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FIXED POINT THEOREM IN FUZZY METRIC SPACE USING WEAKLY COMPATIBLE

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ABSTRACT

In this paper we prove a common fixed point theorem for three mappings in Fuzzy metric space using the notion of weakly compatible.

Keyword: *Fuzzy metric space, Common fixed point, Compatible maps, weakly compatible maps, Continuous t-norm.*

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1. INTRODUCTION:

The study of common fixed points of mappings in a fuzzy metric space satisfying certain contractive conditions has been at the center of vigorous research activity. In 1965, the concept of fuzzy sets was introduced by Zadeh [36]. With the concept of fuzzy sets, the fuzzy metric space was introduced by O. Kramosil and J. Michalek [25] in 1975. Helpern [19] in 1981 first proved a fixed point theorem for fuzzy mappings. Also M. Grabiec [17] in 1988 proved the contraction principle in the setting of the fuzzy metric space. Moreover, A. George and P. Veeramani [16] in 1994 modified the notion of fuzzy metric spaces with the help of t-norm by generalizing the concept of probabilistic metric space to fuzzy situation. Consequently in due course of time some metric fixed point results were generalized to fuzzy metric spaces by various authors.

Gahler in a series of papers [13, 14, and 15] investigated 2-metric space. Sharma, Sharma and Iseki [30] studied for the first time contraction type mappings in 2-metric space.

We know that 2-metric space is a real valued function of a point triples on a set X, which abstract properties were suggested by the area function in Euclidean spaces. Now it is natural to expect 3-Metric space, which is suggested by the volume function.

In this paper we establish a common fixed theorem for three mappings for weakly compatible in fuzzy metric space. First we recall some definitions and known results in Fuzzy metric space.

2. PRELIMINARIES:

Definition: 2.1. A binary operation $*:[0,1]^4 \to [0,1]$ is called a continuous t-norm if $([0,1],*)$ is an abelian topological monoid with unit 1 such that

$a_1 * b_1 * c_1 * d_1 \geq a_2 * b_2 * c_2 * d_2$ Whenever $a_1 \geq a_2$, $b_1 \geq b_2$, $c_1 \geq c_2$ and $d_1 \geq d_2$ for all

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$a_1, a_2, b_1, b_2, c_1, c_2$ and d_1, d_2 are in [0,1].

Definition: 2.2. The 3-tuple $(X, M, *)$ is called a fuzzy 3-metric space if X is an arbitrary set $*$ is continuous t-norm monodies and M is a fuzzy set in $X^4 \times [0, \infty]$ is satisfying the following conditions:

$(FM-1)$: $M(x, y, z, w, 0) = 0$

$(FM-2)$: $M(x, y, z, w, t) = 0, \forall t > 0,$

Only when the three simplex (x, y, z, w) degenerate

$(FM-3)$: $M(x, y, z, w, t) = M(x, w, z, y, t) = M(z, w, x, y, t) = -----$

$(FM-4)$: $M(x, y, z, w, t_1 + t_2 + t_3) \geq M(x, y, z, u, t_1) * M(x, y, u, w, t_2) * M(x, u, z, w, t_3) * M(u, y, z, w, t_4)$

$(FM-5)$: $M(x, y, z, w) : [0,1) \to [0,1]$ is left continuous, $\forall x, y, z, u, w \in X, t_1, t_2, t_3, t_4 > 0.$

Definition: 2.3. Let $(X, M, *)$ be a fuzzy 3-metric space:

(1) A sequence $\{x_n\}$ in fuzzy 3-metric space X is said to be convergent to a point $x \in X$, if

$$\lim_{n \to \infty} M(x_n, x, a, b, t) = 1, \text{ for all } a, b \in X, \text{ and } t > 0.$$

(2) A sequence $\{x_n\}$ in fuzzy 3-metric space X is called a Cauchy sequence if

$$\lim_{n \to \infty} M(x_{n+p}, x_n, a, b, t) = 1, \text{ for all } a, b \in X, \text{ and } t, p > 0.$$

(3) A fuzzy 3-metric space in which every Cauchy sequence is convergent is said to be complete.

Definition: 2.4. A function M is continuous in fuzzy 3-metric space if

$x_n \to x$, $y_n \to y$ then $\lim_{n \to \infty} M(x_n, y_n, a, b, t) = M(x, y, a, t)$, for all $a, b \in X$, and $t > 0$.

Definition: 2.5. Two mapping A and S on fuzzy 3-metric space X are weakly commuting iff

$$M(ASu, SAu, a, b, t) \geq M(Au, Su, a, t) \quad \forall a, b \in X, \text{ and } t > 0.$$

MAIN RESULT:

In this section we prove common fixed point theorems for six three mappings satisfying some conditions.

Theorem: Let $(X, \phi, *)$ be a fuzzy 3-metric space with condition $(FM-6)$ and let A, F and T be self mapping of X and the pair (A, F) and (A, T) are weakly compatible satisfying:

(1.1) $A(X) \subseteq F(X) \bigcap T(X)$

(1.2) for all $x, y \in X$, $k \in (0,1)$,

$$\phi(Ax, Ay, a, b, kt) \geq r[\min\{\phi(Fx, Ty, a, b, t), \phi(Fx, Ax, a, b, t), \phi(Fx, Ay, a, b, t),$$
$$\phi(Ty, Ay, a, b, t), \phi(Ax, Ty, a, b, t), \phi(Fx, Ax, a, b, t)\}]$$

Where $r:[0,1] \to [0,1]$ is continuous function such that $r(t) > t, \quad t > 0.$

(1.3) One of the $A(X)$, $F(X)$ or $T(X)$ is complete subspace of X.

Then A, F and T have a unique common fixed point in X.

Proof: We define a sequence $\{x_n\}$ such that $Fx_{2n+1} = Ax_{2n} = y_{2n}$ and $Tx_{2n+2} = Ax_{2n+1} = y_{2n+1}$, $n = 1,2,3,4..........$. We shall prove that $\{x_n\}$ is a Cauchy sequence for $n = 0,1,2,3,4..........$.

$$G_n = \phi(Ax_n, Ax_{n+1}, t) < 1; \quad n = 0,1,2,3,4..........$$

$$G_{2n} = \phi(Ax_{2n+1}, Ax_{2n}, t)$$
$$\geq r[\min\{\phi(Fx_{2n+1}, Tx_{2n}, a, b, t), \phi(Fx_{2n+1}, Ax_{2n+1}, a, b, t), \phi(Fx_{2n+1}, Ax_{2n}, a, b, t),$$
$$\phi(Tx_{2n}, Ax_{2n}, a, b, t), \phi(Ax_{2n+1}, Tx_{2n}, a, b, t), \phi(Fx_{2n}, Ax_{2n}, a, b, t)\}]$$
$$= r[\min\{\phi(Ax_{2n}, Ax_{2n-1}, a, b, t), \phi(Ax_{2n}, Ax_{2n+1}, a, b, t), \phi(Ax_{2n}, Ax_{2n}, a, b, t),$$
$$\phi(Ax_{2n-1}, Ax_{2n}, a, b, t), \phi(Ax_{2n+1}, Ax_{2n-1}, a, b, t), \phi(Ax_{2n-1}, Ax_{2n}, a, b, t)\}]$$
$$\geq r[\min\{\phi(Ax_{2n}, Ax_{2n-1}, a, b, t), \phi(Ax_{2n}, Ax_{2n+1}, a, b, t), \phi(Ax_{2n}, Ax_{2n}, a, b, t),$$
$$\phi(Ax_{2n-1}, Ax_{2n}, a, b, t), \phi(Ax_{2n+1}, Ax_{2n}, a, b, t), \phi(Ax_{2n}, Ax_{2n-1}, a, b, t), \phi(Ax_{2n-1}, Ax_{2n}, a, b, t)\}]$$
$$= r[\min\{G_{2n-1}, G_{2n}, 1, G_{2n-1}, G_{2n}, G_{2n-1}, G_{2n-1}\}]$$

If $G_{2n-1} > G_{2n}$ then $G_{2n} > r[G_{2n-1}] > G_{2n-1}$, is a contradiction therefore $G_{2n-1} \leq G_{2n}$.

and now $G_{2n} \geq r[G_{2n-1}] \geq G_{2n-1}$ thus $\{G_{2n} : n \geq 0\}$ is increasing sequence of positive real numbers in $[0,1]$ and therefore a limit $L \leq 1$. It is clear that $L = 1$ because if $L < 1$ then on taking $\lim_{n\to\infty}$ and we get $L \geq r(L) > 1$ is a contradiction. Hence $L = 1$.

Now for any integer m,

$$\phi(Ax_n, Ax_{n+m}, a, b, t) \geq \phi(Ax_n, Ax_{n+1}, a, b, \frac{t}{m}) * .... * \phi(Ax_{n+m-1}, Ax_{n+m}, a, b, \frac{t}{m})$$
$$\geq \phi(Ax_n, Ax_{n+1}, a, b, \frac{t}{m}) * .... * \phi(Ax_n, Ax_{n+1}, a, b, \frac{t}{m})$$

$\lim_{n\to\infty} \phi(Ax_n, Ax_{n+1}, t) \geq 1*1*......................*=1.$

because

$\lim_{n\to\infty} \phi(Ax_n, Ax_{n+1}, t) = 1,$ for $t > 0.$

Thus $\{Ax_n\}$ is a Cauchy sequence in X.

Now suppose that $T(X)$ is complete subspace of X. Note that subsequence $\{y_{2n+1}\}$ is contained in $T(X)$ and has a limit in $T(X)$. Call it z. Let $u \in T^{-1}(z)$. Then $Tu = z$. Therefore by (1.2) and using $y = u$ and $x = x_{2n}$ then

$$\phi(Ax, Ay, a, b, kt) \geq r[\min\{\phi(Fx, Ty, a, b, t), \phi(Fx, Ax, a, b, t), \phi(Fx, Ay, a, b, t),$$

$$\phi(Ty, Ay, a, b, t), \phi(Ax, Ty, a, b, t), \phi(Fx, Ax, a, b, t)\}]$$

$$\phi(Ax_{2n}, Au, a, b, kt) \geq r[\min\{\phi(Fx_{2n}, Tu, a, b, t), \phi(Fx_{2n}, Ax_{2n}, a, b, t), \phi(Fx_{2n}, Au, a, b, t),$$

$$\phi(Tu, Au, a, b, t), \phi(Ax_{2n}, Tu, a, b, t), \phi(Fx_{2n}, Ax_{2n}, a, b, t)\}]$$

Taking $\lim_{n\to\infty}$ then we have,

$\phi(z, Au, a, b, kt) \geq r[\min\{\phi(z, z, a, b, t), \phi(z, z, a, b, t), \phi(z, Au, a, b, t),$

$\phi(z, Au, a, b, t), \phi(z, z, a, b, t), \phi(z, z, a, b, t)\}] \geq r[\min\{\phi 1, 1, \phi(z, Au, a, b, t), \phi(z, Au, a, b, t) 1, 1\}]$

Thus,

$$\phi(z, Au, a, b, kt) \geq r.\phi(z, Au, a, b, t)$$

Thus $Au = z.$ Since $Tu = z$ therefore $Tu = Au = z.$

i.e. u is co-incidence point of T and $A.$

Again now suppose that $F(X)$ is complete subspace of X. Note that subsequence $\{y_{2n+1}\}$ is contained in $F(X)$ and has a limit in $F(X)$. Call it z. Let $v \in F^{-1}(z)$. Then $Fv = z$. Therefore by (1.2) and using $x = v$ and $y = x_{2n+1}$ then

$\phi(Ax, Ay, a, b, kt) \geq r[\min\{\phi(Fx, Ty, a, b, t), \phi(Fx, Ax, a, b, t), \phi(Fx, Ay, a, b, t),$

$$\phi(Ty, Ay, a, b, t), \phi(Ax, Ty, a, b, t), \phi(Fx, Ax, a, b, t)\}]$$

$\phi(Av, Ax_{2n+1}, a, b, kt) \geq r[\min\{\phi(Fv, Tx_{2n+1}, a, b, t), \phi(Fv, Av, a, b, t), \phi(Fv, Ax_{2n+1}, a, b, t),$

$$\phi(Tx_{2n+1}, Ax_{2n+1}, a, b, t), \phi(Av, Tx_{2n+1}, a, b, t), \phi(Fv, Av, a, b, t)\}]$$

Taking $\lim_{n\to\infty}$ then we have,

$$\phi(Av, z, a, b, kt) \geq r[\min\{\phi(z, z, a, b, t), \phi(z, Av, a, b, t), \phi(z, z, a, b, t),$$

$$\phi(z, z, a, b, t), \phi(Av, z, a, b, t), \phi(z, Av, a, b, t)\}]$$

$$\phi(Av, z, a, b, kt) \geq r[\min\{1, \phi(z, Av, a, b, t), 1, 1, \phi(Av, z, a, b, t), \phi(z, Av, a, b, t)\}]$$

$$\phi(Av, z, a, b, kt) \geq r.\phi(z, Av, a, b, t)$$

Thus $Av = z.$ Since $Fv = z$ therefore $Av = Fv = z.$
i.e. v is co-incidence point of F and $A.$

Now we shall prove that $Az = z.$ Now since the pair $\{A, T\}$ is weakly compatible therefore for some $u \in X$,

$ATu = TAu$ whenever $Az = Tz.$ Therefore by (1.2) and using $y = u$ and $x = x_{2n}$ then

$$\phi(Ax, Ay, a, b, kt) \geq r[\min\{\phi(Fx, Ty, a, b, t), \phi(Fx, Ax, a, b, t), \phi(Fx, Ay, a, b, t),$$

$$\phi(Ty, Ay, a, b, t), \phi(Ax, Ty, a, b, t), \phi(Fx, Ax, a, b, t)\}]$$

$$\phi(Ax_{2n}, Au, a, b, kt) \geq r[\min\{\phi(Fx_{2n}, Tu, a, b, t), \phi(Fx_{2n}, Ax_{2n}, a, b, t), \phi(Fx_{2n}, Au, a, b, t),$$

$$\phi(Tu, Au, a, b, t), \phi(Ax_{2n}, Tu, a, b, t), \phi(Fx_{2n}, Ax_{2n}, a, b, t)\}]$$

$$\phi(ATx_{2n}, Az, a, b, kt) \geq r[\min\{\phi(Fx_{2n}, Tz, a, b, t), \phi(Fx_{2n}, ATx_{2n}, a, b, t), \phi(Fx_{2n}, Az, a, b, t),$$

$$\phi(Tz, Az, a, b, t), \phi(ATx_{2n}, Tz, a, b, t), \phi(Fx_{2n}, ATx_{2n}, a, b, t)\}]$$

$$\phi(ATz, Az, a, b, kt) \geq r[\min\{\phi(Fz, Tz, a, b, t), \phi(Fz, ATz, a, b, t), \phi(Fz, Az, a, b, t),$$

$$\phi(z, Tz, a, b, kt) \geq r[\min\{\phi(z, Tz, a, b, t), \phi(z, z, a, b, t), \phi(z, Tz, a, b, t),$$

$$\phi(Tz, Tz, a, b, t), \phi(z, Tz, a, b, t), \phi(z, z, a, b, t)\}]$$

$$\phi(z, Tz, a, b, kt) \geq r[\min\{\phi(z, Tz, a, b, t), 1, \phi(z, Tz, a, b, t), 1, \phi(z, Tz, a, b, t), 1\}]$$

$$\phi(z, Tz, a, b, kt) \geq r.\phi(z, Tz, a, b, t)$$

This is contradiction. So $\phi(Tz, z, a, b, kt) = 1.$

Hence z is fixed point of $T.$
i.e. z is common fixed point of T and $A.$

Again since the pair $\{A,F\}$ is weakly compatible therefore for some $v \in X$, $AFv = FAv$ whenever $Az = Fz$. Therefore by (1.2) and using $x = v$ and $y = x_{2n+1}$ then

$$\phi(Ax, Ay, a, b, kt) \geq r[\min\{\phi(Fx, Ty, a, b, t), \phi(Fx, Ax, a, b, t), \phi(Fx, Ay, a, b, t),$$
$$\phi(Ty, Ay, a, b, t), \phi(Ax, Ty, a, b, t), \phi(Fx, Ax, a, b, t)\}]$$

$$\phi(Av, Ax_{2n+1}, a, b, kt) \geq r[\min\{\phi(Fv, Tx_{2n+1}, a, b, t), \phi(Fv, Av, a, b, t), \phi(Fv, Ax_{2n+1}, a, b, t),$$
$$\phi(Tx_{2n+1}, Ax_{2n+1}, a, b, t), \phi(Av, Tx_{2n+1}, a, b, t), \phi(Fv, Av, a, b, t)\}]$$

$$\phi(Az, AFx_{2n+1}, a, b, kt) \geq r[\min\{\phi(Fz, Tx_{2n+1}, a, b, t), \phi(Fz, Az, a, b, t), \phi(Fz, AFx_{2n+1}, a, b, t),$$
$$\phi(Tx_{2n+1}, AFx_{2n+1}, a, b, t), \phi(Az, Tx_{2n+1}, a, b, t), \phi(Fz, Az, a, b, t)\}]$$

$$\phi(Az, AFz, a, b, kt) \geq r[\min\{\phi(Fz, Tz, a, b, t), \phi(Fz, Az, a, b, t), \phi(Fz, AFz, a, b, t),$$
$$\phi(Tz, AFz, a, b, t), \phi(Az, Tz, a, b, t), \phi(Fz, Az, a, b, t)\}]$$

$$\phi(Fz, z, a, b, kt) \geq r[\min\{\phi(Fz, z, a, b, t), \phi(Fz, Fz, a, b, t), \phi(Fz, z, a, b, t),$$
$$\phi(z, z, a, b, t), \phi(Fz, z, a, b, t), \phi(Fz, Fz, a, b, t)\}]$$

$$\phi(Fz, z, a, b, kt) \geq r[\min\{\phi(Fz, z, a, b, t), 1, \phi(Fz, z, a, b, t), 1, \phi(Fz, z, a, b, t), 1\}]$$

$$\phi(Fz, z, a, b, kt) \geq r.\phi(Fz, z, a, b, t)$$

This is contradiction. So $\phi(Fz, z, a, b, kt) = 1$.

Hence z is fixed point of F.
i.e. z is common fixed point of F and A.

UNIQUENESS:
Suppose that there is an another point $v \neq u$ then

$$\phi(Ax, Ay, a, b, kt) \geq r[\min\{\phi(Fx, Ty, a, b, t), \phi(Fx, Ax, a, b, t), \phi(Fx, Ay, a, b, t),$$
$$\phi(Ty, Ay, a, b, t), \phi(Ax, Ty, a, b, t), \phi(Fx, Ax, a, b, t)\}]$$

$$\phi(Au, Av, a, b, kt) \geq r[\min\{\phi(Fu, Tv, a, b, t), \phi(Fu, Au, a, b, t), \phi(Fu, Av, a, b, t),$$
$$\phi(Tv, Av, a, b, t), \phi(Au, Tv, a, b, t), \phi(Fu, Au, a, b, t)\}]$$

$$\phi(u, v, a, b, kt) \geq r[\min\{\phi(u, v, a, b, t), \phi(u, u, a, b, t), \phi(u, v, a, b, t),$$
$$\phi(v, v, a, b, t), \phi(u, v, a, b, t), \phi(u, u, a, b, t)\}]$$

$\phi(u,v,a,b,kt) \geq r[\min\{\phi(u,v,a,b,t),1,\phi(u,v,a,b,t),1,\phi(u,v,a,b,t),1\}]$

$$\phi(u,v,a,b,kt) \geq r.\phi(u,v,a,b,t)$$

This is contradiction. So $v = u.$

Hence A, F and T have unique common fixed point.

Corollary: Let $(X,\phi,*)$ be a fuzzy 3-metric space with condition $(FM-6)$ and let F and T be continuous mapping of X in X. Let A be a self mapping of X satisfying the pair (A,F) and (A,T) are R- weakly commuting and

(1.1) $A(X) \subseteq F(X) \bigcap T(X)$

(1.2) for all $x, y \in X$,

$$M(Ax,Ay,a,b,t) \geq r[\min\{M(Fx,Ty,a,b,t), M(Fx,Ax,a,b,t), M(Fx,Ay,a,b,t),$$
$$M(Ty,Ay,a,b,t), M(Ax,Ty,a,b,t), M(Fx,Ax,a,b,t)\}]$$

Where $r:[0,1] \to [0,1]$ is continuous function such that $r(t) > t,\ \ 0 \leq t \leq 1$ and $r(t) = 1$ for $t = 1$ and $a,b \in X$. Then the sequence $\{x_n\}$ and $\{y_n\}$ in X such that

$x_n \to x,\ \ y_n \to y \Rightarrow M(x_n, y_n, a,b,t) \to M(x,y,a,b,t)$ Where $t > 0.$

Then A, F and T have a unique common fixed point in X.

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