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HYDROMAGNETIC FREE CONVECTIVE OSCILLATORY COUETTE FLOW THROUGH A POROUS VERTICAL CHANNEL WITH PERIODIC WALL TEMPERATURE

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ABSTRACT

The unsteady Couette flow of a viscous incompressible fluid through a highly porous medium bounded by two vertical parallel porous plates in the presence of transversely applied magnetic field has been studied. One of the plate is suddenly moved from rest with a free stream velocity that oscillates in time about a constant mean. Assuming periodic temperature at the moving plate, the approximate solution for the velocity field, the temperature field, the skin friction and the rate of heat transfer are obtain by using perturbation technique and discussed with the help of graphs and tables.

Keywords: *Hydromagnetic, unsteady free convective, periodic wall temperature.*

1. INTRODUCTION

The phenomenon of free convection arises in fluid when temperature changes cause density variation leading to buoyancy forces acting on the fluid elements. It can be observed in our daily life in atmospheric flow, which is driven by temperature differences. Free convective flow past a vertical plate has been studied extensively by Ostrach [1, 2] and many other researchers. These studies are confined to steady flow only. In case of unsteady free convective flows, Soundalgekar [3] studied the effect of viscous dissipation on the flow past an infinite vertical porous plate. It was assumed that the plate temperature oscillates in such a way that its amplitude is small. In fluid dynamics, Couette flow refers to the laminar flow of a viscous fluid in the space between two parallel plates, one of which is moving relative to other. The flow is driven by virtue of viscous drag force acting on the fluid and the applied pressure gradient parallel to the plates. This type of flow is named in honor of Maurice Marie Alfred Couette, a Professor of Physics at French University of Angers in the late 19^{th} century. Couette flow is frequently used in under graduate Physics and Engineering courses to illustrate shear driven fluid motion. Some important application areas of couette motion are magnetohydrodynamics power generators and pumps, polymer technology, petroleum industry and purification of crude oil. Singh [4] studied unsteady free convective flow of an incompressible viscous fluid between two vertical parallel plates, in which one is stationary and other is impulsively moving in its own plane. The steady state couette flow with viscous dissipating heat was studied by Kearsley [5]. The natural convection in unsteady couette flow confined between two vertical plates in the presence of thermal radiation has been investigated by Narahari [6].

Flows of fluid through porous media are of practical interest because these are quite prevalent in nature. Such flows have attracted the attention of a number of scholars due to their application in many branches of science and technology. Raptis [7] studied the unsteady free convective flow through a porous medium. Raptis and Peridikis [8] further studied the unsteady free convection flow through a highly porous medium bounded by an infinite porous plate. The convection in a porous medium with inclined temperature gradient has been studied by Nield [9]. An exact solution of unsteady free convective couette flow of a viscous incompressible heat generating/ absorbing fluid confined between two vertical plates in a porous medium has been studied by Deka and Bhattacharya [10]. Comprehensive literature on various aspects of free convection flows and its application could be found in Nield and Bejan [11]. On the other hand the research works in magnetohydrodynamics (MHD) have been advanced significantly during last three decades in natural sciences and engineering disciplines after the work of Hartmann in liquid metal duct flow under the influence of a strong external magnetic field. This fundamental investigation has provided basic knowledge for development of several MHD devices such as MHD pumps, generators, brakes and flow meters. The study of flow for an electrically conducting fluid has many applications in engineering problems such as plasma studies, nuclear reactors, geothermal energy extraction and the boundary layer control in the field of aerodynamics. MHD Couette flows are frequently encountered in many scientific and environmental processes, such as astrophysical flow, heat and cooling of chambers

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and solar power technology. Jha [12] has studied the natural convection in unsteady MHD couette flow. Jain *et al* [13] investigated the three dimensional Couette flow with transpiration cooling through a porous medium in slip flow regime. The three dimensional Couette flow and heat transfer in the presence of transverse magnetic field has been analyzed by Das *et al* [14]. Unsteady free convection oscillatory couette flow through a porous medium with periodic wall temperature has been analyzed by Sharma *et al* [15].

In this work we purpose to study the effects of free convection on Hydromagnetic oscillatory Couette flow of viscous, incompressible and electrically conducting fluid through a highly porous medium bounded by two infinite vertical porous plates when the temperature of moving plate oscillates with time.

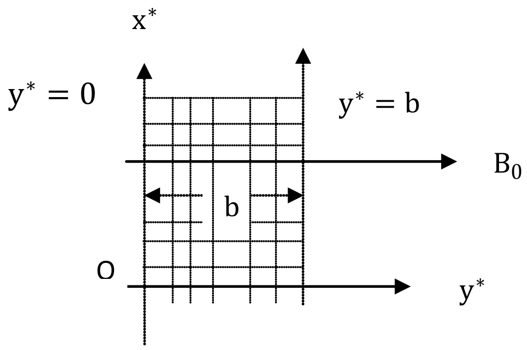
2. FORMULATION OF THE PROBLEM

We consider the unsteady Couette flow of a viscous, incompressible and electrically conducting fluid through a highly porous medium bounded between two infinite vertical porous plates separated by a distance b. One of the plate is suddenly moved from the rest with a free stream velocity which oscillates with time about a constant mean. Choose a Cartesian coordinates systems with x-axis along one of moving plate in vertically upward direction. The other stationary vertical plate is situated at $y^* = b$. Further it is assumed that the temperature of moving plate fluctuates with time about a non-zero constant mean, and the temperature of the other plate is held constant. A magnetic field (fixed relative to moving plate) of uniform strength B_0 is assumed to be applied transversely to the plates. Also, it is assumed that the magnetic Reynolds number is so small that induced magnetic field can be neglected in comparison to externally applied magnetic field.

Boundary conditions

$u^* = U_0\left(1 + \varepsilon e^{i\omega^* t^*}\right)$,

$T^* = T_n^* + \varepsilon(T_n^* - T_s^*)e^{i\omega^* t^*}$



Boundary conditions

$u^* = 0, T^* = T_s^*$

Fig. (1): The physical configuration of the problem.

We consider the free stream velocity distribution of the form $U^*(t^*) = U_0\left(1 + \varepsilon e^{i\omega^* t^*}\right)$ (1)

where U_0 is mean free stream velocity, ω^* is the frequency of oscillations and t^* is time. The equations governing the problem are

Momentum equation

$$\rho\frac{\partial u^*}{\partial t^*} = -\frac{\partial P}{\partial x^*} + \mu\frac{\partial^2 u^*}{\partial y^{*2}} - \rho^* g - \frac{u^*\mu}{k^*} - \sigma B_0^2 u^* \tag{2}$$

and equation of energy is

$$\frac{\partial T^*}{\partial t^*} = \alpha\frac{\partial^2 T^*}{\partial y^{*2}} \tag{3}$$

And boundary conditions are

$$y^* = 0:\ u^* = U_0\left(1 + \epsilon e^{i\omega^* t^*}\right), T^* = T_n^* + \varepsilon(T_n^* - T_s^*)e^{i\omega^* t^*},$$

$$y^* = b:\ u^* = 0, T^* = T_s^* \tag{4}$$

where u^*, U^*, ρ, μ, P, g, β, k^*, α, T^*, T_n^*, T_s^*, B_0 are respectively, velocity, free stream velocity, density, viscosity, pressure, gravity, volumetric coefficient of thermal expansion, permeability parameter, thermal diffusivity, temperature of fluid in the boundary layer, temperature of moving plate, temperature of the stationary plate and magnetic field . The (*) stands for dimensional quantities.

Equation (2), for the free stream, is reduced to

$$\rho\frac{dU^*}{dt^*} = -\frac{\partial P}{\partial x^*} - \rho_\infty^* g - \frac{U^*\mu}{k} - \sigma B_0^2 U^* \tag{5}$$

From equations (2) and (5), we get

$$\rho\frac{\partial u^*}{\partial t^*} = \rho\frac{dU^*}{dt^*} + \mu\frac{\partial^2 u^*}{\partial y^{*2}} + g(\rho_\infty^* - \rho^*) - \frac{(u^*-U^*)\mu}{k^*} - \sigma B_0^2(u^* - U^*) \tag{6}$$

By using the constitutive equation

$$g(\rho_\infty^* - \rho^*) = g\beta\rho(T^* - T_s^*),$$

where β is the volumetric coefficient of thermal expansion and ρ_∞ the density of the fluid far away the surface.

Introducing the following non-dimensional quantities

$$y = \frac{y^*}{b}, u = \frac{u^*}{U_0}, U = \frac{U^*}{U_0}, \omega = \frac{\omega^* b^2}{v}, \ t = \omega^* t^*, \theta = \frac{(T^*-T_s^*)}{(T_n^*-T_s^*)}, k = \frac{k^*}{b^2}, M = B_0 b\sqrt{\frac{\sigma}{\mu}}$$

$$G_r\,(\text{Grashoff number}) = \frac{g\beta b^2(T_n^*-T_s^*)}{vU_0}, \quad P_r\ (\text{Prandtl number}) = \frac{\nu}{\alpha}$$

Using non-dimensional quantities the equations (6) and (3) become

$$\omega\frac{\partial u}{\partial t} = \omega\frac{dU}{dt} + \frac{\partial^2 u}{\partial y^2} + G_r\theta - \frac{(u-U)}{k} - M^2(u - U) \tag{7}$$

$$\omega Pr\frac{\partial\theta}{\partial t} = \frac{\partial^2\theta}{\partial y^2} \tag{8}$$

The corresponding boundary conditions become

$$y = 0 : u = 1 + \varepsilon e^{it} . \theta = 1 + \varepsilon e^{it},$$
$$y = 1 : u = 0 : \ \theta = 0 \tag{9}$$

3. **METHOD OF SOLUTION**

Since the amplitudes of free-stream velocity and temperature variation $\varepsilon(\ll 1)$ is very small, we now assume the solution of the following form:

$$u(y,t) = u_0(y) + \varepsilon u_1(y)e^{it},$$
$$\theta(y,t) = \theta_0(y) + \varepsilon\theta_1(y)e^{it}, \tag{10}$$

The free-stream velocity is given by the expression

$$U = 1 + \varepsilon e^{it} . \tag{11}$$

Substituting equation (10) and (11) in equation (7) and (8), comparing the coefficients of identical power of ε and neglecting those of ε^2, we get following equations

$$\frac{\partial^2 u_0}{\partial y^2} - \frac{u_0}{k} - M^2 u_0 = -G_r\theta_0 - \frac{1}{k} - M^2 \tag{12}$$

$$\frac{\partial^2\theta_0}{\partial y^2} = 0 \tag{13}$$

$$\frac{\partial^2 u_1}{\partial y^2} - \left(i\omega + \frac{1}{k} + M^2\right)u_1 = -G_r\theta_1 - \left(i\omega + \frac{1}{k} + M^2\right) \tag{14}$$

$$\frac{\partial^2\theta_1}{\partial y^2} - i\omega Pr\theta_1 = 0 \tag{15}$$

with the corresponding boundary conditions

$$y = 0 : \ u_0 = 1, u_1 = 1, \theta_0 = 1, \theta_1 = 1$$
$$y = 1 : \ u_0 = 0, u_1 = 0, \ \theta_0 = 0, \ \theta_1 = 0 \tag{16}$$

Solutions of these equations under the corresponding boundary conditions are given by the following expression.

$$\theta_0(y) = 1 - y \tag{17}$$

$$\theta_1(y) = Ae^{\lambda(1+i)y} + Be^{-\lambda(1+i)y} \tag{18}$$

$$u_0(y) = A_1e^{a_1y} + B_1e^{-a_1y} + \frac{G_r(1-y)}{M^2+\frac{1}{k}} + 1 \tag{19}$$

$$u_1(y) = A_3e^{(a_2+ia_3)y} + B_2e^{-(a_2+ia_3)y} - A_2Ae^{\lambda(1+i)y} - A_2Be^{-\lambda(1+i)y} + 1 \tag{20}$$

Where constants used above have been listed in the appendix.

4. RESULTS AND DISCUSSION

In order to point out the effect of permeability and convection on the velocity, when the moving plate is subjected to oscillating free stream velocity, fluctuating wall temperature and the entire system is exposed to transversely applied magnetic field, numerical calculations are carried out for different values of Grashoff number (G_r), Prandtl number (P_r), the frequency of oscillation and the permeability parameter (k) for small ($M = 2$) and large ($M = 10$) value of magnetic field parameter i.e. Hartmann number (M). The values of Prandtl number are chosen as 0.71 and 7.0 approximately, which represents air and water at 20°C. The values of G_r and k are selected arbitrarily.

(a) MEAN FLOW

The mean flow velocity is given by equation (19). The variation in mean flow velocity for various values of Grashoff number and permeability of porous medium is presented in Figure 2. It is observed from this figure that the mean velocity increases with an increase in free convection parameter i.e. Grashoff number whereas it decreases with increasing permeability parameter because the porous material offers resistance to the fluid flow. The above pattern remains unaltered for small and large value of magnetic field parameter ($M = 2$ and $M = 10$)

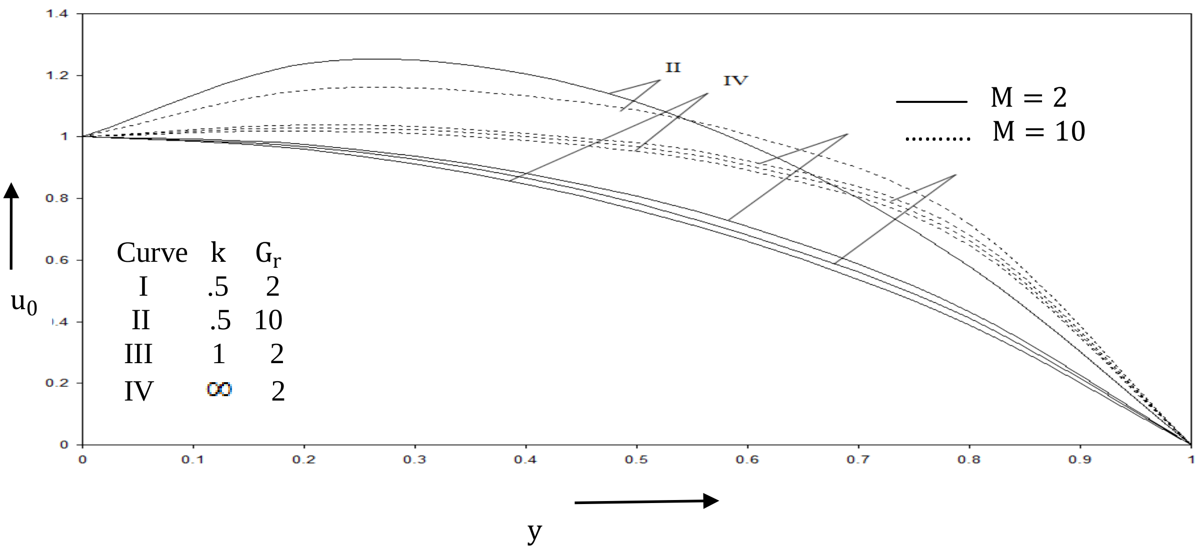


Fig. (2): The mean flow velocity profiles

Knowing the mean velocity field, from the practical point of view, it is important to know the effects of Grashoff number and Hartmann number on mean-skin friction. It is given by:

$$\tau^* = \mu\left(\frac{du^*}{dy^*}\right)_{y^*=0}$$

And in non-dimensional form it is given by:

$$\tau = \frac{\tau^* b}{\mu U_0} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left(\frac{\partial u_0}{\partial y}\right)_{y=0} + \varepsilon\left(\frac{\partial u_1}{\partial y}\right)_{y=0} e^{it} \tag{21}$$

Denoting the mean skin friction

$$\tau_m = \left(\frac{\partial u_0}{\partial y}\right)_{y=0} \tag{22}$$

Substituting equation (19) in equation (22), we have

$$\tau_m = a_1 A_1 - a_1 B_1 - \frac{Gr}{\frac{1}{k} + M^2}$$

The numerical values of τ_m are entered in Table 1.

Table-1: The mean skin friction (τ_m).

M	k	∞	1	2	3	4	5
2	$G_r = 2$	-0.01413	0.03160	0.00982	0.00213	-0.00184	-0.00424
2	$G_r = 5$	0.79185	0.80425	0.79864	0.79652	0.79540	0.79472
10	$G_r = 2$	0.25265	0.25310	0.25291	0.25283	0.25279	0.25276
10	$G_r = 5$	0.73271	0.72611	0.72940	0.73050	0.73105	0.73138

It can be interpreted from this table that the mean friction first increases and there after decreases with increasing value of permeability parameter (k) and free convection parameter (G_r). This pattern is preserved for both small and large value of magnetic field parameter(M). It is interesting to note that mean skin friction is more affected by increase in G_r, which may be attributed to the presence of porous medium.

UNSTEADY FLOW

The velocity and temperature fields given by equations (17) to (20) respectively can be expressed in terms of fluctuating parts as follows:

$$u(y,t) = u_0(y) + \varepsilon e^{it}(m_r + i m_i), \tag{23}$$

$$\theta(y,t) = \theta_0(y) + \varepsilon e^{it}(T_r + i T_i), \tag{24}$$

where

$$m_r + i m_i = u_1(y) \text{ and } T_r + i T_i = \theta_1(y)$$

We can write expression for transient velocity and transient temperature from equation (23) and (24) as follows

$$u\left(y, \frac{\pi}{2}\right) = u_0(y) - \varepsilon m_i \tag{25}$$

$$\theta\left(y, \frac{\pi}{2}\right) = \theta_0(y) - \varepsilon T_i \tag{26}$$

The variation in transient velocity profiles for small ($M = 2$) and large ($M = 10$) value of magnetic field parameter is shown in figure 3. The study of this figure reveals that the transient velocity increases with increasing Grashoff number (G_r) for both the situation i.e. for $P_r = 0.71$ (air) and $P_r = 7.0$ (water), because the buoyancy force increases in the upward direction.

The above pattern is maintained for both small and large value of magnetic parameter(M). Clearly the impact of increasing magnetic field is to enhance the transient velocity profiles. Further it is also evident from this figure that the transient velocity decreases due to increase of permeability parameter (k) in both cases ($P_r = 0.71$ and $P_r = 7.0$), which physically can be interpreted as true because the permeability of the porous medium exert retarding influence on fluid motion. It is also observed from this figure that transient velocity profiles increases with the frequency of oscillation parameter (ω). The values of transient velocity is more for large value of magnetic field parameter ($M = 10$) as compare to its small value ($M = 2$).

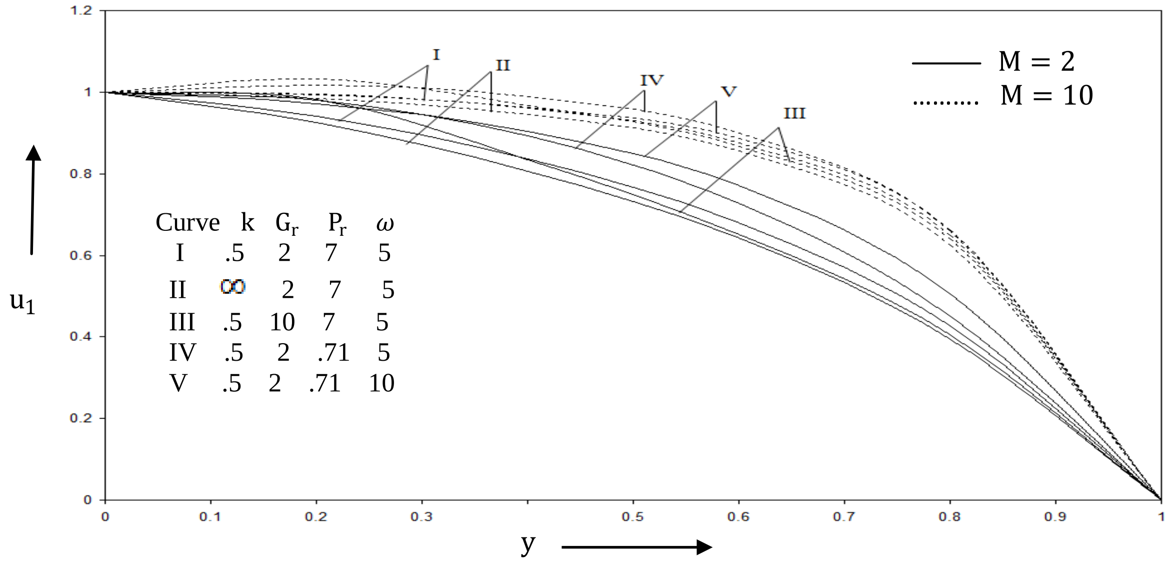


Fig. (3): The transient velocity profiles for $t = \frac{\pi}{2}$ and $\varepsilon = 0.2$

Fig. (4) depicts the variation of temperature with distance from moving plate. It is observed that transient temperature increases with increasing ω for air ($P_r = 0.71$), while reverse phenomena is observed for water ($P_r = 7.0$). It is interesting to note that the values of transient temperature are greater near the moving pate for water while reverse effect is observed as we move towards the stationary plate. It is now proposed to study the behavior of amplitude and phase of skin-friction, from equation (20) and (21) we have,

$$\tau = \tau_m + \varepsilon e^{it}\left[(a_2 + ia_3)A_3 - (a_2 + ia_3)B_2 - AA_2\lambda(1+i) + BA_2\lambda(1+i)\right] \tag{27}$$

Equation can be expressed (27) in terms of the amplitude and phase of skin-friction as

$$\tau = \tau_m + \varepsilon|m|\cos(t + \emptyset) \tag{28}$$

where

$m = m_r + im_i =$ Coefficient of εe^{it} in equation (27)

$|m| = \sqrt{m_r^2 + m_i^2}$, and $\tan\phi = \frac{m_i}{m_r}$

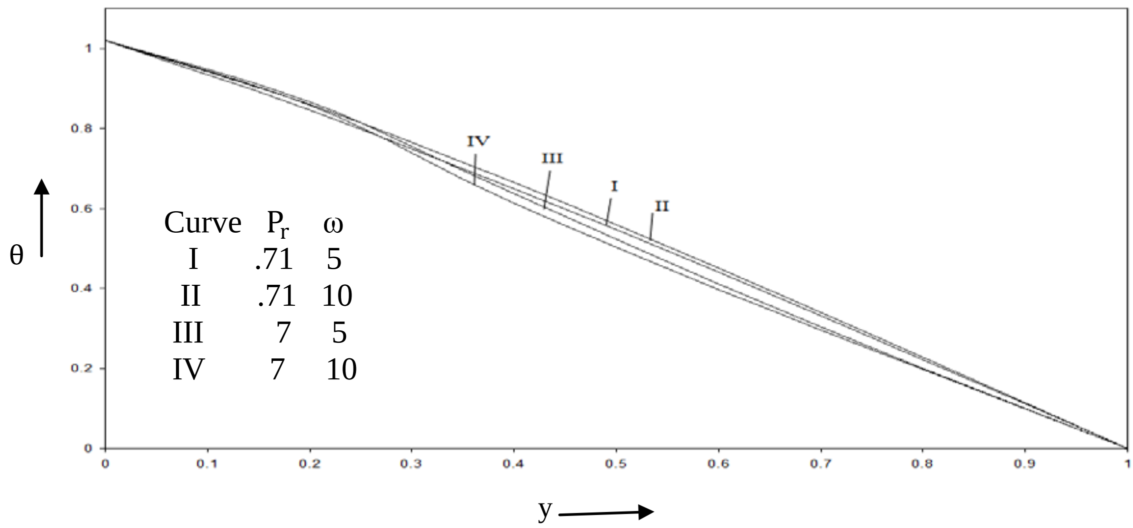


Fig. (4): The transient temperature profiles for $t = \frac{\pi}{2}$ and $\epsilon = 0.2$ Curve P_r ω

Table-2: The values of ($|m|$) for k = 0.5

M	ω	$G_r = 2$ $P_r = 7$	$G_r = 1$ $P_r = 7$	$G_r = 2$ $P_r = 0.71$	$G_r = 1$ $P_r = 0.71$
2	2	0.12980	0.26088	0.07415	0.18060
2	4	0.15702	0.28276	0.099312	0.18791
2	6	0.17702	0.28527	0.12982	0.19764
2	8	0.19451	0.28647	0.16379	0.20829
2	10	0.20921	0.29459	0.19710	0.21831
10	2	0.18590	0.06559	0.25119	0.09701
10	4	0.14622	0.045256	0.24536	0.09470
10	6	0.12400	0.03802	0.23704	0.09165
10	8	0.10932	0.027451	0.22770	0.08867
10	10	0.09756	0.025797	0.21863	0.08647

The numerical value of $|m|$ are presented in Table – 2. for M = 2 and M = 10. It is observed from this table that for M = 10, $|m|$ decrease with increasing frequency of oscillation ω for water and air (for same value of G_r), whereas for M = 2, reverse pattern is observed. Also with increasing value of convection parameter G_r, the amplitude of skin friction $|m|$, decreases for M = 2 (for air and water), whereas the $|m|$ increases with increasing convection parameter G_r, for M = 10 (for air and water).

We now study the effect of ω on the rate of heat transfer. The rate of heat transfer in terms of the Nusselt Number can be obtained as

$$Nu = \frac{q_\omega^* b}{k(T_n^* - T_s^*)} = \left(\frac{\partial\theta}{\partial y}\right)_{y=0} = \left(\frac{\partial\theta_0}{\partial y}\right)_{y=0} + \varepsilon\left(\frac{\partial\theta_1}{\partial y}\right)_{y=0} e^{it}, \tag{29}$$

$$Nu = -1 + \varepsilon e^{it}[A\lambda(1+i) - B\lambda(1+i)]. \tag{30}$$

We can express (30) in terms of amplitude and phase of heat transfer as

$$Nu = -1 + \varepsilon|H|\cos(t+\varphi) \tag{31}$$

Where
$H = H_r + iH_i$ = Coefficients of εe^{it} in expression (30)
$|H| = \sqrt{H_r^2 + H_i^2}$ and $\tan\varphi = \frac{H_i}{H_r}$.

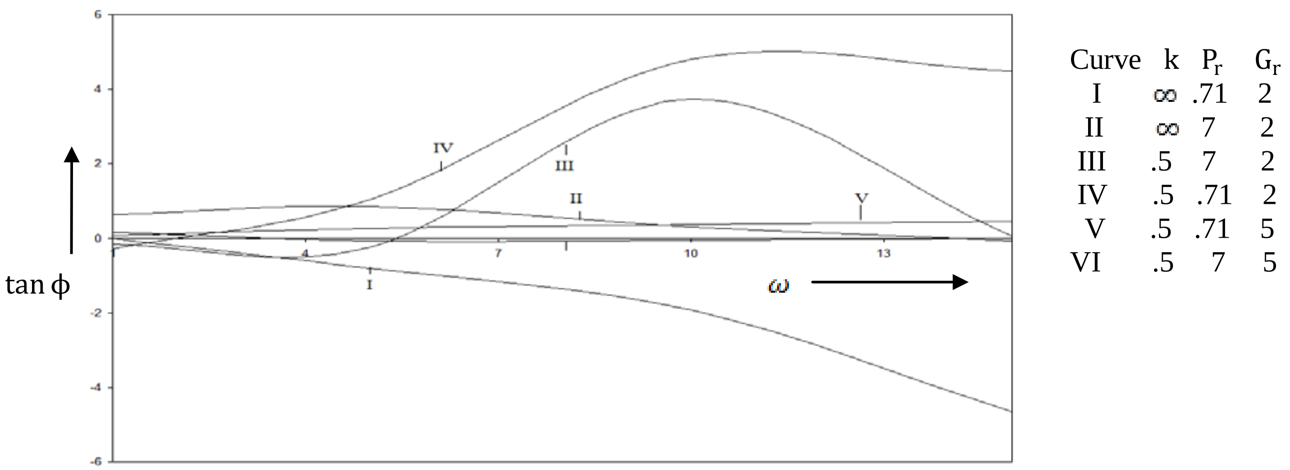


Fig.(5) : The tangent of phase tanϕ of skin friction.

The variation in tangent of phase of skin-friction is exhibited in figure 5. It is observed from this figure that the phase of skin-friction decreases with the increase in G_r in both the cases ($P_r = 0.71$ and $P_r = 7.0$) and there is always a phase lead for small value of k.

Table- 3: The amplitude and phase for heat transfer.

ω	|H|		tan φ	
	$P_r = 0.71$	$P_r = 7.0$	$P_r = 0.71$	$P_r = 7.0$
2	1.1438	3.7623	0.44771	1.01700
4	1.4745	5.2937	0.77310	0.99791
6	1.8575	6.4794	0.95328	0.99989
8	2.2280	7.4832	1.03170	1.00010
10	2.5668	8.3667	1.05570	1.00000

The values of amplitude and phase of heat transfer is listed in Table 3. We observe from the Table 3 that amplitude of heat transfer increases with increasing P_r and ω both. The value of amplitude is greater in case of water than in case of air. It is clear that there is always a phase lead for $P_r = 0.71$ (air), and the phase of heat transfer almost remains constant the case of water ($P_r = 7.0$)

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Appendix

$$\lambda = \sqrt{\frac{\omega P_r}{2}},\ A = -\frac{e^{-\lambda(1+i)}}{e^{\lambda(1+i)} - e^{-\lambda(1+i)}},\ B = \frac{e^{-\lambda(1+i)}}{e^{\lambda(1+i)} - e^{-\lambda(1+i)}},\ a_1 = \sqrt{\frac{1}{k} + M^2},\ a_2 = \frac{1}{\sqrt{2}} + \left[\sqrt{\omega^2 + \left(\frac{1}{k} + M^2\right)^2} + \left(\frac{1}{k} + M^2\right)\right]^{\frac{1}{2}},$$

$$a_3 = \frac{1}{\sqrt{2}} + \left[\sqrt{\omega^2 + \left(\frac{1}{k} + M^2\right)^2} - \left(\frac{1}{k} + M^2\right)\right]^{\frac{1}{2}}$$

$$A_1 = -\frac{1}{e^{a_1} - e^{-a_1}}\left(1 - \frac{Gr\, e^{-a_1}}{\frac{1}{k} + M^2}\right),\ B_1 = \frac{1}{e^{a_1} - e^{-a_1}}\left(1 - \frac{Gr\, e^{a_1}}{\frac{1}{k} + M^2}\right), A_2 = \frac{Gr}{\lambda^2(1+i)^2 - \left(i\omega + \frac{1}{k} + M^2\right)}$$

$$B_2 = \frac{-\left[AA_2\left(e^{(a_2+ia_3)} - e^{\lambda(1+i)}\right) + BA_2\left(e^{(a_2+ia_3)} - e^{-\lambda(1+i)}\right) + 1\right]}{e^{-(a_2+ia_3)} - e^{(a_2+ia_3)}} \quad A_3 = \frac{\left[AA_2\left(e^{-(a_2+ia_3)} - e^{\lambda(1+i)}\right) + BA_2\left(e^{-(a_2+ia_3)} - e^{-\lambda(1+i)}\right) + 1\right]}{e^{-(a_2+ia_3)} - e^{(a_2+ia_3)}}$$

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