, then f_α is (1,2)*-a-continuous for each α.

Proof: For each β let p_β: ΠY_α → Y_β be the projection function. Then f_β = p_β o f where p_β is (1, 2)*-continuous and f is (1,2)*-a-continuous.

Theorem: 5.14 If f: X → Y is (1,2)*-a-continuous and B is a (1,2)*-g-closed subset of X, then f/B: B → Y is (1,2)*-a-continuous.

Proof: Assume A is a (1, 2)*-g-closed subset of B and V is an $\sigma_{1,2}$-open subset of Y for which A ⊆ $(f/B)^{-1}$ (V). Then A ⊆ f^{-1}(V) ∩ B. By Theorem 2.10, A is (1,2)*-g-closed relative to X. Since f is (1,2)*-a-continuous. $\tau_{1,2}$-cl(A) ⊆ f^{-1}(V). Then $\tau_{1,2}$-cl(A) ∩ B ⊆ f^{-1}(V) ∩ B and hence $\tau_{1,2}$-cl_B(A) ⊆ $(f/B)^{-1}$. Thus f/B: B → Y is (1, 2)*-a-continuous.

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