International Journal of Mathematical Archive-2(3), Mar. - 2011, Page: 1-15
Available online through www.ijma.info ISSN 2229 – 5046

# EFFECT OF THERMAL DISPERSION AND VISCOUS DISSIPATION ON MHD RADIATIVE NON-DARCY MIXED CONVECTION IN FLUID SATURATED POROUS MEDIUM WITH RESPECT TO THE PERFORMANCE OF A SECOND ORDER FLUID PAST AN INFINITE PLATE WITH VARIABLE SUCTION UNDER THE INFLUENCE OF MAGNETIC FIELD

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*(Received on: 26-01-11; Accepted on: 20-02-11)*

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*ABSTRACT*

*In this paper the effect of thermal dispersion and viscous dissipation on MHD radiative non-Darcy mixed convection in a fluid saturated porous medium (part a) and the class solutions of a second order fluid past a porous boundary and on the performance of fluid flow past an infinite plate with variable suction under the influence of magnetic field have been examined (part b). The salient features in both the cases have been compared to the best possible extent. Interestingly, some of the parameters in both the cases cause similar effect. It is found that, ( $f$ ) in thermal study is analogues to the velocity profiles in case of sinusoidal oscillations of the bounding surface while the dispersion parameter in the thermal system is equivalent to the frequency of excitation ( $\sigma$ ). Further, the effect of porosity in case of visco elasticity parameter has similar effect as that of dispersion and radiation parameters. Also, it is observed that, the porosity of the bounding surface and magnetic effects are similar to that of radiation and dispersion parameters in case of thermal effects. However, it is noticed that, the investigations stated above are applicable in the boundary layer region.*

**Key words:** *Thermal dispersion, Viscous dissipation, Porous medium, Elastico viscous fluid, Sinusoidal disturbances, Variable suction*

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**NOMENCLATURE IN PART-A:**

$A$ : Constant
$a$ : Absorption coefficient
$b$ : Constant
$B$ : Constant
$B_0$ : Magnetic field strength
$C$ : Empirical constant
$c_p$ : Specific heat at constant pressure
$D$ : Dispersion parameter
$d$ : Mean particle diameter
$Ec$ : Eckert number
$F$ : Inertia parameter
$g$ : Gravitational acceleration
$h$ : Heat transfer coefficient
$K$ : Permeability
$k$ : Thermal conductivity of medium
$M^2$ : Square of the Hartmann number
$Pe$ : Peclet number
$\Pr$ : Prandtl number
$q$ : Heat flux
$R$ : Radiation parameter
$Ra$ : Rayleigh number
$\mathrm{Re}$ : Reynolds number
$T$ : Temperature
$u, v$ : Velocity components in the direction of x and y directions
$U_\infty$ : Free stream velocity

**GREEK SYMBOLS:**

$\alpha$ : Effective thermal diffusivity of the porous medium

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$\beta$ : Coefficient of thermal expansion
$\gamma$ : Thermal dispersion coefficient
$\varepsilon$ : Mixed convection parameter
$\eta$ : Dimensionless similarity variable
$\mu$ : Viscosity
$\upsilon$ : Kinematic viscosity
$\rho$ : Density
$\theta$ : Dimensionless temperature
$\sigma$ : Fluid electrical conductivity
$\sigma_R$ : Stefan – Boltzmann constant
$\psi$ : Stream function

**SUBSCRIPTS:**
$d$ : dispersion
$e$ : effective
$m$ : molecular
$x$ : local
$w$ : wall
$\infty$ : free stream conditions

**NOMENCLATURE IN PART-B:**
$A_i$ : Acceleration component in i th direction
$a_i$ : Non dimensional acceleration in i th direction
$E_{ij}^{(1)}, E_{ij}^{(2)}$ : Strain tensor in the dimensional form
$e_{ij}^{(1)}, e_{ij}^{(2)}$ : Strain tensor in the non-dimensional form
$F_X, F_Y, F_Z$ : External forces applied along $X, Y$ and $Z$ directions
$g(s)$ : Given history
$g_\alpha(s)$ : Retarded History
$K$ : Permeability of the porous bed
$k$ : Non dimensionalised porosity factory
$L$ : Characteristic length
$M$ : Magnetic intensity (dimensional form)
$m$ : Magnetic intensity (non-dimensional form)
$P$ : Indeterminate hydrostatic pressure
$p$ : Non dimensional indeterminate pressure
$r$ : Polar coordinate
$S_{ij}$ : Stress tensor
$T$ : Dimensional time parameter
$t$ : Non-dimensional time parameter
$U, V$ : Dimensional velocity component
$U_i$ : Dimensional velocity component in i th direction
$u, v$ : Non-dimensional velocity component
$u'$ : Non- dimensional free stream velocity
$u_0'$ : Non dimensional form of Magnification factor for free stream velocity
$v_0'$ : Non dimensional form of non zero mean suction velocity
$X, Y$ : Co-ordinate axes (dimensional form)
$x, y$ : Co-ordinate axes (non-dimensional form)

**GREEK SYMBOLS:**
$\alpha$ : Retardation factor
$\beta$ : Visco elasticity parameter
$\delta$ : Polar coordinate
$\varepsilon$ : Suction parameter
$\phi_1$ : Coefficient of viscosity
$\phi_2$ : Coefficient of elastico viscosity
$\phi_3$ : Coefficient of cross viscosity
$\vartheta$ : Non dimensionalised cross viscosity factor
$\upsilon_c$ : Non dimensionalised cross viscosity parameter
$\mu$ : Coefficient of viscosity
$\rho$ : Density of fluid
$\sigma$ : Non-dimensional frequency of excitation

## INTRODUCTION:

Heat transfer in porous medium is gaining utmost importance due its applicability in geothermal energy extraction, nuclear waste disposal, fossil fuels detection, regenerator bed etc... In plasma physics, liquid metal flow, magneto hydrodynamic accelerators and power generation systems, there is acute necessity of studying MHD combined dispersion and dissipation effects in porous medium. Understanding the development of hydrodynamic and thermal boundary layers along with the heat transfer characteristics is the basic requirement to further investigate the problem.

In 1997 Cheng & Minkowycz [1] have analyzed the steady free convection about a vertical plate embedded in porous medium applied to heat transfer from dike. Murthy and Singh [2] using method of similarity solution studied the influence of lateral mass flux and thermal dispersion on non-Darcy natural convection over a vertical plate in porous medium. They have discussed the combined effect of thermal dispersion and fluid injection on heat transfer. Hassanien *et al.* [3] have studied the effects of thermal dispersion and stratification on non-Darcy mixed convection from a vertical plate in porous medium and investigated the flow and temperature fields. Murthy [4] has studied the dispersion and dissipation effects on non-Darcy mixed convection problems and established the trend of heat transfer rate. Kuznetsov [5] investigated the effect of transverse thermal dispersion on forced convection in porous media and identified the situations favorable to heat transfer under dispersion effects. Mohammadien and El-Amin [6] studied the dispersion and radiation effects in fluid saturated porous medium on heat transfer rate for both Darcy and non-Darcy medium. Chamkha and Quadri [7] examined the heat and mass transfer characteristics under mixed convective conditions with thermal dispersion without taking MHD into consideration. Cheng and Lin [8] in their observation pointed out that rate of unsteady heat transfer can be accelerated by thermal dispersion. Wang *et al.* [9] applied an explicit analytical technique namely homotopy analysis to solve the non-Darcy natural convection over a horizontal plate with surface mass flux and thermal dispersion and obtained a totally analytic and uniformly valid solution. El-Amin [10] obtained the velocity, temperature and

concentration profiles with double dispersion. Chamkha *et al.* [11] highlighted the physical aspects of simultaneous heat and mass transfer with thermal dispersion effect. Ibrahim *et al.* [12] studied the radiative and thermal dispersion effects for non-Newtonian fluid from an impermeable vertical plate embedded in a fluid saturated porous medium. However, in the geohydrology, soil physics, magma detection and Magneto hydrodynamic power generation, the magnetic field effects also play a vital role and only few authors have studied the combined effects of MHD and dispersion in porous medium with variable wall temperature. Sobha and Ramakrishna [13] presented the effects of Hartmann number and porosity factor on temperature field, Nusselt number for a vertical plate in porous medium with applied magnetic field without considering the dispersion and inertial effects. The non-Darcy effect will have considerable impact in the transition flow regime between Re 1 to 10.When the internal effect is prevalent the thermal dispersion in porous media is expected to become important [14]. Visco elasticity is the property of the fluid while, the thermal dispersion can alter the nature of the property and the magnetic effect can influence the nature of flow entities. Similar effects which are found by taking into account the thermal dispersion and viscous dissipation under the influence of magnetic field can also be obtained in a suitably equivalent mathematical model (of course) but under different context. A similar such situation has been examined in this paper. Sobha, Ramakrishna *et al.* [15] studied the effect of thermal dispersion and viscous dissipation on MHD radiative non-Darcy mixed convection in a fluid saturated porous medium. During the course of analysis it is found that, the transverse field applied to the medium suppresses the effect of dispersion while decreasing fluid motion due to applied magnetic field and can be compensated by increasing the dispersion parameter. Further, the increase in inertia causes a reduction in the velocity under unaided flow conditions and also the effect of radiation is found to be dominant in the absence of dispersion.
Ramana Murthy and Kulkarni [16] studied on the class of exact solutions of an incompressible second order fluid flow by creating the sinusoidal disturbances. In course of analysis it is found that, as the porosity effects the velocity profiles and the frequency of excitation shows the decreasing trend on the velocity, also it is noted that, the elastico viscosity parameter has profound effect on the magnification factor.

The characteristic performance of elastico viscous fluid past an infinite plate with variable suction under the influence of magnetic field has been investigated by Ramana Murthy and Kulkarni [17]. It is found that, the decrease in the porosity of bounding surface reduces the amplitude for a constant elastico viscosity parameter and the frequency of excitation of the bounding surface when the magnetic field applied on the system is maintained constant. Further, it is also observed that, as the visco elasticity of the fluid reduces the amplification factor also reduces.
Noll [18] defined a simple material as a substance for which stress can be determined with the entire knowledge of the history of the strain. This is called a simple fluid, having the property that at all local states, with the same mass density, intrinsically equal in response, with all observable differences in response being due to definite differences in the history. For any given history g(s), a retarded history $g_\alpha(s)$ can be defined as:

$$g_\alpha(s) = g(\alpha s); \qquad 0 < s < \infty, \qquad 0 < \alpha \le 1 \tag{1}$$

$\alpha$ being termed as a retardation factor. Assuming that the stress is more sensitive to recent deformation that to the deformations at distant past, Coleman and Noll [19] proved that, the theory of simple fluids yield the theory of perfect fluids as $\alpha \to 0$ and that of Newtonian Fluids as a correction (up to the order of $\alpha$) to the theory of the perfect fluids. Neglecting all the terms of the order of higher than two in $\alpha$, we have incompressible elastic viscous fluid of second order type whose constitutive relation is governed by:

$$S_{ij} = -P\delta_{ij} + \phi_1 E_{ij}^{(1)} + \phi_2 E_{ij}^{(2)} + \phi_3 E_{ij}^{(1)^2} \tag{2}$$

where

$$E_{ij}^{(1)} = U_{i,j} + U_{j,i} \tag{3}$$

and

$$E_{ij}^{(2)} = A_{i,j} + A_{j,i} + 2U_{m,i}U_{m,j} \tag{4}$$

In the above equations, $S_{ij}$ is the stress-tensor, $U_i$, $A_i$ are the components of velocity and acceleration in the direction of the i-th coordinate $X_i$ , P is indeterminate hydrostatic pressure and the coefficients $\phi_1, \phi_2$ and $\phi_3$ are material constants.
The constitutive relation for general Rivlin-Ericksen [20] fluids also reduces to equation (2) when the squares and higher orders of $E^2$ are neglected, the coefficients being constants. Also the non-Newtonian models considered by Reiner [21] could be obtained from equation (2) when $\phi_2 = 0$, naming $\phi_3$ as the coefficient of cross viscosity. With reference to the Rivlin – Ericksen fluids $\phi_2$ may be called as the coefficient of viscosity. It has been reported that, a solution of poly-iso-butylene in cetane behaves as a second order fluid and Markovitz [22] determined the constants $\phi_1, \phi_2$ and $\phi_3$.
Viscous fluid flow over wavy wall had attracted the attention of relatively few researchers although the analysis of such flows finds application in different areas such as transpiration cooling of re-entry vehicles and rocket boosters, cross hatching ablative surfaces and film vaporization in combustion chambers. Especially the stream, where the heat and mass transfer takes place in the
 

chemical processing industry. The problem by considering the permeability of the bounding surface in the reactors assumes greater significance.

In view of several industrial and technological importances, Ramacharyulu [23] studied the problem of the exact solutions of two dimensional flows of a second order incompressible fluid by considering the rigid boundaries. Later, Lekoudis *et al.* [24] presented a linear analysis of the compressible boundary layer flow over a wall. Subsequently, Shankar and Sinha [25] studied the problem of Rayleigh for wavy wall. The effect of small amplitude wall waviness upon the stability of the laminar boundary layer had been studied by Lessen and Gangwani [26]. Further, the problem of free convective heat transfer in a viscous incompressible fluid confined between vertical wavy wall and a vertical flat wall was examined by Vajravelu and Shastri [27] and thereafter by Das and Ahmed [28]. The free convective flow of a viscous incompressible fluid in porous medium between two long vertical wavy walls was investigated by Patidar and Purohit [29]. Rajeev Taneja and Jain [30] had examined the problem of MHD flow with slip effects and temperature dependent heat in a viscous incompressible fluid confined between a long vertical wall and a parallel flat plate.

**MATHEMATICAL FORMULATION AND SOLUTION:**

**PART-A:**
Consider a semi infinite vertical flat plate subjected to convective environment as shown in Figure 1. The fluid surrounding the plate is considered as gray, emitting and absorbing subjected to transverse applied magnetic field. The variation of the plate wall temperature is considered as proportional to $x^{\lambda}$. The velocity and radiation effect in the x-direction is considered negligible. Further, it is assumed that, the convective fluids in the surrounding porous medium are isotropic in nature and have constant physical properties. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected. In addition there is no applied electric field and hence the Hall effect and Joule heating are neglected.

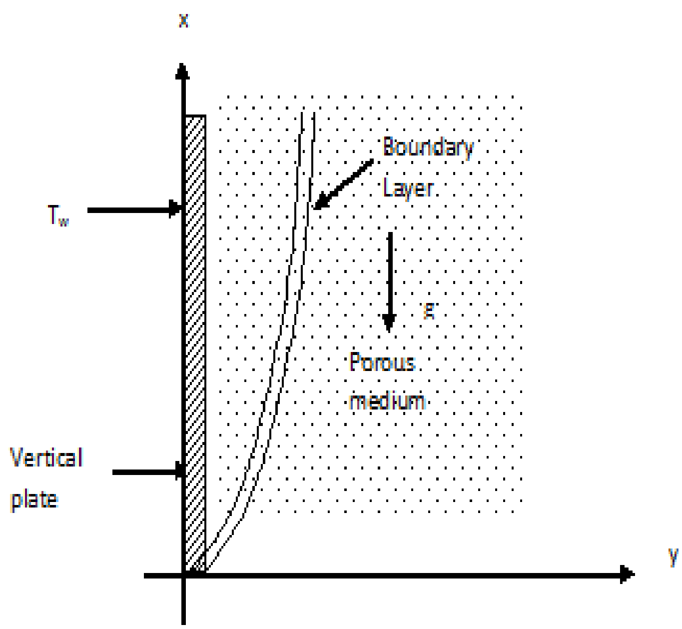


**Figure 1: Physical model of the problem**

Under boundary layer approximations the continuity, momentum and energy equations are written as follows.
The continuity equation is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{5}$$

The momentum equation is

$$\frac{\mu}{K}\left(\frac{\partial u}{\partial y}\right)+\sigma B_0^2\left(\frac{\partial u}{\partial y}\right)+\frac{2C\sqrt{K}}{\upsilon}\frac{\mu}{K}u\left(\frac{\partial u}{\partial y}\right)=\rho_\infty g\beta\left(\frac{\partial T}{\partial y}\right) \tag{6}$$

The energy equation is

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha_e\frac{\partial T}{\partial y}\right)+\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho c_p}\frac{\partial q}{\partial y} \tag{7}$$

where $$\frac{\partial q}{\partial y}=-16a\sigma_R T_\infty^{\ 3}\left(T_\infty-T\right).$$

The momentum equation includes both the inertial forces and magnetic influence. In situations of fluid flow and heat transfer in porous media, the effective thermal diffusivity is modeled as $\alpha_e=\alpha_m+\alpha_d$, where $\alpha_m$ and $\alpha_d$ are the molecular thermal diffusivity and thermal diffusivity of the porous medium due to thermal dispersion respectively. Following the linear model proposed by Plumb [31], the dispersion diffusivity is considered to be proportional to the stream-wise velocity component i.e. $\alpha_d=\gamma ud$, where $\gamma$ is the dispersion coefficient, which is a function of the structure of the porous medium.

The boundary conditions for the problem are

$$\left.\begin{aligned}&at\ y=0, T=T_w=T_\infty+Ax^\lambda\ and\ v=ax^{\frac{\lambda-1}{2}}\\ &as\ y\to\infty, T=T_\infty, u=U_\infty=Bx^\lambda\end{aligned}\right\} \tag{8}$$

Introducing the stream functions $u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}$ the above equations (6) and (7) can be reformulated as

$$\frac{\mu}{K}\frac{\partial^2\psi}{\partial y^2}+\sigma B_0^2\frac{\partial^2\psi}{\partial y^2}+\frac{2C\sqrt{K}}{\upsilon}\frac{\mu}{K}\left(\frac{\partial\psi}{\partial y}\right)\left(\frac{\partial^2\psi}{\partial y^2}\right)=\rho_\infty g\beta\frac{\partial T}{\partial y} \tag{9}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha_e\frac{\partial T}{\partial y}\right)+\frac{\upsilon}{c_p}\left(\frac{\partial^2\psi}{\partial y^2}\right)^2+\frac{16a\sigma_R T_\infty^3}{\rho c_p}\left(T_\infty-T\right) \tag{10}$$

Introducing the similarity variables as

$$\psi=f(\eta)\left(\alpha_m U_\infty x\right)^{1/2},\quad \eta=\left(\frac{U_\infty x}{\alpha_m}\right)^{1/2}\left(\frac{y}{x}\right),\quad U_\infty=Bx^\lambda, \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, f_w=\frac{-2b}{\left(\alpha_m B\right)^{1/2}}$$

Equations (9) and (10) now reduce to

$$\left(1+M+Ff'\right)f''=\varepsilon\theta' \tag{11}$$

$$\left(1+Df'+\frac{4}{3}R\right)\theta''+\left(Df''+\frac{\lambda+1}{2}f\right)\theta'-\lambda f'\theta+\Pr Ecf''^2=0 \tag{12}$$

where $$D=\frac{\gamma u_\infty d}{\alpha_m}$$

The transformed boundary conditions are now redefined as

$$\left.\begin{aligned}&at\ \eta=0,\ \theta=1\ and\ \ f=\frac{f_w}{\lambda+1}\\ &as\ \eta\to\infty,\ \theta=0\ and\ \ f'(\infty)=1\end{aligned}\right\} \tag{13}$$

 

The system of equations (11) and (12) are split into system of first order ordinary differential equations. Using the boundary conditions as in equation (13) the above set of equations are solved by using Runge-Kutta method by applying shooting technique. The solution thus obtained is matched with the given values at $f^{'}(\infty)$ and $\theta(\infty)$.

**PART-B:**

In this part the aim of the problem is to investigate a class of exact solutions for the flow of incompressible second order fluid by taking into account the porosity factor of the bounding surface when it is subjected to sinusoidal disturbances and then to compare the results with those of in Newtonian case and also to find the analogy of the thermal effects as examined earlier. The effects of the disturbance due to sinusoidal oscillation of the bottom of a semi infinite depth are examined. The results are expressed in terms of a non-dimensional porosity parameter $K$, which depends on the non-Newtonian coefficient $\phi_2$ and the frequency of excitation $\sigma$. It is noticed that, the flow properties are identical with those of in the Newtonian case $(K=\infty)$.

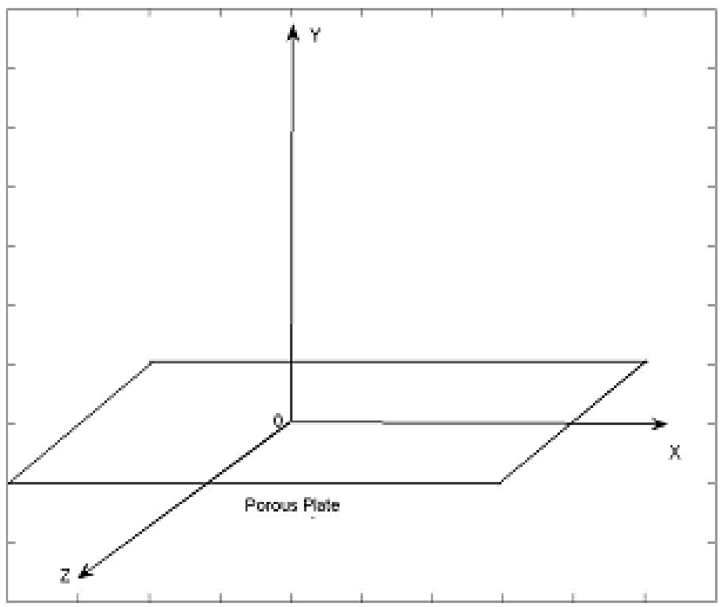


**Figure 2: Geometry of the fluid over porous bed**

In general, the equations (in the dimensional form) of motions in the X, Y and Z directions are

$$\rho\frac{DU_1}{DT}=\rho F_X+\frac{\partial S_{XX}}{\partial X}+\frac{\partial S_{XY}}{\partial Y}+\frac{\partial S_{XZ}}{\partial Z}-\frac{\mu}{k}U_1 \tag{14}$$

$$\rho\frac{DU_2}{DT}=\rho F_Y+\frac{\partial S_{YX}}{\partial X}+\frac{\partial S_{YY}}{\partial Y}+\frac{\partial S_{YZ}}{\partial Z}-\frac{\mu}{k}U_2 \tag{15}$$

$$\rho\frac{DU_3}{DT}=\rho F_Z+\frac{\partial S_{ZX}}{\partial X}+\frac{\partial S_{ZY}}{\partial Y}+\frac{\partial S_{ZZ}}{\partial Z}-\frac{\mu}{k}U_3 \tag{16}$$

Introducing the following non-dimensional variables as:

$$U_i=\frac{\phi_1 u_i}{\rho L}\qquad T=\frac{\rho L^2 t}{\phi_1}\qquad \phi_2=\rho L^2\beta\qquad P=\frac{\phi_1^2 p}{\rho L^2}$$

$$\frac{X_i}{L} = x_i \quad \frac{Y_i}{L} = y_i \quad \phi_3 = \rho L^2 \upsilon_c \quad A_i = \frac{\phi_1^2 a_i}{\rho^2 L^3}$$

$$S_{ij} = \frac{\phi_1^2 s_{ij}}{\rho L^2} \quad E_{ij}^{(1)} = \frac{\phi_1 e_{ij}^{(1)}}{\rho L^2} \quad E_{ij}^{(2)} = \frac{\phi_1^2 e_{ij}^{(2)}}{\rho^2 L^4} \quad k = \frac{\rho L^3}{\phi_1^2 K}$$

where $T$ the (dimensional) time is variable, $\rho$ is the mass density and $L$ is a characteristic length. We consider a class of plane flows given by the velocity components

$$u_1 = u(y,t) \text{ and } u_2 = 0 \text{ while } u_3 = 0 \tag{17}$$

The flow characterized by the velocity in the non-dimensional form is given by

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \beta \frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{K}u \tag{18}$$

where $K$ is the non-dimensional porosity constant. It may be noted that, the presence of $\beta$ changes the order of differential from two to three. The oscillations of a classical viscous liquid on the upper half of the plane $y \geq 0$ with the bottom oscillating with a velocity $\alpha e^{i\sigma t}$ then

$$u(0,t) = \alpha e^{i\sigma t} \qquad \text{and} \qquad u(\infty,t) = 0 \tag{19}$$

Assuming the trial solution as

$$u(y,t) = \alpha e^{i\sigma t} f(y) \tag{20}$$

$$f''(y) = p^2 f(y) \tag{21}$$

where

$$p^2 = \frac{\frac{1}{K} + i\sigma}{1 + i\beta\sigma} = \frac{\left(\beta\sigma^2 + \frac{1}{K}\right) + i\left(\sigma - \frac{\beta\sigma}{K}\right)}{\left(1 + \beta^2\sigma^2\right)} \tag{22}$$

When expressed in the polar form

$$p = r\left(\cos\left(\frac{\pi}{4} - \frac{\delta}{2}\right) + i\sin\left(\frac{\pi}{4} - \frac{\delta}{2}\right)\right) \tag{23}$$

where

$$r = \frac{\left[\left(\beta\sigma^2 + \frac{1}{K}\right)^2 + \left(\sigma - \frac{\beta\sigma}{K}\right)^2\right]^{\frac{1}{4}}}{\sqrt{\left(1 + \beta^2\sigma^2\right)}}, \quad \delta = \tan^{-1}(Q) \text{ and } Q = \frac{\left(\beta\sigma^2 + \frac{1}{K}\right)}{\left(\sigma - \frac{\beta\sigma}{K}\right)}$$

Also the conditions satisfied are:

$$f(0) = 1, \qquad f(\infty) = 0 \tag{24}$$

This yields the solution

$$f\left(y\right)=e^{-yr\left(\cos\left(\frac{\pi}{4}-\frac{\delta}{2}\right)+i\sin\left(\frac{\pi}{4}-\frac{\delta}{2}\right)\right)} \tag{25}$$

and hence

$$u\left(y,t\right)=\alpha e^{\left(i\sigma t-yr\left(\cos\left(\frac{\pi}{4}-\frac{\delta}{2}\right)+i\sin\left(\frac{\pi}{4}-\frac{\delta}{2}\right)\right)\right)} \tag{26}$$

The flow is thus represented by standing transverse wave with its amplitude rapidly diminishing with increasing distance from the plane. This phenomenon is independent of $\upsilon_c$ as is noticed for all two-dimensional flows.

The magnification factor $A^*$ of the amplitude this wave, with respect to the amplitude of the disturbance $(\alpha)$, may be expressed as

$$A^*=\left(real\ part\ of\ u\left(y,t\right)\right)^2+\left(\mathrm{Im}aginary\ part\ of\ u\left(y,t\right)\right)^2$$

$$A^*=\alpha e^{-y\sqrt{r}\cos\left(\frac{\pi}{4}-\frac{\delta}{2}\right)} \tag{27}$$

which is in the form of $A^*=e^{-\chi y^*}$ where

$$\chi y^*=\frac{y\sqrt{r}}{\sqrt{2}}\left[\cos\left(\frac{\delta}{2}\right)+i\sin\left(\frac{\delta}{2}\right)\right] \tag{28}$$

where
$$\chi=\frac{1}{\left(1+\beta^2\sigma^2\right)^{\frac{1}{4}}}\sqrt{\frac{Q+\sqrt{\left(1+Q^2\right)}}{1+Q^2}} \tag{29}$$

and
$$y^*=\frac{y\left(1+\beta^2\sigma^2\right)^{\frac{1}{4}}}{\sqrt{2}}\frac{\left[\left(\beta\sigma^2+\frac{1}{K}\right)^2+\left(\sigma-\frac{\beta\sigma}{K}\right)^2\right]^{\frac{1}{4}}}{\sqrt{\left(1+\beta^2\sigma^2\right)}} \tag{30}$$

Further, a situation is considered where the fluid is bounded by a porous boundary and a uniform magnetic field is introduced which is normal to the bounding surface with variable suction. As considered earlier, the bounding surface is subjected to sinusoidal oscillations.

The governing equation of motion in non-dimensional form is given by

$$\frac{\partial u}{\partial t}-v_0'\left(1+\varepsilon Ae^{i\omega t}\right)\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+\beta\left(\frac{\partial^3 u}{\partial y^2\partial t}-v_0'\left(1+\varepsilon Ae^{i\omega t}\right)\frac{\partial^3 u}{\partial y^3}\right)-\left(\frac{1}{k}+m\right)u \tag{31}$$

where $\omega$ is the frequency of the fluctuating stream as well $\sigma$ the frequency of excitation of the bounding surface together with the conditions

$$u\left(0,t\right)=\alpha e^{i\sigma t} \quad \text{and} \quad u\left(\infty,t\right)=0 \tag{32}$$

The solution of the governing equation of motions under the boundary conditions stated above is given by

$$u\left(y,t\right)=\alpha e^{i\sigma t}\left(1+v_0'\left[1+y\left(1-p^2\right)Q\right]\right)e^{-py} \tag{33}$$

where
$$p^2=\frac{\left(i\sigma+\frac{1}{k}+m\right)}{\left(1+i\beta\sigma\right)}=\frac{\left(\beta\sigma^2+\frac{1}{k}+m\right)+i\left(\sigma-\beta\sigma\left(\frac{1}{k}+m\right)\right)}{\left(1+i\beta^2\sigma^2\right)} \tag{34}$$

when expressed in polar form

$$p = r\left(\cos\left(\frac{\pi}{4}-\frac{\delta}{2}\right)+i\sin\left(\frac{\pi}{4}-\frac{\delta}{2}\right)\right) \tag{35}$$

where $r = \dfrac{\left[\left(\beta\sigma^2+\frac{1}{k}+m\right)^2+\left(\sigma-\beta\sigma\left(\frac{1}{k}+m\right)\right)^2\right]^{\frac{1}{4}}}{\sqrt{\left(1+\beta^2\sigma^2\right)}}$, $\delta = \tan^{-1}\left(\phi\right)$ and $\phi = \dfrac{\left(\beta\sigma^2+\frac{1}{k}+m\right)}{\left(\sigma-\beta\sigma\left(\frac{1}{k}+m\right)\right)}$

The magnification factor $A^*$ of the amplitude of this wave, with respect to the amplitude of the disturbance $(\alpha)$, may be written as

$$A^* = \left(\mathrm{Re}\, al\ part\ of\ u\left(y,t\right)\right)^2+\left(\mathrm{Im}\, aginary\ part\ of\ u\left(y,t\right)\right)^2$$

$$A^* = \alpha e^{-y\sqrt{r}\cos\left(\frac{\pi}{4}-\frac{\delta}{2}\right)} \tag{36}$$

which is in the form of $A^* = e^{-\chi y^*}$ where

$$\chi y^* = \frac{y\sqrt{r}}{\sqrt{2}}\left[\cos\left(\frac{\delta}{2}\right)+i\sin\left(\frac{\delta}{2}\right)\right] \tag{37}$$

$$\chi = \frac{1}{\left(1+\beta^2\sigma^2\right)^{\frac{1}{4}}}\sqrt{\frac{\phi+\sqrt{\left(1+\phi^2\right)}}{1+\phi^2}} \tag{38}$$

and

$$y^* = \frac{y\left(1+\beta^2\sigma^2\right)^{\frac{1}{4}}}{\sqrt{2}}\frac{\left[\left(\beta\sigma^2+\frac{1}{k}+m\right)^2+\left(\sigma-\beta\sigma\left(\frac{1}{k}+m\right)\right)^2\right]^{\frac{1}{4}}}{\sqrt{\left(1+\beta^2\sigma^2\right)}} \tag{39}$$

**RESULTS AND DISCUSSIONS:**

**1.** Figure 3 shows the combined effect of dispersion and magnetic field on velocity profile for both aided flow ($\varepsilon = 1$) and opposing flow ($\varepsilon = -1$) conditions. It is observed that, the velocity at a given location increases due to hydrodynamic mixing of fluid at pore level. Since the increase in dispersion decreases the slope, it indicates an increase in the boundary layer thickness. However, the increase in magnetic field under the same dispersion causes a decrease in velocity, since the magnetic field produces a resistive force that decelerates the motion of the particles. It is further clear from the figure that, the rate of increase in velocity due to dispersion is less in the presence of magnetic field when compared to without magnetic field. It is also clear that, when transverse magnetic field is acting on the system, the reduction in fluid motion is compensated by increase in dispersion parameter. The effect is found opposite in aided flow with that of opposed flow.

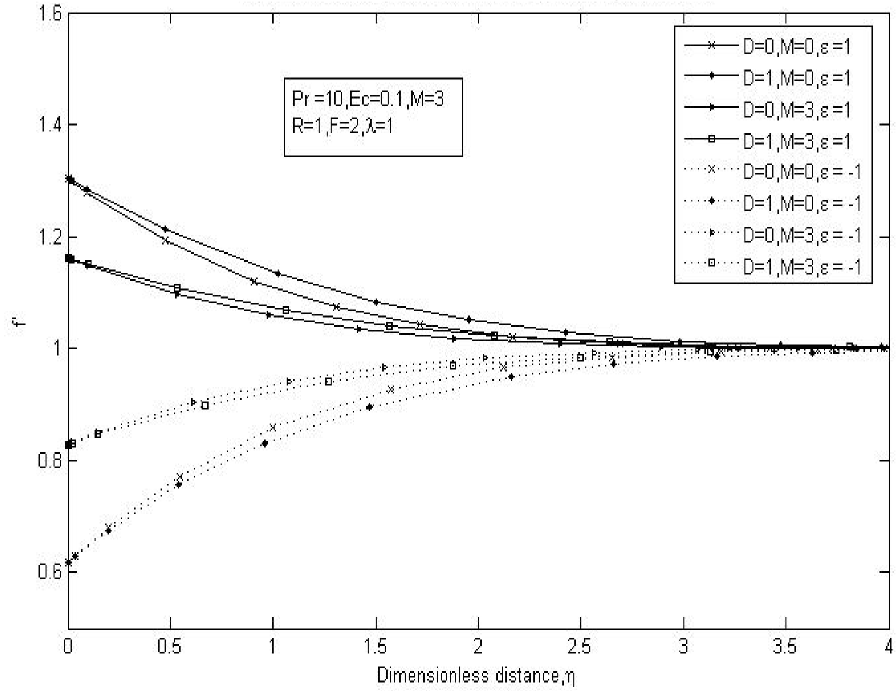


**Figure 3: Effect of dispersion and magnetic field on velocity profile.**

The nature of the velocity profiles for different values of frequency of excitation ( $\sigma$ ) is illustrated in Figure 4. It is observed that, as the frequency of the excitation is increased, there is a decreasing trend in the velocity at the boundary region. Further, it is also observed that, there is a backflow in the neighborhood of the plate which subsequently settles down as we move away from the plate. The effect is found more dominant at near the plate surface.

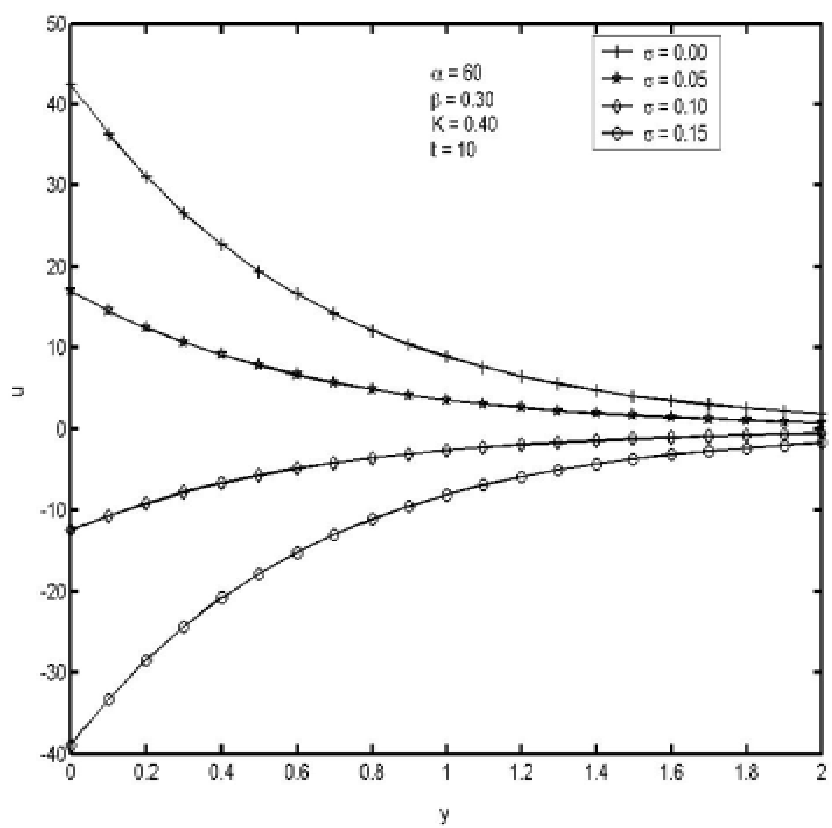


**Figure 4: Effect of frequency of excitation ( $\sigma$ ) on the velocity profiles**

The effect of time ($t$) on the nature of velocity profiles is seen in Figure 5. As $t$ increases, there is a decreasing trend in the velocity profiles near the boundary layer. Further, for certain values of $t$ even backflow is also observed. However, the flow field settles down as we move away from the plate.

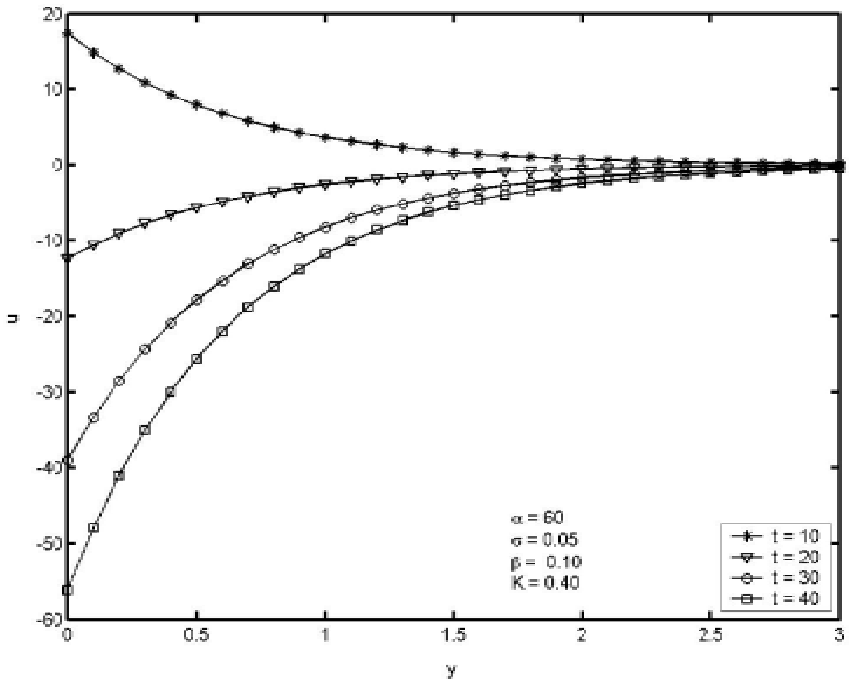


**Figure 5: Effect of time ($t$) on the velocity profiles.**

From the above figures it is quite evident that, $f$ in thermal study is analogous to the velocity profiles in case of sinusoidal oscillations of the bounding surface. And it can be seen that, the dispersion parameter in the thermal system is equivalent to the frequency of excitation, when compared with the latter case. Further, from Figure 5 it can also be noted that, the dispersion parameter also produces the same effect as that of time parameter $t$ in case of visco elastic fluids.

**2.** The dispersion and radiation effects on velocity profiles under aided flow conditions are shown in Figure 6. It is clear that, both of them tend to increase the velocity in the boundary. However, the rate of increase in velocity under combined effect is considerably less when applied individually. Thus, the dispersion effect is slightly suppressed in the presence of radiation.

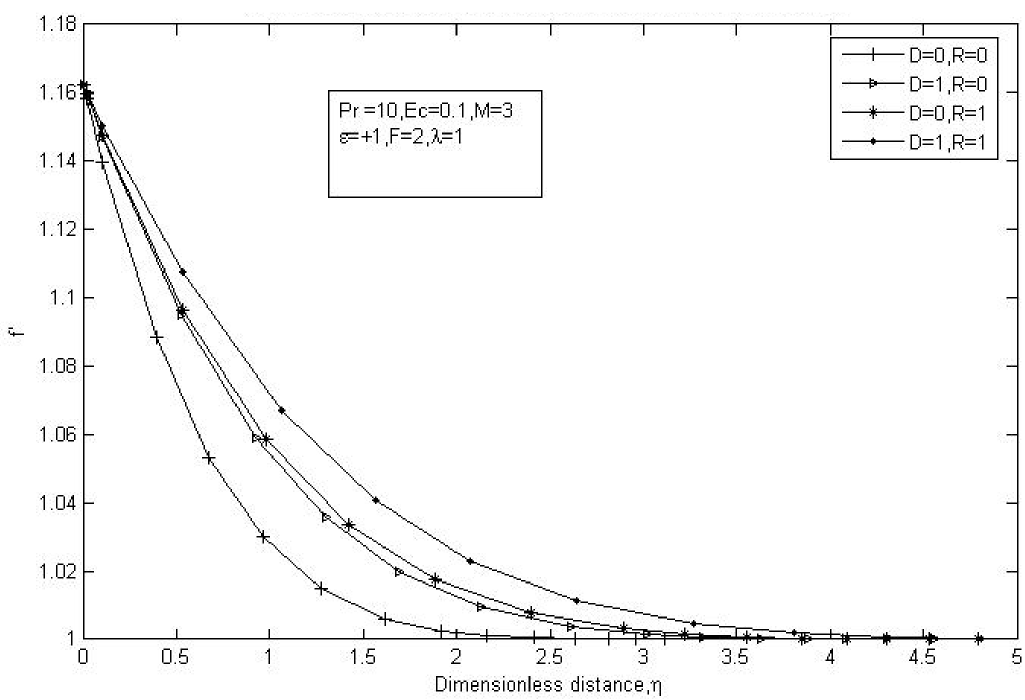


**Figure 6: Effect of dispersion and radiation on velocity profile in aided flow.**

Figure 7 illustrates, the effect of elastico viscosity parameter ($\beta$) on the magnification factor ($A^*$). It is seen that, as $\beta$ increases, there is an increase trend in the magnification factor.

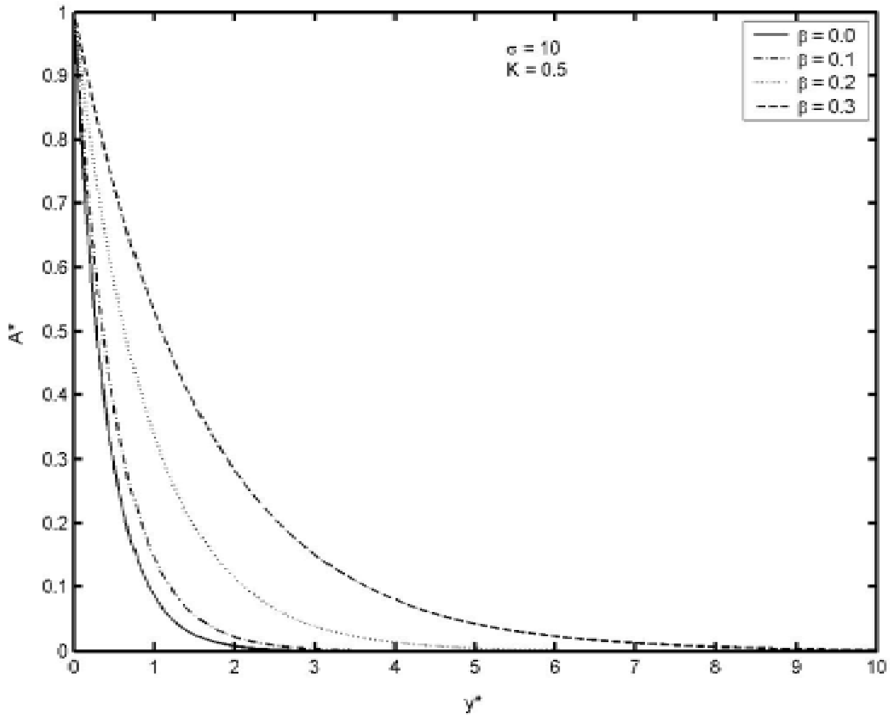


**Figure 7: Effect of elastico viscosity ($\beta$) on the magnification factor**

Figure 8 shows the effect of porosity on the magnification factor $A^*$. It is observed that, as the porosity decreases, the amplitude of the propagated wave into the medium also decreases when the viscosity of the medium is maintained constant i.e. the boundary layer thickness decreases.

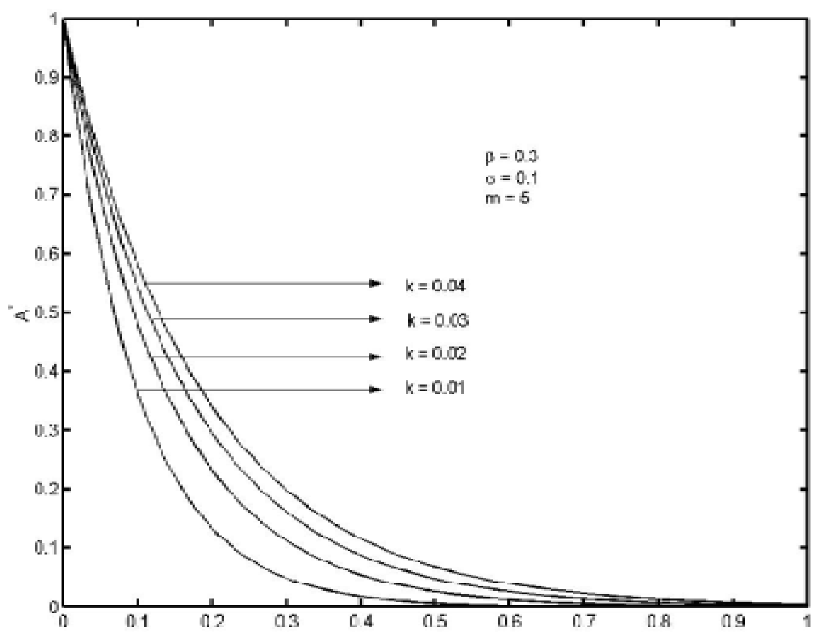


**Figure 8: Variation of magnification parameter ($A^*$) with respect to different porosity parameters**

The influence of magnetic field on the magnification parameter $A^*$ has been illustrated in Figure 9. As the magnetic field increases, the magnification also increases, while all other parameters are maintained constant.

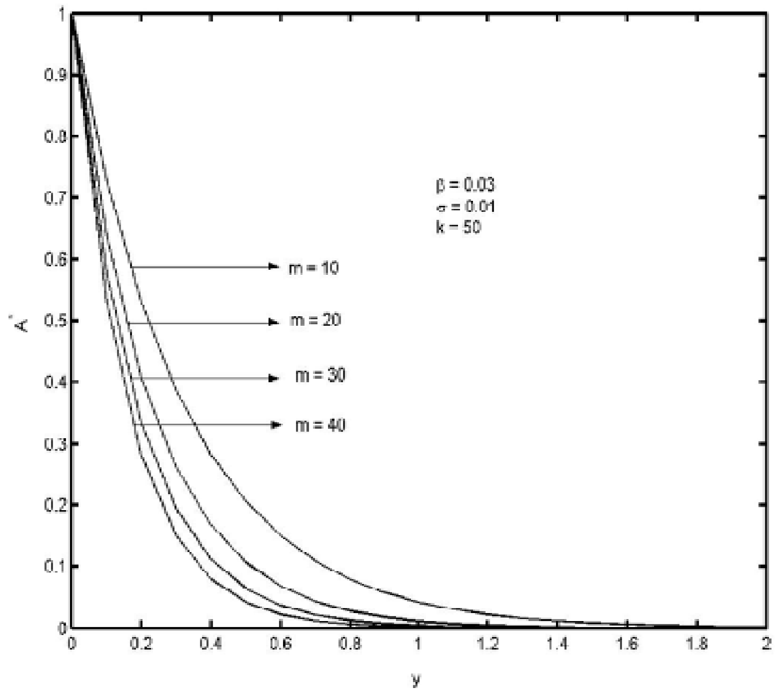


**Figure 9: Effect of magnetic parameter on magnification ( $A^*$ )**

The effect of visco elasticity parameter $(\beta)$ on the magnification parameter $A^*$ has been illustrated in Figure 10. As the visco elasticity reduces, there is a decreasing trend in the magnification parameter $A^*$. This can be attributed to the fact that, less molecular forces present in the fluid medium tend to cause the decrease in $A^*$

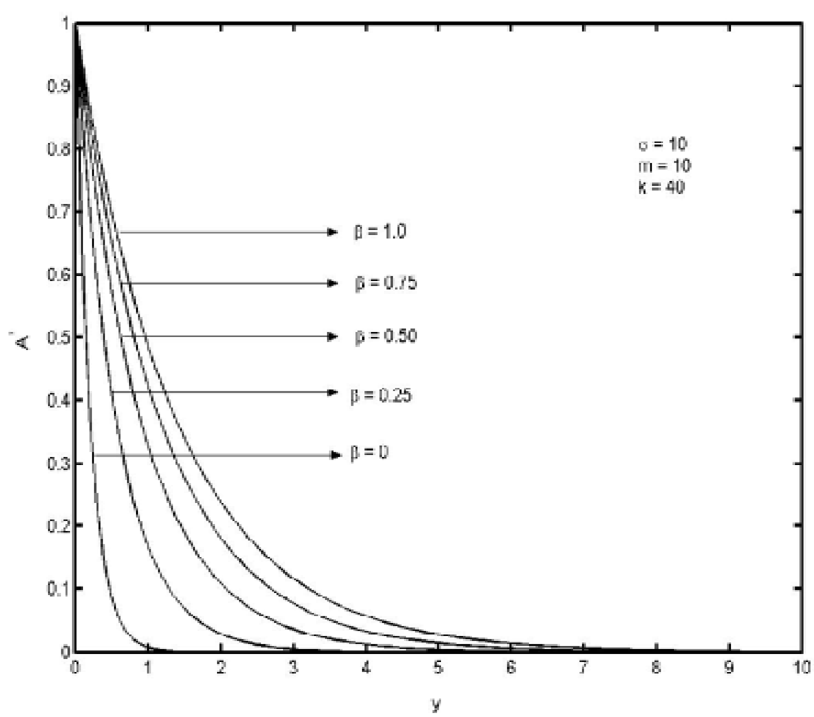


**Figure 10: Effect of elastico viscosity ( $\beta$ ) on magnification ( $A^*$ )**

On comparing the Figures 6, 7, 8, 9 and 10 it can be seen that, the amplification factor $A^*$ in case of the visco elastic fluids is similar to that of $f$ when thermal effects are considered. In both the cases, the profiles show the decreasing trend as we move away from the boundary surface. Further, an analogy can be drawn that, the effect of porosity in case of visco elastic fluid has similar effect as that of dispersion parameter and radiation parameters.

**3.** It is found from Figure 11 that, increase in dispersion and radiation increases local temperature due to increase in convective moment within the boundary layer and increases the boundary layer thickness. Further, the increasing effect of radiation is found dominant in the absence of dispersion than with dispersion.

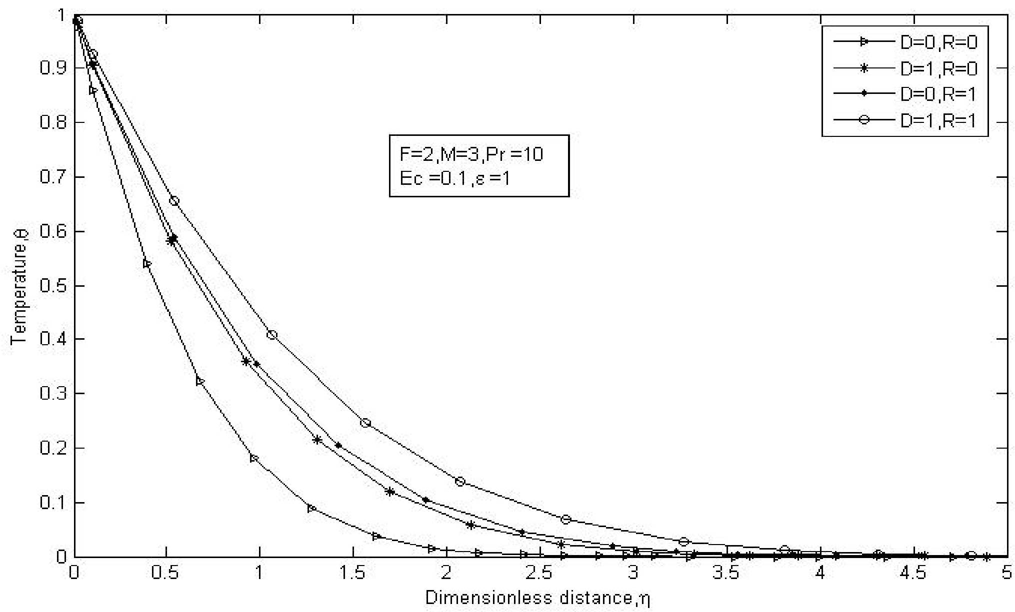


**Figure 11: Effect of dispersion and radiation on temperature profile.**

On comparing Figure 11 with Figure 9 and Figure 10 it can be concluded that, the amplitude parameter $A^*$ in case of visco elastic fluid assumes the role of the temperature dissipation. Further, the viscosity of the fluid under consideration and intensity of the magnetic field are as that of dispersion and radiation parameters when the thermal effects are taken into account.

**4.** Figure 12 shows the combined effect of dispersion and buoyancy on temperature profile. It is observed that, as mixed convection parameter increases the thermal boundary layer thickness decreases. Hence, the rate of heat transfer increases with increase in $\varepsilon$ or the buoyancy parameter. However, the effect of buoyancy parameter is significant without dispersion when compared to with dispersion.

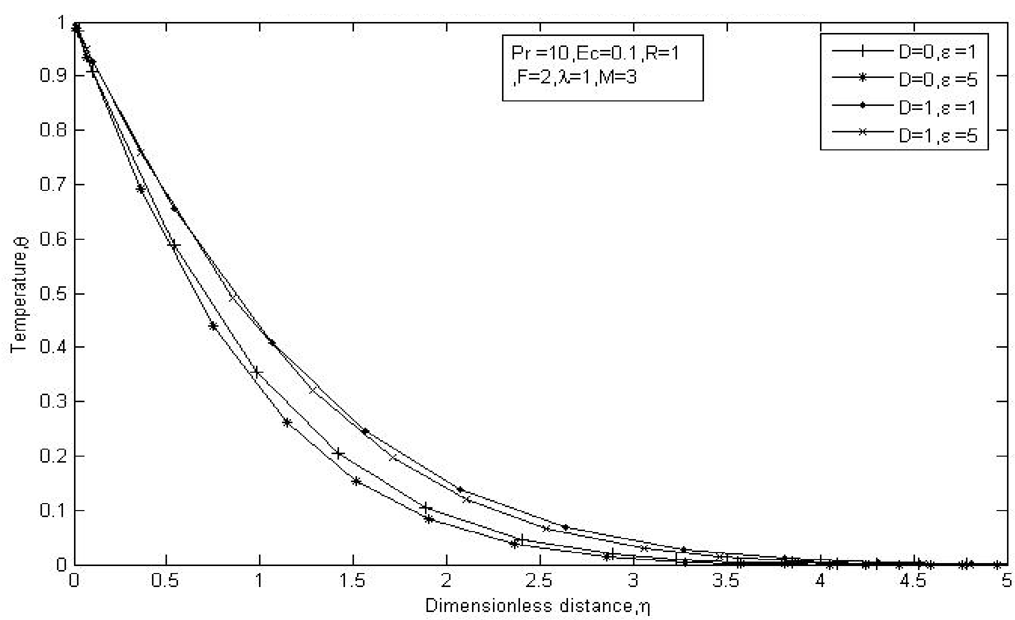


**Figure 12: Effect of buoyancy and dispersion on temperature profile.**

On comparing Figure 12 with Figure 7 it is observed that, the effect of buoyancy and dispersion on temperature profile has the same effect of elastico viscosity on magnification factor. Further, when compared with Figure 8 it is observed that, variation of magnification parameter with respect to different porosity parameters causes the same effect of buoyancy and dispersion on temperature profile and vice-versa.

**CONCLUSIONS:**

1. The increase in magnetic field and in the frequency of excitation causes same effect i.e. decreases the fluid velocity in the boundary region.
2. The dispersion parameter in thermal system is found equivalent to frequency of excitation. It also observed that, dispersion parameter effect and time parameter $(t)$ effect on visco elastic fluid is same.
3. In the presence of radiation, the dispersion effect is suppressed.
4. With increase in visco elastic parameter or magnetic field strength, there is an increasing trend in magnification factor.
5. The amplification factor in case of the visco elastic fluid is similar to that of $(f)$ when thermal effects are considered.
6. The effect of porosity in case of visco elastic fluid has similar effect as that of dispersion and radiation parameters.
7. The effect of buoyancy and dispersion on temperature profiles has the same effect as that of elastico viscosity on magnification factor.
8. Variation of magnification parameter with respect to different porosity parameters causes the same effect of buoyancy and dispersion on temperature profile and vice-versa.

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