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SPECIALLY STRUCTURED FLOWSHOP PRODUCTION SCHEDULING TO MINIMIZE THE RENTAL COST IN FUZZY ENVIRONMENT

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ABSTRACT

This paper is an attempt to introduce the concept of fuzziness in specially structured n jobs, 2 machines flowshop scheduling problem in which the processing time of jobs are uncertain and are represented by trapezoidal fuzzy membership function. In general, the processing time of the jobs are taken to be random. But there are significant cases in which the processing time are not merely random but bears a well-defined relationship to one another. The objective of the paper is to develop a heuristic algorithm to minimize the rental cost of the machines taken on rent under a specified rental policy without violating the value the maximum makespan as proposed by Johnson's for a two stage specially structured flowshop scheduling. A numerical illustration is also given to demonstrate the computational efficiency of the proposed algorithm.

Keywords: *Specially structured flowshop schedule, Rental Policy, Fuzzy Processing Time, Ranking function, Utilization Time.*

Mathematical Subject Classification: *90B30, 90B35.*

1. INTRODUCTION

Flow shop problems concerns the sequencing of a given number of jobs through a series of machines in the exact same order on all machines with the aim to satisfy a set of constraints as much as possible, and optimize a set of objectives. A large number of deterministic scheduling algorithms have been proposed in last decades to deal with flow shop scheduling problems with various objectives and constraints. However, it is often difficult to apply those algorithms to real-life flow shop problems. For example, in practice the processing times of jobs can be uncertain due to incomplete knowledge or uncertain environment. Fuzzy sets and logic can be used to tackle uncertainties inherent in actual flow shop scheduling problems. Zadeh [1965] introduced the term fuzzy logic in his seminal work "Fuzzy sets", which described the mathematics of fuzzy set theory. As per literature review it has been found that processing time of machines is considered to be random in magnitude. There are the cases when processing time of jobs are not random but follow some well-defined structural conditions and hence, the concept of specially structured flowshop scheduling is significant. One of the earliest results in flow shop scheduling theory is an algorithm given by Johnson [1954] for two stage flow shop scheduling problem. Gupta J.N.D [1975] gave an algorithm to find the optimal sequence for specially structured flow shop scheduling. MacCahon and Lee [1990] discussed the job sequencing with fuzzy processing time. Ishibuchi and Lee [1996] addressed the formulation of fuzzy flowshop scheduling problem with fuzzy processing time. Some of the noteworthy approaches are due to Yager [1981], Shukla and Chen [1996], Yao and Lin [2002], Singh and Gupta [2005], Sanuja and Song [2006], Singh, Sunita and Allawalia [2008], Thorani, Rao and Shankar [2012].

Gupta, D., Sharma, S. and Shashi [2012] studied specially structured two stage flow shop scheduling to minimize the rental cost of machines and. In the present work we have introduced the concept of fuzziness in processing time of jobs and has used trapezoidal fuzzy membership function to describe the uncertainty in the processing times. Fuzzy set theory in the form of approximate reasoning provides decision support and expert systems with powerful reasoning capabilities.

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2. ROLE OF FUZZY LOGIC IN SCHEDULING

A fuzzy system can be thought of an attempt to understand a system for which no model exists, and it does so with the information that can be uncertain in a sense of being vague, or fuzzy, or imprecise, or altogether lacking. From this angle, fuzzy logic is a method to formalize the human capacity of imprecise reasoning. Such reasoning represents the human ability to reason approximately and judge under uncertainty. In fuzzy logic all truths are partial or approximate. In this sense the reasoning has also been termed interpolative reasoning, where the process of interpolating between the binary extremes of truth and false is represented by the ability of fuzzy logic to encapsulate partial truths.

Scheduling is a enduring process where the existence of real time information frequently forces the review and modification of pre-established schedules. The real world is complex; complexity in the world generally arises from uncertainty. From this prospective, the concept of fuzzy environment is introduced in the theory of scheduling.

3. BASICS OF FUZZY SET THEORY

A fuzzy set $\tilde{A}$, defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}} : R \longrightarrow [0, 1]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \bigcap [d, \infty)$.
3. $\mu_{\tilde{A}}(x)$ strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$.
4. $\mu_{\tilde{A}}(x) = 1$ for all $x \in [b, c]$, where $a < b < c < d$.

All information contained in a fuzzy set is described by its membership function. The trapezoidal fuzzy numbers are used to represent fuzzy processing times in our algorithm. The membership value of the trapezoidal fuzzy number A= (a, b, c, d) denoted by $\mu_{\tilde{A}}(x), x \in R^+$, can be calculated according to the formula

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}; a \le x \le b \\ 1, b \le x \le c \\ \dfrac{x-d}{c-d}; c \le x \le d \\ o; otherwise \end{cases}$$

The four basic operations that can be performed on trapezoidal fuzzy numbers are as follows:

If $A = (a_1, b_1, c_1, d_1)$ and $B = (a_2, b_2, c_2, d_2)$ be the two trapezoidal fuzzy number,then

1. $A + B = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$
2. $A - B = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$
3. $kA = (ka_1, kb_1, kc_1, kd_1); k \ge 0$
4. $kA = (kd_1, kc_1, kb_1, ka_1); k \le 0$

Fuzzy set theory is a powerful tool to deal with real life situations. Real numbers can be linearly ordered by $\ge$ or $\le$, however this type of inequality doesnot exist in fuzzy numbers. Since fuzzy numbers are represented by possibility distribution, they can overlap with each other and it is difficult to determine clearly whether one fuzzy number is larger or smaller than other. An efficient approach for ordering the fuzzy numbers is by the use of a ranking function $R: F(R) \to R$, where $F(R)$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number in to the real line, where a natural order exists. Thus, specific ranking of fuzzy numbers is an important procedure for decision--making in a fuzzy environment and generally has become one of the main problems in fuzzy set theory.

In the present work the processing times of jobs are ranked by using ranking function given by Kumar, Singh and Kaur [2010].i.e.

If $A=(a_1,b_1,c_1,d_1)$ be the trapezoidal fuzzy number, then ranking of A is defined as $R(A)=\dfrac{a_1+b_1+c_1+d_1}{4}$.

4. PROBLEM FORMULATION

The following notations will be used throughout the course of present work:

S : Sequence of jobs 1, 2, 3,…., n
M_j : Machine j, j= 1, 2
a_{ij} : Fuzzy processing time of i^{th} job on machine M_j i=1,2,3,....,n; j=1,2
A_{ij} : Ranking function of processing time of i^{th} job on machine M_j
$t_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M_j
$I_{ij}(S_k)$: Idle time of machine M_j for job i in the sequence S_k
$U_j(S_k)$: Utilization time for which machine M_j is required
$R(S_k)$: Total rental cost for the sequence S_k of all machine
C_i : Rental cost of i^{th} machine.

Further, the Completion time of i^{th}job on machine M_j is denoted by t_{ij} and is defined as:
$t_{ij} = \max(t_{i-1,j}, t_{i,j-1}) + a_{ij}$ for $j \geq 2$. where $a_{i,,j}$=fuzzy processing time of i^{th} job on j^{th} machine.

Also, the machines will be taken on rent as and when they are required and are returned as and when they are no longer Required, .i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2^{nd} on rent at time when 1^{st} job is completed on 1^{st} machine.

Let some job i (i=1, 2, 3, ..., n) is to be processed on two machines M_1 and M_2 in the order M_1M_2 such that no passing is allowed. Let a_{ij} be the fuzzy processing time of i^{th}job on j^{th} machine. Let A_{ij}; i=1, 2, 3,.....n; j=1, 2 be the ranking function of the processing times on two machines M_1& M_2 such that either $A_{i1}\geq A_{i2}$ or$A_{i1} \leq A_{i2}$ for all values of i. Our aim is to find the sequence $\{S_k\}$ of the jobs which minimize the rental cost of the machines.

Table 1: Mathematical model of the problem in matrix form

Jobs	Machine M_1	Machine M_2
i	a_{i1}	a_{i2}
1	a_{11}	a_{12}
2	a_{21}	a_{22}
3	a_{31}	a_{32}
4	a_{41}	a_{42}
-	-	-
n	a_{n1}	a_{n2}

Mathematically, the problem is stated as

Minimize $R(S_k)=\sum_{i=1}^{n} a_{i1}(S_k)\times C_1 + U_2(S_k)\times C_2$

Subject to constraint: Rental Policy

Our objective is to minimize rental cost of machines while minimizing total elapsed time.

5. THEOREMS

The following theorems have been developed to find the optimal sequence of jobs processing for a two stage specially structured flowshop scheduling.

5.1. Theorem: If $A_{i1} \leq A_{j2}$ for all i, j,≠i j, then k_1, k_2 …….k_n is a monotonically decreasing sequence, where $K_n=\sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$.

Solution: Let $A_{i1} \leq A_{j2}$ for all i, j , i ≠ j i.e., max $A_{i1} \le$ min A_{j2} for all i, j , $i \neq j$

Let $K_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$

Therefore, we have $k_1 = A_{11}$

Also $k_2 = A_{11} + A_{21} - A_{12} = A_{11} + (A_{21} - A_{12}) \le A_{11}$ ($\because A_{21} \le A_{12}$)
$\therefore k_1 \le k_2$

Now, $k_3 = A_{11} + A_{21} + A_{31} - A_{12} - A_{22}$
$= A_{11} + A_{21} - A_{12} + (A_{31} - A_{22}) = k_2 + (A_{31} - A_{22}) \le k_2$ ($\because A_{31} \le A_{22}$)

Therefore, $k_3 \le k_2 \le k_1$ or $k_1 \ge k_2 \ge k_3$.

Continuing in this way, we can have $k_1 \ge k_2 \ge k_3 \ge \ldots\ldots \ge k_n$, a monotonically decreasing sequence. Corollary: The total rental cost of machines is same for all the sequences.

Proof: The total elapsed time $T(S) = \sum_{i=1}^{n} A_{i2} + k_1 = \sum_{i=1}^{n} A_{i2} + A_{11}$

Therefore total elapsed time and hence total rental cost of machines is same for all the sequences.

5.2. Theorem: If $A_{i1} \ge A_{j2}$ for all $i, j, i \neq j$, then $k_1, k_2 \ldots\ldots k_n$ is a monotonically increasing sequence, where
$K_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$.

Proof: Let $K_n = \sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2}$

Let $A_{i1} \ge A_{j2}$ for all i, j , $i \neq j$ i.e., min $A_{i1} \ge$ max A_{j2} for all i, j , $i \neq j$

Here $k_1 = A_{11}$
$k_2 = A_{11} + A_{21} - A_{12} = A_{11} + (A_{21} - A_{12}) \ge k_1$ ($\because A_{21} \ge A_{j2}$)

Therefore, $k_2 \ge k_1$.

Also, $k_3 = A_{11} + A_{21} + A_{31} - A_{12} - A_{22} = A_{11} + A_{21} - A_{12} + (A_{31} - A_{22})$
$= k_2 + (A_{31} - A_{22}) \ge k_2$ ($\because A_{31} \ge A_{22}$)

Hence, $k_3 \ge k_2 \ge k_1$.

Continuing in this way, we can have $k_1 \le k_2 \le k_3 \ldots\ldots \le k_n$, a monotonically increasing sequence.

Corollary: The total rental cost of machines is same for all the possible sequences.

Proof: The total elapsed time $T(S) = \sum_{i=1}^{n} A_{i2} + k_n = \sum_{i=1}^{n} A_{i2} + \left(\sum_{i=1}^{n} A_{i1} - \sum_{i=1}^{n-1} A_{i2} \right) = \sum_{i=1}^{n} A_{i1} + \left(\sum_{i=1}^{n} A_{i2} - \sum_{i=1}^{n-1} A_{i2} \right) = \sum_{i=1}^{n} A_{i1} + A_{n2}$.

It implies that under rental policy P the utilization time of machine M_2 is same. Therefore total rental cost of machines is same for all the sequences.

6. ALGORITHM

Step 1: Find the ranking function A_{ij} ;i=1,2,3,...,n; j=1,2 of the trapezoidal fuzzy processing times for all the jobs on two machines M_1& M_2.

Step 2: Obtain the job J_1 (say) having maximum processing time on 1st machine.

Step 3: Obtain the job J_n(say) having minimum processing time on 2nd machine.

Step 4: If $J_1 \neq J_n$ then put J_1 on the first position and J_n as the last position & go to step 7, Otherwise go to step 5.

Step 5: Take the difference of processing time of job J_1 on M_1 from job J_2 (say) having next maximum processing time on M_1. Call this difference as G_1.Also, Take the difference of processing time of job J_n on M_2 from job J_{n-1}(say) having next minimum processing time on M_2. Call the difference as G_2.

Step 6: If $G_1 \leq G_2$ put J_n on the last position and J_2 on the first position otherwise put J_1 on 1^{st} position and J_{n-1} on the last position.

Step 7: Arrange the remaining (n-2) jobs between 1^{st} job & last job in any order, thereby we get the sequences $S_1, S_2 \ldots S_r$.

Step 8: Compute the total completion time $CT(S_k)$ k=1, 2…r.

Step 9: Calculate utilization time U_2 of 2^{nd} machine $U_2 = CT(S_k) - a_{11}(S_k)$; k=1,2,…. r.

Step 10: Find rental cost $R(S_k) = \sum_{i=1}^{n} a_{i1}(S_k) \times C_1 + U_2(S_k) \times C_2$, where C_1 & C_2 are the rental cost per unit time of 1^{st}& 2^{nd} machine respectively.

7. NUMERICAL ILLUSTRATION

Consider 5jobs and 2 machine problem to minimize the rental cost in which the uncertain processing times are described by trapezoidal fuzzy numbers as given in the table. The rental costs per unit time for machines M_1 and M_2 are 6 units &5 units respectively.

Table 2: Machines with processing time

Jobs	Machine M_1	Machine M_2
i	a_{i1}	a_{i2}
1	(7,8,9,10)	(6,7,8,9)
2	(9,11,13,15)	(7,8,9,10)
3	(12,13,14,15)	(5,6,7,8)
4	(8,9,10,11)	(4,5,6,7)
5	(10,11,12,13)	(4,6,8,10)

Solution: As per step 1: The ranking function of the processing time of the jobs is as follows:

Table 3: Machines with AHR Processing time

Jobs	Machine M_1	Machine M_2
i	A_{i1}	A_{i2}
1	8.5	7.5
2	12	8.5
3	13.5	6.5
4	9.5	5.5
5	11.5	7

Here each $A_{i1} \geq A_{i2}$ for all i. Also, Max A_{i1} = 13.5 which is for job 3.i.e. J_1=3.

Min A_{i2}= 5.5 which is for job 4.i.e. J_n = 4 .i.e. $J_1 \neq J_n$, therefore J_1 = 3 will be on 1^{st} position and J_n= 4 will be on the last position. Therefore, the optimal sequences are: S_1 =3 – 1 – 2 – 5 – 4, S_2 = 3 – 1 – 5 – 2 – 4, S_3 = 3 – 2 – 5 – 1 – 4, ------

The total elapsed time is same for all these possible 6 sequences $S_1, S_2, S_3, \ldots, S_6$.

The In – Out table for any of these 6 sequences say S_1 = 3 – 1 – 2 – 5 – 4 is

Table 4: In-Out flow table

Jobs	Machine M_1	Machine M_2
i	In - Out	In - Out
3	(0,0,0,0) – (12,13,14,15)	(12,13,14,15) – (17,19,21,23)
1	(12,13,14,15) – (19,21,23,25)	(19,21,23,25) – (25,28,31,34)

2	(19,21,23,25) – (28,32,36,40)	(28,32,36,40) – (35,40,45,50)
5	(28,32,36,40) – (38,43,48,53)	(38,43,48,53) – (42,49,56,63)
4	(38,43,48,53) – (46,52,58,64)	(46,52,58,64) – (50,57,64,71)

The total elapsed time, CT (S_1) = (50, 57, 64, 71)

Utilization time for M_2 , $U_2(S_1)$ = (50,57,64,71) - (12,13,14,15) = (35,43,51,59)

Therefore, total rental cost for each of sequence R (S_k) = (451, 527, 603, 679)

The ranking function of rental cost = 565 units.

Remarks

If we solve the above problem by Johnson's rule, we get the optimal sequence asS = 2 – 1 – 5 – 3 – 4. The In-Out flow table for the sequence S is

Table 5: In-Out flow table

Jobs	Machine M_1	Machine M_2
i	In - Out	In - Out
2	(0,0,0,0) – (9,11,13,15)	(9,11,13,15) – (16,19,22,25)
1	(9,11,13,15) – (16,19,22,25)	(16,19,22,25) – (22,26,30,34)
5	(16,19,22,25) – (26,30,34,38)	(26,30,34,38) – (30,36,42,48)
3	(26,30,34,38) – (38,43,48,53)	(38,43,48,53) – (43,49,55,61)
4	(38,43,48,53) – (46,52,58,64)	(46,52,58,64) – (50,57,64,71)

The total elapsed time, CT(S) = (50, 57, 64, 71)

Utilization time for M_2 , $U_2(S)$ = (50,57,64,71) - (9,11,13,15) = (35,44,53,62)

Therefore, total rental cost for each of sequence R(S) = (451,532,613,694)

The ranking function of rental cost = 572.5 units.

8. CONCLUSION

A heuristic algorithm to minimize the rental cost of the machines for a specially structured two stage flow shop scheduling is discussed irrespective of their total elapsed time.The algorithm proposed here for specially structured flow shop scheduling problem is more efficient and less time consuming as compared to the algorithm proposed by Johnson [1954] to find the optimal sequence of jobs processing to minimize the utilization time of the machines and hence their rental cost. Fuzzy logic is an excellent mathematical tool to handle the uncertainty arising due to vagueness. From this prospective, the concept of fuzzy environment is introduced in the theory of scheduling. The study may further be extended by using various constraints of flow shop scheduling problems like setup time, transportation time, Weightage of jobs, Breakdown Interval etc.

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